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Anomalous Magnetic Moments of Nucleons

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Calculations are presented for the anomalous magnetic moment of nucleons in which the excited nucleon states with spin 3/2 and isotopic spin 3/2 are included. Divergent expressions were obtained which were regularized with the aid of Feynman factors. The cut-off parameter can be so selected that agreement between theory and experiment is obtained.

1. INTRODUCTION

THE usual theory for the interaction of π-mesons with nucleons explains qualitatively the anomalous magnetic moment of nucleons. Pseudoscalar meson theory gives the correct sign for the magnetic moment of the proton and neutron but does not give quantitative agreement with experimental values. For the case of a mixed pseudoscalar and pseudovector coupling of the meson and nucleon fields in a symmetric meson theory one obtains the following expressions for the anomalous moment[1,2]:

$$\delta\mu_p = \frac{e}{2m}\frac{g^2(4/\Lambda+s^2)}{2\pi^2} \tag{1}$$

$$\times\left[\frac{1}{4}-\frac{3\Lambda}{2}+\frac{5\Lambda-3\Lambda^2}{4}\ln\frac{1}{\Lambda} - \frac{\Lambda^{1/2}(4-11\Lambda+3\Lambda^2)}{2(4-\Lambda)^{1/2}}\cos^{-1}\frac{\Lambda^{1/2}}{2}\right]$$

$$\delta\mu_N = -\frac{e}{2m}\frac{g^2(4/\Lambda+s^2)}{2\pi^2} \tag{2}$$

$$\times\left[1+\frac{\Lambda}{2}\ln\frac{1}{\Lambda}\frac{\Lambda^{1/2}(2-\Lambda)}{(4-\Lambda)^{1/2}}\cos^{-1}\frac{\Lambda^{1/2}}{2}\right].$$

Here gs/μ is the pseudovector and g is the pseudoscalar coupling constant, μ is the meson mass. It follows immediately that

$$|\delta\mu_N/\delta\mu_p| \approx 7. \tag{3}$$

Experimental measurements of the value of the anomalous magnetic proton and neutron moment yield

$$\delta\mu_p^{\text{exp.}} = 1.79\ \mu_0; \quad \delta\mu_N^{\text{exp}} = -1.91\ \mu_0, \tag{4}$$

where μ_0 is the nuclear magneton (in units of $c = 1$, $\hbar = 1$). Consequently,

$$|\delta\mu_N^{\text{exp}}/\delta\mu_p^{\text{exp}}| \approx 1. \tag{5}$$

Thus, a quantitative agreement between theory and experiment is absent.

Progress in the use of isobaric theory of nucleons in problems of pion-nucleon scattering[3] and gamma-pion production[4] raises the need for the consideration of the effects of excited nucleon states with spin 3/2 and isotopic spin 3/2 on the magnetic moments of nucleons. The object of the present paper is to calculate the anomalous magnetic moment of nucleons with the inclusion of these excited states. The calculation is based on a semi-phenomenological theory of π-meson and nucleon interactions, as developed by Tamm, Gol'fand and Fainberg[3]. All quantities are written in Feynman's notation[5]. For clarity, expressions of the form $a_\mu\gamma_\mu$ are indicated by the sign $\hat{}$ i.e., $\hat{a} \equiv a_\mu\gamma_\mu$.

2. LAGRANGIAN SYSTEM. EQUATIONS OF MOTION

The Lagrangian system for nucleons and mesons in an electromagnetic field in symmetric pseudoscalar meson theory with mixed pseudoscalar and pseudovector coupling between the mesonic and nucleonic fields has the form

$$L = L_0 + L' \tag{6}$$

$$L_0 = \bar{\psi}(i\hat{\nabla}-m)\psi - \bar{B}_\mu(i\hat{\nabla}-M_1)B_\mu$$
$$\left(\bar{B}_\mu\frac{\partial D}{\partial x_\mu}-\frac{\partial\bar{D}}{\partial x_\mu}B_\mu\right)-$$
$$-\frac{3}{2}i\bar{D}\hat{\nabla}D - 3M_1\bar{D}D + \frac{1}{2}\left[\left(\frac{\partial\varphi_i}{\partial x_\nu}\right)^2-\mu^2\varphi_i^2\right];$$

$$L' = -e\bar{\psi}\frac{1+\tau_3}{2}\hat{A}\psi + e\bar{B}_\mu Q\hat{A}B_\mu$$
$$+\frac{g\sqrt{4\pi}}{\mu}\bar{\psi}\left(\gamma_\nu\frac{\partial\varphi_i}{\partial x_\nu}+i\mu s\varphi_i\right)\tau_i\gamma_5\psi$$
$$+\frac{g_1\sqrt{4\pi}}{\mu}(\bar{\psi}S_iB_\nu+\bar{B}_\nu S_i^+\psi)\frac{\partial\varphi_i}{\partial x_\nu}+\frac{ie\varepsilon}{m}(\bar{\psi}\gamma_\nu\gamma_5 f_{\nu\mu}NB_\mu$$
$$-\bar{B}_\mu\gamma_\nu\gamma_5 f_{\nu\mu}N^+\psi)+eA_\nu\left(\varphi_1\frac{\partial\varphi_2}{\partial x_\nu}-\varphi_2\frac{\partial\varphi_1}{\partial x_\nu}\right)$$

(repeated Greek indices imply a summation from 1 to 4 and for Latin indices, 1 to 3) where τ_i is the isotopic spin matrix of the nucleon, Q is the charge operator of the isobar (isotopic spin operator) $(1+\tau_3)/2$ is the charge operator of the nucleon, viz.,

$$Q = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad \frac{1+\tau_3}{2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{7}$$

S_i are the matrices introduced in Refs. 3 and 6; they change the charge state of a nucleon from the ordinary to the isobar and back; ϵ is the constant for the additional interaction of the nucleon with the electromagnetic field which is responsible for transitions from the ordinary to the isobaric state and back; N is an operator which accomplishes these transitions in isotopic space. Its form and the form of ϵ is given by Ritus[4].

Applying the variational principle with the auxiliary condition that $\bar{B}_\mu \gamma_\mu = 0$, we obtain the following equations for the wave functions ψ and B_μ:

$$(i\hat{\nabla} - m)\psi = \Big[e\frac{1+\tau_3}{2}\hat{A}_t - \frac{gV\overline{4\pi}}{\mu}\Big(\gamma_\nu\frac{\partial\varphi_i}{\partial x_\nu} + i\mu s\varphi_i\Big)\tau_i\gamma_5\Big]\psi - \Big[\frac{g_1V\overline{4\pi}}{\mu}S_i\frac{\partial\varphi_i}{\partial x_\nu} + i\frac{e\varepsilon}{m}\gamma_\mu\gamma_5 f_{\mu\nu}N\Big]B_\nu; \tag{8}$$

$$(i\hat{\nabla} - M_1)B_\mu - \frac{i}{2}\gamma_\mu\frac{\partial B_\nu}{\partial x_\nu} = eQ\Big[\hat{A}\delta_{\mu\nu} - \frac{1}{2}\gamma_\mu A_\nu\Big]B_\nu + \Big[\frac{g_1V\overline{4\pi}}{\mu}\Big(\frac{\partial\varphi_i}{\partial x_\mu} - \frac{1}{4}\gamma_\mu\gamma_\nu\frac{\partial\varphi_i}{\partial x_\nu}\Big)S_i^+ - \frac{ie\varepsilon}{m}\Big(\gamma_\nu\gamma_5 f_{\nu\mu} - \frac{1}{4}\gamma_\mu\gamma_\sigma\gamma_\tau\gamma_5 f_{\tau\sigma}\Big)N^+\Big]\psi$$

$$-\Big(\frac{\partial^2}{\partial x_\mu^2} + \mu^2\Big)\varphi_i = 2eA_\nu e_{is}\frac{\partial\varphi_S}{\partial x_\nu}.$$

The symbol e_{is} is specified by the identities

$$-e_{12} = e_{21} = -1, \quad e_{11} = e_{22} = 0.$$

Eq. (8) can be written in the form

$$\sum_{\beta=0}^{4} L_{\alpha\beta}\Phi_\beta = \sum_{\beta=0}^{4} A_{\alpha\beta}\Phi_\beta; \quad \alpha = 0, 1, 2, 3, 4.$$

$$\Phi_0 = \Psi; \quad \Phi_\mu = B_\mu; \quad \mu = 1, 2, 3, 4;$$

$$A_{00} = e\frac{1+\tau_3}{2}\hat{A} - \frac{gV\overline{4\pi}}{\mu}\Big(\gamma_\nu\frac{\partial\varphi_i}{\partial x_\nu} + i\mu s\varphi_i\Big)\tau_i\gamma_5; \tag{9}$$

$$A_{0\mu} = -\Big(\frac{g_1V\overline{4\pi}}{\mu}S_i\frac{\partial\varphi_i}{\partial x_\mu} + i\frac{e\varepsilon}{m}\gamma_\nu\gamma_5 f_{\nu\mu}N\Big);$$

$$A_{\mu 0} = \frac{g_1V\overline{4\pi}}{\mu}\Big(\frac{\partial\varphi_i}{\partial x_\mu} - \frac{1}{4}\gamma_\mu\gamma_\nu\frac{\partial\varphi_i}{\partial x_\nu}\Big)S_i^+ - \frac{ie\varepsilon}{m}\Big(\gamma_\nu f_{\nu\mu} - \frac{1}{4}\gamma_\mu\gamma_\sigma\gamma_\tau f_{\tau\sigma}\Big)\gamma_5 N^+.$$

The system (8) can be solved by the Feynman method with the help of the inverse operators $(L^{-1})_{\alpha\beta} = K_{\alpha\beta}$, where

$$K_{00} = (\hat{p} - m)^{-1}; \tag{10}$$

$$K_{\mu\nu} = (\hat{p} - M_1)^{-1}[\delta_{\mu\nu} - (1/_6 M_1^2)(2\gamma_\mu\hat{p} + \hat{p}\gamma_\mu + 3M_1\gamma_\mu)p_\nu];$$

$$K_{0\mu} = K_{\mu 0} = 0.$$

3. CALCULATIONS OF MATRIX ELEMENTS

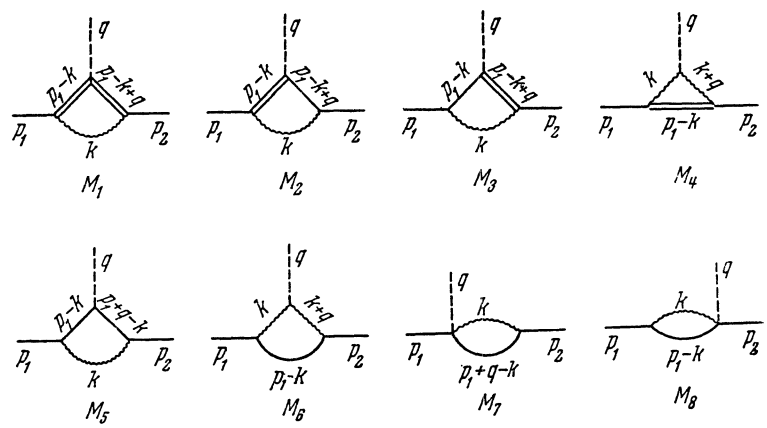
One must calculate the contribution to the anomalous magnetic moment of nucleons of processes whose corresponding diagrams are illustrated in the Figure.

In view of the strong singularity of the inverse operator $K_{\mu\nu}$, the matrix elements M diverge rapidly (4th order divergence). This divergence can be removed by the introduction of the Feynman cut-off factors.

As a simplification in the calculations, the cut-off is not applied to all matrix elements at once but to each component separately, i.e., for each divergence of degree n in the meson momentum, i.e., k^n, one needs a separate cut-off factor $[-\lambda^2/(k^2 - \mu^2 - \lambda^2)]^{N_n}$, where N_n is the smallest integer for which the regularization integral for the given degree k^n is finite. This procedure corresponds to the fact that the cut-off does not change the convergence in the absence of a cut-off value, i.e., the convergence for $\lambda = \infty$.

The very unwieldy expressions in the integrand for M are transformed in a direct fashion: the quantities $K_{\mu\nu}$, K_{00}, $S_{\nu\tau}$, Q_σ are intermultiplied and the results grouped according to powers of k; denominators of the form

$$(k^2 - 2kp_1 - \Delta_1)(k^2 - 2kp_2 - \Delta_2)^{-1}$$



Diagrams of third-order contributions to the anomalous magnetic moment of nucleons. In all diagrams, $p_2 = p_1 + q$; – – – – = external electromagnetic field; ———— = nucleon in the ordinary state; ═══ = nucleon in isobaric state; ∿∿∿∿ = meson.

are transformed by the Feynman formula

$$\frac{1}{ab} = \int_0^1 \frac{dx}{ax + b(1-x)},$$

the integration is then performed with quantities of order μ^2/m^2 being dropped. Finally, we have, isolating expressions of the form $\delta_\mu(\hat{A}\hat{q} - \hat{q}\hat{A})$ in M,

$$\begin{aligned}
\delta\mu^{\mathrm{isob}} = {} & \frac{e}{2m}\left(\frac{m}{\mu}\right)^2 \Bigl\{-\frac{g_1^2}{2\pi^2}(3 + 5\tau_3) \\
& \times\Bigl[\lambda^4\left(\frac{1}{85M_1^4} + \frac{1}{16m^3 M_1}\right) \\
& + \lambda^2\left(-\frac{m^2}{24M_1^4} + \frac{2m}{3M_1^3} + \frac{1}{3M_1^2} - \frac{1}{5mM_1}\right) \\
& - \frac{m^4}{4M_1^4} - \frac{m^3}{3M_1^3} + \frac{m^2}{5M_1^2} + \frac{m}{3M_1} + \frac{3}{64} \\
& + \frac{M_1}{16m} - \frac{M_1^2}{64m^2} - \Bigl(\frac{1}{16} - \frac{m}{6M_1} + \frac{m^2}{10M_1^2} \\
& + \frac{13m^3}{12M_1^3} + \frac{m^4}{3M_1^4}\Bigr)\ln\left(1 + \frac{\lambda^2}{\mu^2}\right) - \frac{2gg_1\varepsilon}{\pi^2}\tau_3\Bigl[\frac{\lambda^2}{m^2}\Bigl(\frac{5m^2}{24M_1^2}. \\
& + \frac{9m}{20M_1^2} + \frac{3}{8} + \frac{3\mu s}{4M_1} + \frac{2\mu sm}{3M_1^2}\Bigr) + \Bigl(\frac{3m}{10M_1} + \frac{3}{64} + \frac{M_1}{16m} \\
& + \frac{\mu s}{20} - \frac{\mu sm}{16M_1^2}\Bigr)\ln\left(1 + \frac{\lambda^2}{\mu^2}\right) + \frac{m^2}{2M_1^2} + \frac{7m}{9M_1} + \frac{25}{24} \\
& + \frac{M_1}{12m} - \frac{\mu s}{32m} + \frac{7\mu s}{2M_1} + \frac{5\mu sm}{2M_1^2}\Bigr] + \frac{g_1^2}{\pi^2}\tau_3\Bigl[-\lambda^2\Bigl(\frac{1}{4M_1^2} \\
& + \frac{3}{16mM_1}\Bigr) + \frac{5m^3}{32M_1^3} + \frac{5m^2}{3M_1^2} + \frac{m}{8M_1} + \frac{5m^4}{32M_1^4} + \frac{1}{24} + \\
& + \frac{M_1}{24m} + \left(\frac{m^2}{18M_1^2} + \frac{m}{36M_1}\right)\ln\left(1 + \frac{\lambda^2}{\mu^2}\right)\Bigr]\Bigr\}.
\end{aligned} \tag{11}$$

This is now to be inserted into the expression for the anomalous magnetic moment derived from the regular theory as discussed in the introduction (Eq. (2).

4. NUMERICAL RESULTS. DISCUSSION

The numerical results for $\delta\mu$ depend upon the sign of the pseudovector coupling constant g (the sign of the constant ϵg_1 was determined by Ritus[4]). If one employs the constant values as determined by Tamm *et al.*[3] and Ritus[4],

$$g^2 = 0.2; \quad g_1^2 = 0.13; \quad s = 2;$$

$$M_1 = m + 2.25\ \mu; \quad \varepsilon = 1.61$$

and assumes that $g > 0$, then for $\lambda \simeq m$ we have

$$\delta\mu_p \approx 1.5\ \mu_0; \quad \delta\mu_N \approx -1.3\ \mu_0.$$

As λ increases, the absolute magnitudes of $\delta\mu_p$ and $\delta\mu_N$ increase while their ratio remains approximately one (changes slowly). Thus, with a suitably selected cut-off parameter, one can obtain sufficiently good agreement between the theory and the experimental data.

Kanazawa and Sugawara[6] obtained an approximately similar result when they also computed the effect of isobaric states on the magnetic moment of nucleons. Our calculation differs from theirs in its greater accuracy. Actually, they do not include the additional interaction of the nucleons with the electromagnetic field (coupling constant ϵ). Calculations indicate that this additional interaction is of the same order as other types of interactions and its inclusion is necessary. Besides this in the evaluation of the matrix elements M, the authors replaced the inverse operator $K_{\mu\nu}$ by a "nonrelativistic" approximation obtained by neglecting quantities of the type k/M_1 in comparison to 1. This is permissible in those cases if $\lambda/M_1 \ll 1$ but for satisfactory agreement between theory and experiment one must select $\lambda/M_1 \simeq 1$. Consequently, the "nonrelativistic" approximation as used by Kanazawa and Sugawara[6] is inapplicable.

In conclusion, I use this opportunity to express my gratitude to I. E. Tamm for suggesting this problem and for his continuing aid, and to Iu. A. Gol'fand, V. Ia. Fainberg and V. P. Silin for valuable discussions relating to this problem.

[1] K. M. Case, Phys. Rev. 76, 1 (1949).

[2] T. Hamada, Prog. Theor. Phys. 10, 309 (1953).

[3] Tamm, Gol'fand and Fainberg, J. Exptl. Theoret. Phys. (U.S.S.R.) 26, 649 (1954).

[4] V. I. Ritus, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 660 (1954).

[5] R. P. Feynman, Phys. Rev. 76, 749, 769 (1949).

[6] A. Kanazawa and M. Sugawara, Prog. Theor. Phys. 11, 231 (1954).

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Theory of Isothermal Galvanomagnetic and Thermomagnetic Effects in Semiconductors

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Isothermal galvanomagnetic and thermomagnetic effects in isotropic semiconductors are treated theoretically in the case of intermediate and strong magnetic fields.

ONE of the most effective methods of investigating the properties and parameters of semiconductors is the study of galvanomagnetic and thermomagnetic effects. A theory of these effects has been developed by a number of authors[1-7]. Most of the authors start with a quadratic dependence of the energy on the momentum, and with weakness of the magnetic field. Meanwhile, experiment has revealed many cases in which it is not legitimate to consider the effective magnetic field* φ small.

Even at room temperature, it is often necessary to deal with intermediate effective magnetic fields ($\varphi^2 \sim 1$), and sometimes even with strong ones ($\varphi^2 \gg 1$). Thus, for example, at $T = 300°$ K and $H = 10^4$ oe, $\varphi^2 \approx 1.5$ for HgSe, and $\varphi^2 = 36$ for InSb. At low temperatures we quite often deal with intermediate and strong effective magnetic fields. Davydov and Shmushkevich[3] obtained formulas for the Hall effect and for the change of electrical conductivity in the case $\varphi \gg 1$, and Madelung[6] considered the same phenomena in the case $\varphi \gtrsim 1$, but only for semiconductors with an atomic lattice.

The present work concerns the extension of the theory to the region of intermediate and strong magnetic fields, for various types of interaction of the carriers with the crystal lattice. We also determine which features of the galvanomagnetic and thermomagnetic effects depend on the statistics and on the scattering law. We consider only isothermal effects; for, as Tolpygo[5] showed, the adiabatic effects differ little in magnitude from the isothermal.

1. SEMICONDUCTORS WITH CARRIERS OF A SINGLE TYPE

Transport Equations

The kinetic equation for the distribution function $f(\mathbf{r}, \mathbf{p})$ of the carriers, in momentum ($\mathbf{p}$) and coordinate ($\mathbf{r}$) space, has in the stationary case the well-known form

$$\mathbf{v}\nabla_{\mathbf{r}} f + \mathbf{F}\nabla_{\mathbf{p}} f = -(f - f_0)/\tau. \quad (1)$$

Here $\mathbf{v}$ is the velocity of a carrier, $\mathbf{F}$ is the external force acting on it, $f_0\left(\frac{\epsilon-\mu}{kT}\right)$ is the equil-

* By "effective magnetic field" we shall understand the dimensionless quantity $\varphi = uH/c$, which essentially determines the effect of the magnetic field H on the carriers of current in a semiconductor. Here u is the mobility of the carriers, and c is the speed of light.