The experimental data on the interaction of slow K-particles with nuclei seem to indicate that the first of the two capture processes

$$K^- + \mathrm{N} \rightarrow \Lambda^0 + \pi, \quad K^- + \mathrm{N} \rightarrow \Sigma + \pi \qquad (4)$$

is more probable than the second[4] by an order of magnitude. In connection with these, on the one hand, a study of the reaction of type (3) may be fruitful; but, on the other hand, it is of interest to examine other possibilities for obtaining information about the spins of the new particles. Some information can be obtained from a comparison of the probabilities of exchange collisions of K-particles with hydrogen and deuterons[8].

Another possibility of obtaining information about the spins of hyperons and K-particles is connected with the fact that beams of particles produced in nuclear interactions are partly polarized. As is well known, the scattering cross section of a polarized beam depends on the angle θ as well as on the azimuth φ. But if the θ dependence is connected with which transitions play a role, then the nature of the φ dependence is determined by the spin of the particle. Thus for particles with spin ½ the characteristic dependence is proportional to $\cos\varphi$, for spin 1, to $\cos\varphi + A\cos 2\varphi$, and for spin 3/2, to $\cos\varphi + a\cos 2\varphi + b\cos 3\varphi$. For particles with arbitrary spin the φ dependence is characterized by the expression

$$\sum_{n=1}^{2s} A_n \cos n\varphi,$$

where s is the spin of the particle. The origin of such a dependence is connected with the fact that the orbital angular momentum does not have a z-component ($m = 0$), so that the values of the z-component of the total angular momentum agree with the allowed values of the z-components of the spin of the particles.

Knowing, e.g., the mode of decay, it is possible to ascertain whether the particles are bosons (integral spin) or fermions (half-integral spin). Consequently, knowing, e.g., that the K-particle appears to be a boson, it is necessary to have the possibility to separate experimentally the absence of a dependence on φ for $s = 0$ from the existence of the dependence of the form $\cos\varphi + A\cos 2\varphi$ for $s = 1$.

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Spin-Orbit Interaction in Nuclear Magnetic Multipole Radiation

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IN Moszkowski's review[1] of nuclear multipole radiation, esitmates are given for the probabilities of radiative transitions in the single-particle nuclear model. For ML-radiation with a nucleonic transition from the state (n_1, l_1, j_1, μ_1) to the state (n_2, l_2, j_2, μ_2) with the following changes of the nucleonic moments: $\Delta j = L$; $\Delta l = L + 1$ (for example, $M1$-radiation from the $d_{3/2} - s_{1/2}$ transition), the formulas which are given are unsuitable. As has been noted in Ref. 2, in proton transitions of this type, the proton spin-orbit interaction makes an essential contribution, so that the perturbation operator contains the additional term $-(e/c)\Phi \times (v)\,(\sigma \cdot \mathbf{r} \times \mathbf{A})$. In a neutron transition this term does not appear because the neutron bears no charge. However, even in this case an ML-transition is not absolutely forbidden, as is assumed in Refs. 1 and 2. Its intensity is lower [by the factor $\sim (4E^2/1840 \cdot (2L+3))$] than the intensity of the corresponding proton transition (E is the photon energy in mev), but it is comparable with the probability of the electric multipole transition $E(L+1)$ for energies of the order of 5 mev with $A = 100$, and for energies < 1 mev in the case of transition with $L > 1$.

References 1 and 2 do not contain estimates of the probabilities for ML transitions of the type $\Delta j = L$, $\Delta l = L + 1$ when the spin-orbit coupling of the nucleon is taken into consideration. The results of such a calculation are given below. (All quantities are given in terms of the relativistic units $\hbar = m_e = c = 1$.) The transition probability is determined by the matrix element of the perturbation operator:

$$\hat{H} = -\mu_0 (e/2M)(\sigma \mathbf{H}) - e\Phi(\mathbf{r})(\sigma \cdot \mathbf{r} \times \mathbf{A}_L^0)\delta,$$

where μ_0 is the algebraic value of the magnetic moment of the nucleon in nuclear Bohr magnetons; $\Phi(r)$ is the spin-orbit interaction potential. The symbol δ is equal to 1 for a proton and 0 for a neutron. $\mathbf{A}_L^0$ is the potential of a ML-multipole photon with the wave vector $\mathbf{k}$ and polarization ϵ:

$$\mathbf{A}_L^0 = 4\pi \sqrt{2\pi/k}\, i^L \sum_M (\varepsilon Y_{LM}^{0*}(\theta_k, \Phi_k))\, \mathbf{Y}_{LM}^0(\theta, \varphi) f_L(kr) e^{ikt}$$

Using the relations between spherical harmonics[3] and expanding the Bessel spherical functions $f_L(kr)$ in series for $kr \ll 1$ we obtain for the perturbation operator of an ML-transition with $\Delta j = L$, $\Delta l = L + 1$:

$$\hat{H} = i^{L+1} e 4\pi \sqrt{2\pi/k} \left[\frac{(kR)^L R}{(2L+1)!!}\right] \times \left\{\frac{\mu_0}{2M}\frac{k^2}{2L+3} x^{L+1} - \Phi(x) x^{L+1}\delta\right\} \times \sqrt{\frac{L}{2L+1}} \sum_M (\varepsilon \mathbf{Y}_{LM}^{0*}(\theta_k, \Phi_k))(\sigma \mathbf{Y}_{LM}^{+1}(\theta, \varphi)),$$

where $x = r/R$, R is the nuclear radius and $0 \leq x \leq 1$. From perturbation theory the transition probability is

$$W_{ML} = \frac{2\pi}{2j_1+1} \sum_{\mu_1, \mu_2, \varepsilon} |\langle \Psi_{n_2 j_2 \mu_2}^+ | \hat{H} | \Psi_{n_1 j_1 \mu_1} \rangle|^2 \rho_k; \quad \rho_k = \frac{k^2}{2\pi^2},$$

where the wave function of the nucleon is taken in the form

$$\Psi_{nlj\mu} = R_{jln}(x)\, \Omega_{j\mu}(\theta, \varphi) = R_{jln}(x) \sum_{m\alpha} C_{lm\sigma\alpha}^{j\mu} Y_{lm\varphi\sigma\alpha}.$$

The matrix element is separated into radial and angular parts, and we obtain for the transition probability

$$W_{ML} = 2e^2 \frac{(kR)^{2L+1} R}{[(2L+1)!!]^2} \left|\langle R_{j_2 l_2 n_2}^*(x) \left| \frac{\mu_0}{2M}\frac{k^2}{2L+3} x^{L+1} - \Phi(x) x^{L+1}\delta \right| R_{j_1 l_1 n_1}\rangle\right|^2 S,$$

where

$$S = \frac{L}{(2L+1)} \frac{(4\pi)^2}{(2j_1+1)} \tag{12}$$

$$\sum_{\mu_1 \mu_2 \varepsilon} |\langle \Omega_{j_2\mu_2}^+ | \sum_M (\varepsilon \mathbf{Y}_{LM}^{0*}(\theta_k, \Phi_k))(\sigma \mathbf{Y}_{LM}^{+1}(\theta, \varphi) | \Omega_{j_1\mu_1}\rangle|^2$$

$$^3/_4 L (2l_1+1)(2L+3)(2j_2+1)(C_{l_1 0 L+1 0}^{l_2 0})^2 \cdot \times \left[\sum_j (2j+1) W(j_2 j_1 L+1 1; Lj)\, W(l_1 j_1 \sigma 1; j)\, W(\sigma j_2 l_1 L+1; l_2 j)\right]^2.$$

In the practically important cases $l_1 = 0$ and $l_2 = 0$, the formula for S is considerably simplified:

for $l_1 = 0$ $\quad S = {}^3/_4 L (2j_2+1) W^2(j_1 \sigma L l_2; 1 j_2)$;

for $l_2 = 0$ $\quad S = {}^3/_4 L (2j_2+1) W^2(j_2 \sigma L l_1; 1 j_1)$.

For the purpose of evaluating the radial part of the transition matrix element we set:

$$\langle R_{j_2 l_2 n_2}^*(x) | \Phi(x) x^{L+1} | R_{j_1 l_1 n_1}(x)\rangle \approx \langle \Phi(x)\rangle_{av} \langle R_{j_2 l_2 n_2}^* | x^{L+1} | R_{j_1 l_1 n_1}\rangle \approx \frac{3}{L+4}\langle\Phi\rangle_{av},$$

where $\langle\Phi\rangle_{av}$ is the diagonal element of the spin-orbit coupling potential. According to Ref. 3, $\langle\Phi(x)\rangle_{av} \simeq -9.6 A^{1/3}$ (in our units), where A is the mass number of the nucleus. Using this value we obtain for the probability of an ML-transition with $\Delta j = L$, $\Delta l = L + 1$:

$$W_{ML} = 1.8 e^2 \frac{(kR)^{2L+1} R}{[(2L+1)!!]^2} \frac{9}{(L+4)^2} \times \left[\frac{\mu_0}{2}\frac{1}{1840}\frac{k^2}{2L+3} + 9{,}6 A^{-2/3}\delta\right]^2 S \cdot 10^{21}\,\text{sec}^{-1},$$

where k is the photon energy in the units $m_e c^2 = 0.511$ mev, $e^2 = 1/137$, and R is the nuclear radius: $R = 0.85 A^{1/3}$ 274 ($R = 1.25 \times 10^{-13} A^{1/3}$ cm).

As can easily be ssen, the spin-orbit term for $k < 16$ (< 8 mev) is considerably larger than the first term. Indeed, for $k = 16$ we obtain $1/2(\mu_0/1840)k^2/(2L+3) \leq 1/5(256/1840) = 0.028$, which is smaller than $9.6 A^{-2/3}$ even when $A = 200$; therefore, the first term can be neglected in proton transitions. For neutron transitions the second term disappears and the transition probability is determined entirely by the first term in the square brackets. It is interesting to compare the probability of and $E\ (L+1)$ transitions of neutrons:

$$W_{ML}/W_{E(L+1)} = S_{ML}/S_{E(L+1)}$$

$$(\mu_0 kA^L/2z)^2 \leqslant (S_{ML}/S_{E(L+1)})\ (2kA^{L-1}/1840)^2.$$

When $k \approx 10$ and $A = 100$ the probabilities W_{M1} and W_{E2} are of the same order of magnitude, and the radiation probabilities of the higher multipoles $M2$ and $M3$ are comparable even at low energies with the probabilities for $E3$ and $E4$ transitions.

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A Dispersion Relation for All Scattering Angles

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A DISPERSION relation for forward scattering has been obtained by Goldberger *et al.*[1,2]. In the present note a relation is obtained between the imaginary and real parts of the scattering amplitude for all angles.

To obtain this relation one can use Goldberger's method[1], in which it is simplest to employ the coordinate system in which the combined momentum of the nucleons (incident and scattered) is zero. However, we shall derive our dispersion relation here by using some results obtained by Nambu[3].

According to Nambu[3], the Feynman matrix element for the scattering of bosons (with momentum k and charge index α) by fermions (with momentum p and additional quantum numbers λ) can be represented independently of the type of interaction in the form

$$F_{\alpha\beta}(k, \alpha; p, \lambda; k', \beta; p', \lambda') \tag{1}$$

$$= \bar{u}^{\lambda'}(p') \sum_{n=0,1} \{(\hat{k})^{n_1} (\tau_{\alpha\beta})^{n_2} \rho_{n_1 n_2}((p-p')^2, (p+k)^2)$$

$$+ (-k)^{n_1} (\tau_{\beta\alpha})^{n_2} \rho_{n_1 n_2}((p'-p)^2,$$

$$(p-k')^2)\}\, u^\lambda(p)\, \delta(p+k-p'-k'),$$

where

$$\rho(p^2, k^2) = \int \nu_{n_1 n_2}(p^2, u)\, \delta_+(k^2+u)\, du, \tag{2}$$

$$k'^2 = k^2 = -\mu^2, \quad \tau_{\alpha\beta} = \frac{1}{2}[\tau_\alpha, \tau_\beta],$$

in which the values of u are given by $u = m^2$ and $u \geq (m+\mu)^2$.

The type of interaction affects only the dependence of ν on the arguments. By dividing $\delta_+(k^2)$ into $i\pi\delta(k^2)$ and Pk^{-2} we obtain the imaginary and real parts, respectively, of the scattering amplitude $F_{\alpha\beta}$. It is convenient to continue our investigation in the coordinate system where the combined momentum of the nucleons is zero (i.e., $\mathbf{p}' + \mathbf{p} = 0$). Taking the z axis in the direction of the vector $\mathbf{p}$ we have

$$p\,(E = \sqrt{m^2+p^2},\ 0, 0, p);\ p'\,(E, 0, 0-p);$$

$$k\,(k_0 = \omega$$

$$= \sqrt{p^2 + k_x^2 + k_y^2 + \mu^2}; k_x, k_y, -p);\ k'\,(\omega, k_x, k_y, p).$$

From (1) and (2) we obtain (hereafter $\delta(p+k-p'-k')$ will be omitted]:

$$F_{\alpha\beta} = D_{\alpha\beta} + i\tilde{A}_{\alpha\beta};\ \tilde{A}_{\alpha\beta} = a_{\alpha\beta} + b_{\alpha\beta}; \tag{3}$$

$$D_{\alpha\beta}(k, p, \lambda'\lambda) = \bar{u}(p')^\lambda \tag{4}$$

$$\sum_{n=0,1} \left\{ (\hat{k})^{n_1} (\tau_{\alpha\beta})^{n_2} \int \frac{\nu_{n_1 n_2}((p-p')^2, u)\, du}{(p+k)^2 + u} \right.$$

$$\left. + (-k)^{n_1} (\tau_{\beta\alpha})^{n_2} \int \frac{\nu_{n_1 n_2}((p-p')^2, u)}{(p'-k)^2 + u}\, du \right\} u^\lambda (p')^\lambda;$$

$$a_{\alpha\beta} = \pi \bar{u}^{\lambda'}(p') \tag{5}$$

$$\times \sum_{n=1,0} \{(\hat{k})^{n_1} (\tau_{\alpha\beta})^{n_2} \nu_{n_1 n_2}((p'-p)^2, -(p+k)^2)\}\, u^\lambda(p);$$

ERRATA TO VOLUME 4

	reads	should read
P. 218, column 2, Eq. (10)	$\ldots \xi(\sqrt{3}+2)(2-\sqrt{3})$	$\ldots \xi^{(\sqrt{2}+2)/(2-\sqrt{3})} \ldots$
P. 219, column 1, Eq. (11)	$\ldots (t\xi)^{\sqrt{3/2}} \ldots$	$\ldots (t\xi)]^{\sqrt{3/2}} \ldots$
P. 219, column 1, Eq. (12)	$y^2 = \rho^{2/3}$	$y^2 - \rho^{2/3} >> 1$
P. 223, column 1, Eq. (45)	$\ldots (E_0\mu^{3/4})^{\sqrt{3/4}}$	$\ldots (E_0\mu^{3/4})^{\sqrt{3}/4}$
P. 223, column 2, Eq. (46)	$\ldots \mu^{3\sqrt{3/4}} \ldots$	$\ldots \mu^{3\sqrt{3/4}} \ldots$
P. 225, column 1, 3 lines above Eq. (1.1)	transversality	cross section
P. 225, column 1, 3 lines above Eq. (1.2)	transversality	cross section
P. 256, column 1, Eq. (37)	$\ldots \frac{55\sqrt{3}}{48} \ldots$	$\ldots \frac{55}{\sqrt{3}\,48} \ldots$
P. 289, column 2, Eq. (2)		$I = \sum_n \frac{1}{2n+1} A_n \sum_{\nu=-n}^{n} \frac{1}{1+i\omega\tau} Y_{n\nu}(n_1) Y_{n\nu}(n_2)$
P. 377, column 1, last line	$\delta_{35} = \eta - 21 \times \eta^5$	$\delta_{35} - 21\eta^5$
P. 436–7	Figures 2 and 3 should be exchanged.	
P. 449, column 1, last Eq.	$\ldots Y_{lm}\ \varphi\sigma\ \alpha$	$\ldots Y_{lm}\ \varphi_\sigma\ \alpha$
P. 449, column 2, Eq. (12)	$\ldots W(l, j, \sigma\ 1;\ j) \ldots$	$\ldots W(l, j, \sigma\ 1; \sigma\ j) \ldots$
P. 451, column 1, Eq. (7)	$\ldots D_{\alpha\ \beta}^{(1)}(p, 0, \lambda'\ \lambda) = \ldots$	$\ldots D_{\alpha\ \beta}^{(1)}(p, \omega_0, \lambda', \lambda) = \ldots$
P. 541, column 1, Eq. (28)	$M_{++}^{*\,monex}$	$M_{+}^{*\,monex}$
P. 543, column 2, Eq. (35)	$\ldots \int_{\rho^2 - \tau^2 + l_0^2}$	$\ldots \int_{\rho^2 < \tau^2 + l_0^2} \ldots$