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On the Theory of the Stripping Reaction

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This investigation presents a simple derivation of the Butler formula for the angular distribution in the (d, p) reaction. The scattering of the deuteron and proton waves in the nuclear field is included. The emamined theory leads to Serber's results in the limiting case of large energies for the impinging deuterons.

1. AS is known, the stripping reaction is an important means for investigating nuclear properties. From the angular distribution of the products of the (d, p) and (d, n) reactions it is possible to determine the spin and parity of the corresponding state of the residual nucleus, if the spin and parity of the ground state of the initial nucleus are known. The stripping reaction theory for bombarding deuterons of moderate energies (~ 10 mev) was first presented by Butler[1], who determined the angular distribution of stripping reaction products by using the continuity condition of the wave functions on the nuclear surface. Since Butler's derivation is extremely complicated, there are several other investigations[2-6] in which the same results are obtained by different means. In particular, Bhatia[2] made use of the Born approximation, which led to correct results, though there is little justification for the application of such an approximation to the energy region mentioned. Incomplete agreement of the theoretical data with the results of experiments indicates the importance of considering the nuclear and Coulombic scattering of the particles participating in the reaction. This was done in some of the investigations[5-7]. It may be that an important role is also played by interference between the stripping reaction and the processes of compound nucleus formation.

In this article, using the method developed by Landau and Lifshitz[8], we determine the angular distribution of protons in the (d, p) reaction, and calculate the scattering of deuteron and proton waves in the nuclear field. Calculation of the scattering reveals the presence of partial polarization of the freed protons. If a neutron is captured by the nucleus on the virtual level, the stripping reaction cross section is proportional to the level width. When the impinging deuterons have enough energy, the total cross section of the stripping reaction is identical with the cross section predicted by Serber[9]. In this case the distribution in angles and energy of the freed protons corresponds to that of the "transparent model" in the Serber theory[9].

2. Schrödinger's equation, which describes the motion of a neutron + proton in a field due to the presence of a nucleus, may be written as

$$\{T_n + T_p + V_n + V_p + V_{np} - E\} \Psi = 0, \quad (1)$$

where T_n and T_p are the kinetic energy operators of the neutron and proton, V_n and V_p are the interaction potentials of neutron and proton with the nucleus, V_{np} is the nuclear interaction potential of a neutron with a proton, and E is the total energy in the system.

To solve Eq. (1), we expand the unknown function Ψ in wave functions of the proton released by the deuteron disintegration. These wave functions, which we shall designate as $\psi_{\mathbf{k}_p}$ ($\mathbf{k}_p$ being the wave vector of the outgoing proton), satisfy the equation

$$\{T_p + V_p - E_p\} \psi_{\mathbf{k}_p} = 0 \quad (2)$$

($E_p = \hbar^2 k_p^2/2M$ is the proton energy), with $\psi_{\mathbf{k}_p}$ being composed of a plane wave and converging spherical wave at infinity. Let us assume that the function $\psi_{\mathbf{k}_p}$ is subject to the following normalization condition:

$$\int \psi_{\mathbf{k}_p} \psi^*_{\mathbf{k}'_p} d\mathbf{r}_p = \delta_{\mathbf{k}_p \mathbf{k}'_p}. \quad (3)$$

The solution of Eq. (1) can be represented as

$$\Psi(\mathbf{r}_n, \mathbf{r}_p) = \sum_{\mathbf{k}_p} a(\mathbf{r}_n, \mathbf{k}_p) \psi_{\mathbf{k}_p}(\mathbf{r}_p), \quad (4)$$

where $a(\mathbf{r}_n, \mathbf{k}_p)$ are certain functions of the coordinates of the neutron and wave vector of the outgoing proton. Obviously, $a(\mathbf{r}_n, \mathbf{k}_p)$ is the wave function of the neutron formed by the disintegration of a deuteron and corresponds to a proton with wave vector $\mathbf{k}_p$.

Substituting (4) in (1), and making use of the orthogonality of the $\psi_{\mathbf{k}_p}$ functions in (3), we obtain the following equations for the functions $a(\mathbf{r}_n, \mathbf{k}_p)$:

$$\{\Delta_n + (2M/\hbar^2)(E - E_p - V_n)\} \tag{5}$$
$$\times a(\mathbf{r}_n, \mathbf{k}_p) = f(\mathbf{r}_n),$$

$$f(\mathbf{r}_n) = (2M/\hbar^2)\int \psi^*_{\mathbf{k}_p}(\mathbf{r}_p) V_{np} \Psi(\mathbf{r}_n, \mathbf{r}_p)\, d\mathbf{r}_p. \tag{6}$$

Equation (5) is exact. To obtain an approximate solution of this equation we substitute the approximate function Ψ_0 for Ψ, where

$$\Psi_0(\mathbf{r}_n, \mathbf{r}_p) = \varphi(r)\,\psi_{\mathbf{k}d}(\mathbf{r}_d), \tag{7}$$

$\varphi(r)$ is the wave function of the ground state of the deuteron and $\psi_{\mathbf{k}d}(\mathbf{r}_d)$ is the wave function of the deuteron moving as a unit in the nuclear field. At infinity, the function $\psi_{\mathbf{k}d}(\mathbf{r}_d)$ is composed of the impinging plane wave and a scattered diverging spherical wave.

Because of the short range of the nuclear forces, one can use the relation[8]

$$V_{np}\varphi(r) = -(4\pi\hbar^2/M)\sqrt{\alpha/2\pi}\,\delta(\mathbf{r}) \tag{8}$$

in the integral in (6). In this way we obtain

$$f(\mathbf{r}_n) = -8\pi\sqrt{\alpha/2\pi}\,\psi^*_{\mathbf{k}_p}(\mathbf{r}_n)\,\psi_{\mathbf{k}_d}(\mathbf{r}_n). \tag{6'}$$

Henceforth, we shall disregard the possibility of the occurrence of a deuteron as a unit inside the nucleus, and therefore $f(\mathbf{r}_n)$ will differ from zero only when $r_n > R\ (R \approx R_0 + R_d)$, where R_0 is the radius of the nucleus and R_d the radius of the deuteron.

Expanding $a(\mathbf{r}_n, \mathbf{k}_p)$ in spherical functions

$$a(\mathbf{r}_n, \mathbf{k}_p) = \sum_{l,m} a_{lm}(r_n)\, Y_{lm}(\vartheta_n, \varphi_n), \tag{9}$$

we obtain the following equations for the coefficients $a_{lm}(r_n)$:

$$\left\{\frac{d^2}{dr_n^2} + \frac{2}{r_n}\frac{d}{dr_n} - \frac{l(l+1)}{r_n^2} + \frac{2M}{\hbar^2}(E - E_p - V_n)\right\} a_{lm}(r_n) = f_{lm}(r_n) \tag{10}$$

$$f_{lm}(r_n) = \int Y^*_{lm}(\vartheta_n, \varphi_n)\, f(\mathbf{r}_n)\, do_n. \tag{11}$$

The required solution for Eq. (10) must be finite as $r_n \to 0$ and must represent a diverging spherical wave when $r_n \to \infty$.

Let $R_l^{(1)}$ be the solution of Eq. (10) with the right side zero which satisfies the bounded condition for $r_n \to 0$. In the region beyond the action of nuclear forces ($r_n > R$) this solution is clearly of the form

$$R_l^{(1)} = j_l(k_n r_n) + b h_l^{(1)}(k_n r_n),$$
$$r_n > R,\ k_n = \sqrt{(2M/\hbar^2)(E - E_p)}, \tag{12}$$

where b is a coefficient determined by the conditions of continuity and boundedness of the solution. Let $R_l^{(2)}$ designate the second independent homogeneous solution of Eq. (10), which represents the divergent spherical wave at infinity. In the outside region this solution has the form

$$R_l^{(2)} = h_l^{(1)}(k_n r_n), \quad r_n > R. \tag{13}$$

Knowing $R_l^{(1)}$ and $R_l^{(2)}$, one can construct a solution of the inhomogeneous equation [i.e., Eq. (10)] which satisfies the given boundary conditions, viz.,

$$a_{lm}(r_n) = A R_l^{(2)}(r_n)\int_0^{r_n} R_l^{(1)}(r)\, f_{lm}(r)\, r^2 dr + A R_l^{(1)}(r_n)\int_{r_n}^{\infty} R_l^{(2)}(r)\, f_{lm}(r)\, r^2 dr, \tag{14}$$

where the coefficient A is determined from the condition

$$r^2\{R_l^{(1)} R_l^{(2)\prime} - R_l^{(1)\prime} R_l^{(2)}\} = 1/A. \tag{15}$$

Substituting solutions (12) and (13) into (15), we find that $A = -ik_n$.

Since $f_{lm}(r) = 0$ if $r < R$, then with the use of Eqs. (11) and (6), we finally obtain a solution for points $r_n < R$; thus,

$$a_{lm}(r_n) = i8\pi\sqrt{\alpha/2\pi}\, k_n R_l^{(1)}(r_n)\, I_l^m, \quad r_n < R, \tag{16}$$

$$I_l^m = \int_{r>R} h_l^{(1)}(k_n r)\, Y^*_{lm}(\vartheta, \varphi)\, \psi^*_{\mathbf{k}_p}(\mathbf{r})\, \psi_{\mathbf{k}_d}(\mathbf{r})\, d\mathbf{r}. \tag{17}$$

3. We now develop the general formula for the (d, p) stripping reaction cross section. For this purpose we compute the neutron flux S through the nuclear surface in the normal direction,

$$S = \frac{i\hbar}{2M} r_n^2 \int \left\{ a^*(\mathbf{r}_n, \mathbf{k}_p) \frac{\partial a(\mathbf{r}_n, \mathbf{k}_p)}{\partial r_n} - a(\mathbf{r}_n \mathbf{k}_p) \frac{\partial a^*(\mathbf{r}_n, \mathbf{k}_p)}{\partial r_n} \right\} do_n \Big|_{r_n = R}. \quad (18)$$

Substituting $a(\mathbf{r}_n, \mathbf{k}_p)$ as given by (9) and using (16), we obtain

$$S = 32\pi\alpha\, |k_n|^2 \times \sum_l \frac{i\hbar}{2M} R^2 \left\{ R_l^{(1)*} \frac{dR_l^{(1)}}{dr_n} - R_l^{(1)} \frac{dR_l^{(1)*}}{dr_n} \right\}_{r_n = R} \sum_m |I_l^m|^2. \quad (19)$$

Since the proton wave function $\psi_{\mathbf{k}_p}$ is normalized to a δ-function in wave vector space, then by multiplying S by the element of solid angle do_p we obtain the number of neutrons absorbed per unit of time that are associated with the protons emitted within do_p. Dividing this figure into the flux density of impinging deuterons, which is $\hbar k_d/2M$, we obtain the following equation for the stripping reaction cross section:

$$d\sigma = 32\pi \frac{\alpha}{k_d} |k_n|^2 R^2 \times \sum_l i \left\{ R_l^{(1)*} \frac{dR_l^{(1)}}{dR} - R_l^{(1)} \frac{dR_l^{(1)*}}{dR} \right\} \sum_m |I_l^m|^2 do_p. \quad (20)$$

Thus, the problem of determining the angular distribution of the freed protons is reduced to the computation of the integral, Eq. (17), in which the deuteron and proton wave functions are given by

$$\psi_{\mathbf{k}_d}(\mathbf{r}) = e^{i\mathbf{k}_d \mathbf{r}} + 2\pi \times \sum_{l,m} i^l (\beta_l^d - 1)\, h_l^{(1)}(k_d r)\, Y_{lm}^*(\vartheta_{k_d}, \varphi_{k_d})\, Y_{lm}(\vartheta, \varphi), \quad (21)$$

$$\psi_{\mathbf{k}_p}(\mathbf{r}) = (2\pi)^{-\frac{3}{2}} \Big\{ e^{i\mathbf{k}_p \mathbf{r}} + 2\pi \times \sum_{l,m} i^l (\beta_l^{p*} - 1)\, h_l^{(2)}(k_p r)\, Y_{lm}^*(\vartheta_{k_p}, \varphi_{k_p}) Y_{lm}(\vartheta, \varphi) \Big\}. \quad (22)$$

Complex coefficients β_l^d and β_l^p characterize the scattering of deuteron and proton waves in the nuclear field [11]

Inclusion of the scattering of deuteron and proton waves leads to partial polarization of the freed protons*, which is fully determined by the integrals I_l^m, that is,

$$P = \mp \frac{2}{3(2j+1)} \Big(\sum_m m\, |I_l^m|^2 \Big/ \sum_m |I_l^m|^2 \Big), \quad (23)$$

$$j = l \pm {}^1\!/_2.$$

* The proton polarization caused by proton wave scattering has been investigated by Horowitz and Messiah[12].

Far from resonance, the deuteron and proton experience almost complete reflection at the edge of the nucleus, and consequently, nuclear wave scattering can be regarded approximately as scattering by an impenetrable sphere of radius R[13]. For this case coefficients β_l^d and β_l^p (if we disregard Coulomb interaction) are determined by the equation[13]

$$\beta_l^{d,p} = 1 - 2j_l(k_{d,p}R)\,[j_l(k_{d,p}R) - i n_l(k_{d,p}R)] / [j_l^2(k_{d,p}R) + n_l^2(k_{d,p}R)]. \quad (24)$$

Near energy resonance, resonance scattering plays a major role, in which case

$$\beta_l^{d,p} = 1 - i\Gamma_l^{d,p} \Big/ \Big(E - E_l + \frac{i}{2}\Gamma_l \Big), \quad (25)$$

where E_l is the resonance energy, Γ_l the total width of the resonance level, and Γ_l^d and Γ_l^p are partial widths. However, near resonance the deuteron penetrates into the nucleus with appreciable probability, and therefore, compound nucleus formation will play an important part in the (d, p) process. Interference between the two processes will lead to considerable change in the angular distribution of the reaction products.

In the limiting case of high energies, potential scattering, Eq. (24), becomes diffraction scattering, in which case we may set approximately[11]

$$\beta_l^{d,p} = \begin{cases} 0 & l \leqslant k_{d,p} R \\ 1 & l > k_{d,p} R. \end{cases} \quad (26)$$

However, even in the simplest cases, i.e., Eqs. (24) and (26), the integration of Eq. (17) is feasible only numerically. (In this connection, compare Ref. 14.) However, the integral can be easily calculated if deuteron and proton wave scattering in the nuclear field is disregarded.

$$I_l^m = \sqrt{\frac{2}{\pi}}\, i^l \frac{R^2}{k_n^2 - k^2} \left\{ j_l(kR) \frac{dh_l^{(1)}(k_n R)}{dR} - h_l^{(1)}(k_n R) \frac{dj_l(kR)}{dR} \right\} Y_{lm}^*(\vartheta_k, \varphi_k), \quad (27)$$

$$k = |\mathbf{k}_d - \mathbf{k}_p|.$$

For this case we obtain Butler's result for the angular distribution of the protons, viz.,

$$d\sigma = \frac{16\alpha R^6 |k_n|^2}{\pi k_d [(\mathbf{k}_d - \mathbf{k}_p)^2 - k_n^2]^2} \times \sum_l (2l+1)\, i \left\{ R_l^{(1)*} \frac{dR_l^{(1)}}{dR} - R_l^{(1)} \frac{dR_l^{(1)*}}{dR} \right\} \times \left| j_l(kR) \frac{dh_l^{(1)}(k_n R)}{dR} - h_l^{(1)}(k_n R) \frac{dj_l(kR)}{dR} \right|^2 do_p. \quad (28)$$

Inclusion of the spins causes an additional factor to appear in Eq. (28) in the sum, viz.,

$$2(2j+1)/3(2i+1)(2l+1),$$

where i and j are the spins of the initial and final nucleus and l is the orbital moment transferred by the neutron to the nucleus.

In deriving Eq. (28) we neglected the possibility of the occurrence of a deuteron as a unit inside the nucleus. This assumption is justified for deuterons with impact parameters in excess of the effective nuclear radius R; therefore, the angular distribution found for the products of the stripping reaction will be quite exact for small angles.

4. If the energy of the impinging deuteron is great enough ($E_d \gg \epsilon$), a neutron will be captured by the nucleus on the virtual level (k_n real). In this case it is convenient to express the cross section Eq. (28) in terms of the partial width Γ_l^n which corresponds to an emission of a neutron with an orbital moment of l by the final nucleus. Obviously, the probability of neutron emission per unit time, $\Gamma_l^n/\hbar$, is equal to the neutron flux through the surface of a sphere with radius R in the normal direction[13], i.e.,

$$\Gamma_l^n/\hbar = -(i\hbar/2M)R^2 \times \{R_l^{(1)*}dR_l^{(1)}/dR - R_l^{(1)}dR_l^{(1)*}/dR\} \quad (29)$$

($R_l^{(1)}$ is the radial wave function of the neutron inside the nucleus).

The energy of the residual nucleus lies in the region of the continuous spectrum. In this case, the stripping reaction cross section, with a proton emitted in an element of solid angle do_p and with energy of the nucleus in the interval dE_f, will be

$$d\sigma = (S/v_d)\rho_f dE_f do_p,$$

where ρ_f is the density of the final states of the nucleus. Noting that $dE_f = -dE_p$ and that

$$S = -32\pi\alpha k_n^2 \sum_l \frac{\Gamma_l^n}{\hbar} \sum_m |I_l^m|^2,,$$

we finally obtain

$$d\sigma = \frac{32\alpha M k_n^2 R^4}{\pi\hbar^2 k_d (k^2-k_n^2)^2} \times \sum_l (2l+1)\Gamma_l^n \left| j_l(kR)\frac{dh_l^{(1)}(k_nR)}{dR} - h_l^{(1)}(k_nR)\frac{dj_l(kR)}{dR}\right|^2 \rho_{lf} dE_p do_p. \quad (30)$$

Since there is a large number of levels with different l, angles for which the difference $k^2 - k_n^2$ is quite small play the main role in Eq. (30), in which case

$$\{j_l(kR)\,dh_l^{(1)}(k_nR)/dR - h_l^{(1)}(k_nR)\,dj_l(kR)/dR\} \approx i/k_nR^2.$$

The cross section will now be

$$d\sigma = \frac{M}{\pi^2\hbar^2 k_d}\frac{32\pi\alpha}{(k^2-k_n^2)^2}\sum_l (2l+1)\Gamma_l^n \rho_{lf} dE_p do_p. \quad (31)$$

Availing ourselves of the principle of detailed balance, we can connect the partial width of the disintegration of the residual nucleus, Γ_l^n, to the sticking probability of the neutron to the nucleus, ζ_l, and to the density of the final nuclear states, ρ_l [11]:

$$\Gamma_l^n = \zeta_l/2\pi\rho_l. \quad (32)$$

Substituting (32) into (31) and noting that $k^2 - k_n^2 = 2\{\alpha^2 + (\tfrac{1}{2}\mathbf{k}_d - \mathbf{k}_p)^2\}$, we have

$$d\sigma = \frac{M}{2\pi^3\hbar^2 k_d}\frac{8\pi\alpha}{\{\alpha^2 + (\tfrac{1}{2}\mathbf{k}_d - \mathbf{k}_p)^2\}^2} \times \sum_l (2l+1)\zeta_l dE_p do_p. \quad (33)$$

We see that the deuteron energy is about equally divided between the neutron and proton.

In the case of fast neutrons, they may be considered to be absorbed by the nucleus, provided only that the collision parameter is less than the radius of the nucleus. Inasmuch as we are interested in the stripping process, we need consider only neutrons bound with protons which do not interact with the nucleus. If the average distance between the neutron and proton is R_d, then, obviously, these neutrons will have impact parameters of $l\lambda$ ($\lambda = 2/k_d$) included in the interval between $R - R_d$ and R. Because the effective value of l is considerably greater than unity, the sum in (33) may be replaced by the integral

$$\sum_l (2l+1) = \int_{\frac{1}{2}k_d(R-R_d)}^{\frac{1}{2}k_dR} 2l\,dl = \tfrac{1}{2}k_d^2 RR_d, \quad R_d \ll R. \quad (34)$$

Substituting (34) in (33), we have

$$d\sigma = \frac{2RR_dMk_d\alpha}{\pi^2\hbar^2[\alpha^2 + (\tfrac{1}{2}\mathbf{k}_d - \mathbf{k}_p)^2]^2} dE_p do_p. \quad (35)$$

Integrating (35) over the energies and angles of the outgoing protons, we obtain Serber's formula for the total stripping reaction cross section*

$$\sigma = {}^1/_2 \pi R R_d. \quad (36)$$

The distributions of outgoing protons as to angles and energies are also in accord with Serber's "transparent model"[9], viz.,

$$d\sigma(\vartheta_p) = \sigma \sqrt{\frac{\varepsilon}{E_d}} \frac{\vartheta_p d\vartheta_p}{(\varepsilon/E_d + \vartheta_p^2)^{3/2}}, \quad (37)$$

$$d\sigma(E_d) = \frac{\sigma}{\pi} \frac{\sqrt{\varepsilon E_d}\, dE_p}{[(E_p - {}^1/_2 E_d)^2 + \varepsilon E_d]}.$$

In conclusion, I wish to express my deep gratitude to Prof. A. I. Akhiezer for his valuable advice and for reviewing the results of this investigation.

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* The passage in the limit to the Serber theory has been investigated by Bhatia[2], but he obtained too large a value for the total cross section.

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