this Vlasov's formula for the frequency[1] and Landau's formula for the damping*

$$\gamma = \sqrt{\pi/8}\Omega\,(ka)^{-3}e^{-1/2k^2a^2}e^{-3/2}.$$

In conclusion we express our sincere thanks to Prof. A. I. Akhiezer for his attention, for his assistance, and for the detailed discussion of the results of this work.

* We note that in the expression for γ obtained by Landau[2] the factor $e^{-3/2}$ is missing.

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Investigation of the $Be^9(dn)B^{10}$ Nuclear Reaction

V. I. MAMASAKHLISOV
Physical Institute, Academy of Sciences, Georgian SSR

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An investigation is carried out on the reaction between the nucleus of beryllium and a deuteron in which the latter is captured by the nucleus and the unpaired neutron is ejected. The effective cross section of the process and the angular distribution of the freed neutrons were found. The comparison of the angular distribution with experimental data results in a satisfactory agreement for small angles up to 70°.

1. THE model of the Be^9 nucleus, according to which the unpaired neutron moves in the field of the nuclear remainder Be^8, was applied by many investigators to the problem of the electron and photoelectric disintegration of this nucleus[1-3]. The success of this model is determined first by the weak binding of the unpaired neutron in the Be^9 nucleus, considerably smaller than the mean binding energy per particle , and second, by the relatively long life of the Be^8 nuclear remainder relative to the decay into two α-particles. The current research is dedicated to the investigation of the $Be^9(dn)B^{10}$ reaction on the basis of this model[1].

It is customarily assumed that the (dn) reaction can proceed by the formation of a compound nucleus and stripping of a proton by a nucleus from a deuteron passing nearby. Calculations on the basis of the compound nucleus model are often not feasible in actual cases because the line widths of the corresponding processes are unknown; therefore, most of the theoretical investigations of the (dn) reaction are made from the point of view of the stripping process. The corresponding angular distributions of neutrons are then determined on the basis of Butler's theory[4]. When there is no agreement between this theory and experiment it is pointed out that in such cases the reaction does not proceed by stripping, but by the formation of a compound nucleus, which then undergoes various cascade transitions.

The probability of the stripping process is more or less apparent because the deuteron binding energy is considerably smaller than the binding energy of each particle in the nucleus with which the deuteron collides. However, this specific condition does not hold for the case of the beryllium nucleus. As is known, the binding energy of the unpaired neutron in the Be^9 nucleus amounts to only 1.66 mev, while the binding energy of the deuteron equals 2.23 mev. The low binding energy of the neutron in the Be^9 nucleus points to the assumption that the mean distance of the neutron from the nuclear remainder is relatively large. It is natural, therefore, to assume that the deuteron flying nearby will interact primarily with the unpaired neutron only, without causing any observable excitations in the nuclear remainder Be^8. The method proposed by us for calculating the (dn) reaction in the Be^9 nucleus is in a sense intermediary between the stripping and the compound nucleus methods. We are assuming that in the (dn) reaction the Be^9 nucleus interacts not only with one of the particles in the deuteron, as allowed by the stripping theory, but with both particles, the neutron and proton, whereupon the deuteron does not interact with the entire nucleus but only with its unpaired neutron.

Since the binding energy of the neutron in the Be^9 nucleus is smaller than the deuteron binding energy, there is a definite probability that before the interacting deuteron decays into a proton and a neutron, the unpaired neutron in the Be^9 nucleus will be ejected out of it. In this paper we are considering this very case.

2. It is known that the unpaired neutron in the Be^9 nucleus is found in the p-state. It follows from the shell model that the proton and the neutron in the B^{10} nucleus are also found in the p-state. Furthermore, it is not difficult to see that the binding energy of the last proton in the B^{10} nucleus is equal to

$$\Delta\varepsilon = \varepsilon(B^{10}) - \varepsilon(Be^9) = 6.62 \text{ mev}. \qquad (1)$$

Furthermore, taking into account that the binding energy of the deuteron $\epsilon_d = 2.23$ mev, we obtain for the energy liberated in the reaction under consideration (on the assumption that B^{10} is found in the ground state)

$$Q = (Be^9 d, B^{10} n) = \Delta\varepsilon - \varepsilon_d = 4.29 \text{ mev}. \qquad (2)$$

3. For the interaction energy between the deuteron and the Be^9 nucleus we take the expression

$$V = g[\delta(\mathbf{r}_1 - \mathbf{r}) + \delta(\mathbf{r}_2 - \mathbf{r})], \qquad (3)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the radius vectors of the deuteron's neutron and proton; $\mathbf{r}$ is the radius vector of the neutron in the Be^9 nucleus; $g = (4\pi\hbar/M)^2 a$, where M is the mass of the proton and neutron, and a is the scattering length associated with the effective cross section of neutron scattering on free protons by the formula

$$\sigma = 4\pi a^2.$$

Equation (1) for the interaction energy is correct so long as the wavelength of incident particles is greater than the effective radius of nuclear forces; therefore, we restrict ourselves to those incident deuterons whose energies do not exceed 1-2 mev. The wave functions for the initial and final states are

$$\psi_i = v_d^{-1/2} e^{i\mathbf{k}(\mathbf{r}_1+\mathbf{r}_2)/2}\, \Phi_d(|\mathbf{r}_1 - \mathbf{r}_2|)\, \psi_1(\mathbf{r}), \qquad (4)$$
$$\psi_f = (2\pi\hbar)^{3/2} e^{i\mathbf{k}'\mathbf{r}}\, \psi_2(\mathbf{r}_2)\, \psi_3(\mathbf{r}_1),$$

where $\mathbf{k}$ is the wave vector of the incident deuteron, $\mathbf{k}'$ is the wave vector of the ejected neutron, v_d is the velocity of the incident deuteron, $\psi_1(\mathbf{r})$ is the wave function of the bound neutron in the Be^9 nucleus, ψ_2 and ψ_3 are wave functions of the proton and neutron in the B^{10} nucleus, Φ_d is the wave function of the deuteron's inner state.

Inasmuch as we are assuming that in the reaction under consideration the ejected neutron belongs to the Be^9 nucleus, and the incident neutron, decaying into a neutron and proton, is bound in the nucleus forming B^{10}, then we can consider that the proton-neutron distance is on the average the same in both the deuteron and in the B^{10} nucleus and, therefore, in calculating the matrix element in the deuteron function $\Phi_d(|\mathbf{r}_1 - \mathbf{r}_2|)$ the value $|\mathbf{r}_1 - \mathbf{r}_2|$ can be equated to the more probable neutron-proton distance in the deuteron, equal to

$$r_d = \hbar / \sqrt{\mu\varepsilon_d}.$$

4. As noted above, the unpaired neutron in the Be^9 nucleus, as well as the proton in the B^{10}, are found in the p-state. Therefore, in the matrix element of transition ψ_1, ψ_2 and ψ_3 are replaced by the corresponding equations for p-waves, with the radial wave functions determined from the assumption that the graphs of the neutron-proton interaction in the B^{10} nucleus and for the interaction between the neutron in the Be^9 nucleus and the Be^{10} nuclear remainder have the shape of a square well.

If one transforms into spherical waves the plane waves $e^{i\mathbf{kr}|2}$ and $e^{-i\mathbf{k}'\mathbf{r}}$ corresponding to the incident deuterons and the neutrons ejected from the nucleus, and restricts the transformation to the values of orbital momentum $l = 0$ and $l = 1$ (so as to exclude as far as possible the stripping effect) then one finally gets after calculation of the differential section for the process under study,

$$d\sigma = 512\pi^6 (\mu / M)^2 a^2 \sqrt{2/5 (1 + Q / E_d)} \tag{5}$$

$$\times (r_0 / r_d)^3 e^{-2} (1 + r_0' / r_d)^2$$

$$\times \frac{(k_1 r_0)^2}{(kr_0)^2 (k' r_0)^2 (k_3 r_0)^2} \frac{F_1^2 F_2^2}{g_1 g_2 g_3} \sum_{i=0}^{6} \beta_i \cos^i \vartheta \, d\Omega_n,$$

where μ is the effective mass of the neutron and the proton relative to the nuclear remainder Be^8, E_d is the energy of the incident deuteron, r_0 is the radius of the potential well for the Be^9 and Be^{10} nuclei, r_0' is the neutron-proton interaction radius,

$$k = (6 / \hbar) \sqrt{M E_d / 11}, \tag{6}$$

$$k' = \hbar^{-1} \sqrt{2\mu (E_d + Q)}, \quad \beta_i = a_i / B_{00}^2;$$

$$F_1 = \int_0^{k_2 r_0} J_{1/2}\left(\frac{k}{2k_2} x\right) J_{1/2}\left(\frac{k'}{k_2} x\right) J_{3/2}\left(\frac{k_1}{k_2} x\right) J_{3/2}(x)\, dx \tag{7}$$

$$+ \frac{2}{\pi} \left(\frac{k_1}{\alpha_1}\right)^{1/2} \left(\frac{k_2}{\alpha_1}\right)^{1/2}$$

$$\times \left(\frac{k_2}{\alpha_2}\right)^2 \int_{k_2 r_0}^{\infty} \frac{(1 + \alpha_1 x / k_2)(1 + \alpha_2 x / k_2)}{x^3} J_{1/2}\left(\frac{k}{2k_2} x\right)$$

$$\times J_{1/2}(x)\, e^{-(\alpha_1 + \alpha_2)(x - k_2 r_0)/k_2} dx;$$

$$F_2 = \int_0^{k_3 r_0} x J_{3/2}\left(\frac{k}{2k_3} x\right) J_{3/2}(x)\, dx \tag{8}$$

$$- \sqrt{\frac{2}{\pi}} \left(\frac{k_2}{\alpha_3}\right)^2 \sin k_3 r_0$$

$$\times \int_{k_3 r_0}^{\infty} \frac{(1 + \alpha_1 x / k_3) J_{3/2}(kx / 2k_3)}{\sqrt{x}} e^{-\alpha_3 (x - k_3 r_0)/k_3} dx;$$

$$g_\mu = k_\mu r_0 \tag{9}$$

$$+ \left[(2 + \alpha_\mu r_0) \frac{k_\mu^4}{\alpha_\mu^4} + (1 + \alpha_\mu r_0) \frac{k_\mu^2}{\alpha_\mu^2} - 1\right] \frac{\sin^2 k_\mu r_0}{k_\mu r_0};$$

$$(\mu = 1, 2, 3);$$

$$a_0 = (3/16) B_{00}^2 + (81/200) B_{11}^2 + (3/8) B_{02}^2$$

$$+ (3927/9800) B_{13}^2 + (567/700) B_{11} B_{13};$$

$$a_1 = (15/16) B_{00} B_{11} - (9/10) B_{02} B_{11}$$

$$+ (27/20) B_{02} B_{13};$$

$$a_2 = (513/400) B_{11}^2 + (3/8) B_{02}^2 + (5/16) B_{02}^2$$

$$+ (14/23) B_{13}^2 - (567/175) B_{11} B_{13};$$

$$a_3 = (3/4) B_{02} B_{11} + (189/112) B_{02} B_{13};$$

$$a_4 = (65/96) B_{02}^2; \quad a_5 = -(8663/4200) B_{02} B_{13};$$

$$a_6 = (21/196) B_{13}^2,$$

with

$$B_{ll'} = \int f_l\left(\frac{kr}{2}\right) f_{l'}(k'r) R_1(r) R_2(r) r^2 dr; \tag{10}$$

$$k_i = \hbar^{-1} \sqrt{2\mu (V_i - \varepsilon_i)}, \tag{11}$$

$$\alpha_i = \sqrt{2\mu \varepsilon_i} / \hbar, \quad (i = 1, 2, 3).$$

In the last equations ϵ_1 represents the binding energy of the neutron in the Be^9 nucleus, ϵ_2 and ϵ_3 are the binding energies of the corresponding proton and neutron in the B^{10} nucleus, and V_1, V_2 and V_3 are the corresponding values of the potential well depth, with the assumption that the radius of the well r_0 has the same value for the Be^9 and B^{10} nuclei.

In Eq. (10), $R_1(r)$ and $R_2(r)$ are the radial wave functions of the p-state, i.e.,

$$R_i(r) = b_i (k_i r)^{-1/2} J_{3/2}(k_i r) \quad \text{for} \quad r \leqslant r_0, \tag{12}$$

$$R_i(r) = c_i \alpha_i^{-1/2} (1 + \alpha_i r) r^{-2} e^{-\alpha(r - r_0)} \quad \text{for} \quad r \geqslant r_0,$$

$$f_n(z) = (\pi/2z)^{1/2} J_{n+1/2}(z), \quad (13)$$

where $J_m(n)$ is a Bessel function of order m. The coefficients b_i and c_i are determined from the conditions of continuity and normalization, namely,

$$c_i = -\sqrt{2/\pi}\, b_i \alpha_i^{-3/2} \sin k_i r_0; \quad b_i^2 = \pi k_i^2 / g_i, \quad (14)$$

and the values of k_1, k_2 and k_3 are the roots of the transcendental equation

$$\operatorname{tg} k_i r_0 = \frac{k_i r_0}{1 + (1 + \alpha_i r_0)\, k_i^2 / \alpha_i^2}. \quad (15)$$

The radius of the potential well for the interaction of Be^9 is well known from the investigations on electron and photoelectric disintegrations of beryllium. It is equal to 5×10^{-13} cm. Taking this value for r_0 and requiring that the well depth be in the 10-30 mev range, one can uniquely determine the coefficients k_i from Eq. (15). As a result of solving Eq. (15), $k_1 r_0 = 3.35$, $k_2 r_0 = 3.63$, $k_3 r_0 = 3.64$, with $\alpha_1 r_0 = 1.33$, $\alpha_2 r_0 = 2.66$ and $\alpha_3 r_0 = 3.02$.

After integrating Eq. (5) over the angles, we obtain the complete effective cross section for the reaction under study:

$$\sigma = 4 \cdot 512 \cdot \pi^7 (\mu / M)^2 a^2 (r_0 / r_d)^2 \times e^{-2} (1 + r_0' / r_d)^2 \sqrt{2/5 (1 + Q / E_d)} \times \frac{(k_1 r_0)^2}{(k r_0)^2 (k' r_0)^2 (k_3 r_0)^2} \times \frac{F_1^2 F_2^2}{g_1 g_2 g_3} \left(\beta_0 + \frac{1}{2}\beta_2 + \frac{2}{5}\beta_4 + \frac{3}{7}\beta_6\right). \quad (16)$$

5. In Ref. 5 the experimental investigation was carried out on the angular distribution of neutrons produced in the reaction $Be^9(dn)B^{10}$. The writers of this paper give the following function of neutron intensity versus the scattering angle for the case in which the B^{10} nucleus remains in the unexcited state:

$$f(\vartheta) = 0.87 - 0.28 \cos\vartheta + 0.42 \cos^2\vartheta. \quad (17)$$

The graph of (17) is plotted with six experimental points and $E_d = 0.945$ mev and with the unit intensity taken as the intensity corresponding to $\vartheta = 0$. The second line of the Table gives values calculated from the empirical formula (17).

TABLE

ϑ	0	15°	40°	65°	90°	115°	140°	130°
$f(\vartheta)$	1	0.99	0.90	0.82	0.87	1.06	1.33	1.57
$f'(\vartheta)$	1	1.00	0.97	0.85	0.72	0.62	0.49	0.27

Using Eqs. (5)-(9) one can easily obtain an equation for the angular distribution of the neutrons for incident deuterons with energies of 0.945 mev.

If the intensity corresponding to $\vartheta = 0$ is taken as unity, then we have

$$\sum \beta_i \cos^i \vartheta = 0.347 f'(\vartheta), \quad (18)$$

$$f'(\vartheta) = 0.715 + 0.256 \cos\vartheta + 0.176 \cos^2\vartheta + 0.121 \cos^3\vartheta - 0.251 \cos^4\vartheta - 0.017 \cos^5\vartheta,$$

with the $\cos^6\vartheta$ term rejected because of the small value of the coefficient.

The comparison of the $f'(\vartheta)$ values for various scattering angles obtained by using Eq. (18) with the experimental values found from Eq. (17) shows that there is agreement between theory and experiment for small scattering angles in the range 0-70°. The discrepancies in numerical values are found for angles in the range 70-180°. The theoretical curve for small scattering angles on the whole describes correctly the dependence of the distribution function on the scattering angle.

In the third line of the Table are given the theoretical values of the neutron distribution as a function of the scattering angle. The discrepancy between theory and experiment for relatively large angles can be ascribed to the fact that the binding energy of the deuteron is comparable after all to the binding energy of the neutron in the Be^9 nucleus, so that those cases are also possible, not calculated by us, in which there is an ejection of the neutron belonging to the deuteron and not of the neutron from the Be^9 nucleus. For a complete description of neutron scattering with medium and large angles this fact requires the calculation of the exchange effect between the neutrons of the deuteron and those the Be^9 of nucleus. It is known that the calculation of the exchange results in an increase of the scattering probability for large angles.

As to the value of the complete effective cross section for the (dn) reaction with the Be^9 nucleus, there are no experimental data in the literature, and therefore, for the time being, Eq. (16) cannot be verified for the complete effective cross section.

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Dielectric Properties of Bismuth Titanate

G. I. SKANAVI AND A. I. DEMESHINA
P. N. Lebedev Physical Institute, Academy of Sciences, USSR

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It is shown that the titanates of bismuth possess high dielectric constants ($\epsilon = 70$-120) and comparatively large positive temperature coefficients ($TC\epsilon = 130$-550×10^{-6}) which are evidently brought about by a combination of a favorable internal field and a high ionic polarizability. The formation process of one of the barium titanates is investigated, and the temperature and frequency dependencies of ϵ and $\tan\delta$ of various bismuth polytitanates have been measured.

THE titanates of metals of the second group of the periodic table have an elevated dielectric constant which increases upon increase in the atomic weight of the metal[1]. In this case, for nonpiezoelectric titanates, as well as for many other ceramic dielectrics, this rule holds: the higher the dielectric constant, the more negative the temperature coefficient $(1/\epsilon)(d\epsilon/dT)$[2]. For small dielectric constants the quantity $(1/\epsilon)(d\epsilon/dT)$ has a positive sign. For most dielectric constants, $(1/\epsilon)(d\epsilon/dT)$ has a comparatively large negative value. As shown earlier[3], the high dielectric constant of nonpiezoelectric titanates of barium and rutile (TiO_2) is brought about by a combination of high electronic polarization and a favorable polarization of the internal field produced by the ionic displacement. The ionic polarizability in this case is not large. The electronic polarizability does not depend on the temperature but the electronic polarizability per unit volume decreases with increase in temperature at the expense of a decrease in the number of polarized particles per unit volume in thermal expansion. The ionic polarizability increases with increase in temperature, since the elastic coupling is weakened. In dielectrics with high dielectric constant, the electronic polarizability appreciably exceeds the ionic; therefore, the dielectric constant decreases with increase in temperature $[(1/\epsilon)(d\epsilon/dT) < 0]$. The positive temperature coefficient of ϵ points to the predominance of the ionic polarizability.

In the present work, an attempt was made to realize, in a polycrystalline, nonpiezoelectric dielectric, such conditions under which a high dielectric constant would be combined with a comparatively large positive temperature coefficient. An appreciable ionic polarization takes place in glasses, the dielectric constant of which increases sufficiently rapidly with increase in temperature; therefore, the presence of glassy layers in a polycrystalline dielectric ought to facilitate an increase of ϵ with temperature. On the other hand, it is necessary that relaxation processes connected with inelastic dislocations of the ions not take place in this glassy layer and in defective places in the crystalline lattice, for this would, in turn, lead to greater loss. In this connection, the glassy layer must contain only heavy ions, possessing small mobility.

Starting from these considerations, we investigated the possibility of the formation of a combination of titanium dioxide and bismuth trioxide. Bismuth trioxide possesses weakly basic properties