

Remarks on Bilocal Field Theory

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(Submitted to JETP editor June 14, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 705-706
(October, 1956)

VA. ZHIROV has opposed three arguments against my bilocal field theory: 1) in the early stages of the theory, the formulas are not compatible with themselves; 2) in a later stage of the formulation, the equations are compatible but admit some physically inadmissible solutions, with a space-like energy-momentum 4-vector; 3) in the theory, the question of the introduction of interaction remains open.

In answer to the first argument, I should like to note that the bilocal equations of my theory are always compatible with themselves, and that the apparent "incompatibility" can arise only in the automatic applying of my formulas without consideration of their limitations, or in the confusion among the notations for the field quantities in six-dimensional or eight-dimensional multiplicities. I admit to be guilty of not having sufficiently clarified these points.

The bilocal field quantities are two types of functions of the variables χ_μ and r_μ , where χ_μ denotes the center of the particles and r_μ the intrinsic degrees of freedom. A Fourier transformation enables us to go from the χ_μ to the p_μ variables, the latter being the components of the energy-momentum 4-vector.

The bilocal varieties and the bilocal fields are limited by the following conditions:

$$p_\mu r_\mu = 0, \quad (I); \quad p_\mu^2 + r_\mu^2 = 0 \quad (II) \quad (1)$$

(we use a system of units where $c = \hbar = l = 1$, where l is the fundamental length; therefore, all the magnitudes are dimensionless).

One has to differentiate between the field quantities φ , which depend only on 6 independent variables, and the quantities ψ , defined on an eight-dimensional multiplicity. φ and χ are related by

$$\psi(p, r) = N \delta(p_\mu r_\mu) \delta(p_\mu^2 + r_\mu^2) \varphi(p, r), \quad (2)$$

where N is the normalizing factor. ψ satisfies the following supplementary condition:

$$p_\mu r_\mu \psi = 0, \quad (I); \quad (p_\mu^2 + r_\mu^2) \psi = 0; \quad (II). \quad (3)$$

Moreover, ψ satisfies an equation which is a generalization of the Schrödinger-Fock-Gordon equation

$$(p_\mu^2 + k_\mu^2) \psi = R\psi, \quad (4)$$

where $k_\mu = -i \partial / \partial r_\mu$ and R is an operator taking into account the strength of the reactions which arise as a result of the limitations (1) or (3). Furthermore, one can raise the question about the equations and the supplementary conditions satisfied by φ . The condition (1) means that, if p_μ is time-like, r_μ is space-like, and vice versa. For space-like p_μ , φ is limited by the conditions

$$p_\mu k_\mu \varphi = 0, \quad (I)$$

$$\varphi = \text{zeroth order single-valued function of } r_\mu. \quad (II)$$

Indeed, the condition (I) gives the result that φ does not depend on r_4 in the rest system ($\mathbf{p} = 0$). Using condition (II) one then sees that φ does not depend on $|\mathbf{r}|$, i.e., φ is a function of the angles φ and θ only, in agreement with condition (1). On the other hand, in the case of a space-like p_μ , the vector r_μ is time-like and the relations (I) and (II) have to be replaced by: $(r_\mu \partial / \partial p_\mu) \varphi = 0$ and $\varphi =$ a zeroth order single-valued function of p_μ . The term R in Eq. (4) has to be properly replaced. Therefore, by choosing the conditions (I) and (II) we are bound to choose time-like p_μ 's.

In addition to (I) and (II) we have to introduce another equation, analogous to (4). For time-like p_μ this will simply be

$$(p_\mu^2 + k_\mu^2) \varphi = 0, \quad (5)$$

which differs from (4) by the absence of the R -term. Eq. (5) is obviously compatible with (I) and (II) (but it is not compatible with the conditions chosen in the case of space-like p_μ 's).

Using condition (I), we see that, in the rest system ($\mathbf{p} = 0$), the term $k_\mu^2 \varphi$ is reduced to $\mathbf{k}^2 \varphi$; using condition (II) we see that this term becomes $n(n+1)r^{-2} \varphi$. Finally, applying the second of the conditions (1), we obtain the mass spectrum

$$m_n = [n(n+1)]^{1/2}. \quad (6)$$

The eigenfunctions φ_n of this spectrum are related to the local Fierz field by

$$\varphi_n(p, r) = \varphi(p)_{\mu_1 \mu_2 \dots \mu_n} r_{\mu_1} r_{\mu_2} \dots r_{\mu_n} / (r_\nu^2)^{n/2}. \quad (7)$$

In this sense, the bilocal quantities are the generating functions of the local Fiertz fields. The latter form the whole irreducible unitary representation of the Lorentz group, and there are, therefore, no other solutions which would be mathematically possible but physically impossible. All other solutions either do not satisfy the supplementary conditions or (for space-like p_μ) are not related to the irreducible unitary representation of the Lorentz group. The solutions with time-like p_μ form a closed system and include all the Hilbert space.

Let us finally say a few words about the problem of the interaction. This problem is so far absolutely unclarified, and the usual requirements applied in the local theory are not applicable when the bilocal interaction is introduced. The same considerations allow us to hope that a criterion for the interaction will be found in the future, which will considerably differ from the (unmistakably wrong) criterion used now.

¹ V. A. Zhirov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **30** 425 (1956); *Soviet Phys. JETP* **3**, 452 (1956).

² H. Horvath, *Classical theory of physical fields of second kind in general spaces* (to be published in *Acta Phys. Hungar.*).

Translated by E. S. Troubetzkoy
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Effect of Multiple Thermal Ionization on the Specific Heat of Gases

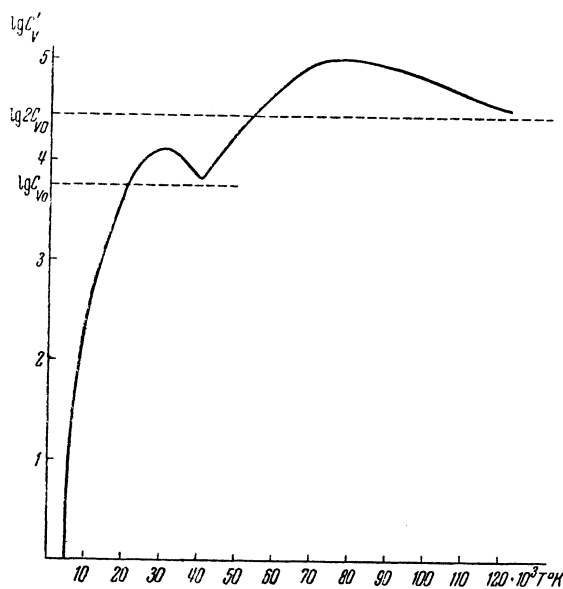
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(Submitted to JETP editor June 18, 1956)
J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 706-707
(October, 1956)

ONE of us¹ has calculated the specific heat of gases at high temperatures, taking into account a single thermal ionization. The purpose of this note is to consider the effect of multiple thermal ionization on the specific heat of gases.

We follow the method used in Ref. 1. The result is that the specific heat of the gas at high temperatures can be calculated from the formula

$$C_V = C_{V0} + C'_V. \quad (1)$$



where $C_{V0} = 3/2 nk$ is the specific heat of the non-ionized gas at constant volume; C'_V is the correction to the specific heat, due to the thermal ionization, with $C'_V = \sum_m C_{Vm}$; in the latter formula, C_{Vm} is the correction to the specific heat, due to the thermal ionization of the gas, with a $(m-1)$ -fold ionization; C_{Vm} is determined by the following expression:

$$C_{Vm} = (k/4f)(kT)^2 L \{ [3kT(2J_m - J_{m-1}) + 2J_m(J_m - J_{m-1})][nf(m+1) - L + 1] + 15/2(kT)^2 + 15/2(kT)^2(1-L) \} \quad (2)$$

$$+ 2^{1/2}nf(m+1)(kT)^2 + 3nf(kT)^2(m-1)L;$$

$$f = (g_{m-1}/2g_m)(2\pi/m')^{3/2}h^3(kT)^{-3/2}\exp(J_m/kT),$$

$$L = [1 + n^2f^2(m^2 + 1 - 2m) + 2nf(m+1)]^{1/2}.$$

It follows from (2) that the specific heat of gases at high temperature has a strong dependence on the temperature. To give an example, we have calculated the effect on the atomic oxygen gas; the correction C'_V is plotted on a logarithmic scale, vs. the temperature, in the range where only single and double ionizations are effective (solid line). One sees from the graph that the dependence of the specific heat on the temperature is represented by a curve showing steps and maxima in the temperature interval where a particular ionization order prevails. These maxima are several times higher than the specific heat of the non-ionized gas. For comparison, the specific heat