that in the case of extreme relativistic energies this difference evidently depends on the behavior characteristics of the spin magnetic moment of the electron. It should be noted, first of all, that in the relativistic case the electron, in a manner of speaking, "loses" its magnetic moment in accordance with the formula

$$\mu \approx \mu_0 mc^2/E, \quad \mu_0 = eh/2mc. \tag{4.15}$$

However, on the other hand, with increasing energy the interaction with the high-frequency parts of the virtual field of the photons plays an increasingly significant role in the radiation.

The matrix elements characterizing the radiation at the expense of the magnetic moment and at the expense of the charge interaction are proportional to the magnitudes $\sim \mu H \sim \mu \varkappa A$ and $\sim eA$, respectively. Consequently, the ratio of the energy of radiation W_μ at the expense of the magnetic moment to the energy of radiation W_e at the expense of the charge interaction is equal to $(W_\mu/W_e) \sim (\mu\omega_{max}/ec)^2$ in order of magnitude. In the case of $\zeta \ll 1$, the maximum frequency is given by $\omega_{max} \approx \omega_0(E/mc^2)^3$, while for $\zeta \gg 1$ it is given by $\omega_{max} \approx E/h$. Hence, we obtain at once

$$W_\mu / W_e \sim \begin{cases} \zeta^2 & \text{for } \zeta \ll 1, \\ 1 & \text{for } \zeta \gg 1. \end{cases}$$

Thus the statement of Sokolov corresponds completely with the results of the present work.

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Two-Nucleon Potential of Intermolecular Type and Nuclear Saturation

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A study is made of the statistical model of the nucleus with uniform density distribution of the nucleons, on the basis of a two-nucleon interaction potential of the type of the Lennard-Jones intermolecular potential with a hard barrier. It is shown that saturation can be obtained with a certain choice of the parameters in the potential.

1. THE explanation of the stability of atomic nuclei is one of the main problems of the theory of nuclear structure, directly related to the explanation of saturation, which consists of the fact that in medium-weight and heavy nuclei the density of nucleons and the binding energy per nucleon are roughly constant. The existence of saturation has always placed restrictions on the choice of one or another kind of theory of the nuclear forces, which it is still impossible to determine uniquely. At first it seemed possible to achieve saturation by means of exchange forces of various kinds.[1] But the data on the scattering of nucleons (n–p and p–p) at moderately high energies ($\gtrsim$ 100 mev)

evidently show that the exchange forces play a considerably smaller part than is needed for the explanation of saturation. It is also impossible to explain saturation if the nuclear forces are only nonexchange forces of attraction, for example in the form of a rectangular well or a Yukawa potential. In such a case the collapsed state would have to be the most stable one. In recent papers[2–4] it has been shown that a two-nucleon central potential, consisting of a spin-exchange force of large magnitude and a repulsive Wigner force of relatively small magnitude (with radial dependences of the form $e^{-k_0 r}/r$), leads to the observed saturation of the nuclear density and binding energy. On the other hand, it has recently been suggested[5] that there is a possibility of explaining saturation by taking into account many-particle forces acting between three and more nucleons. In this case it turned out that the introduction of (non-exchange) many-particle repulsive forces (mainly involving three particles) obtained from the pseudoscalar meson theory leads to a qualitative explanation of the observed saturation. It is still of interest to examine in more detail the influence of two-particle repulsive forces on the saturation.

In the present paper we show the possibility of explaining saturation on the basis of a semi-phenomenological two-nucleon interaction potential of the type of the intermolecular potential of Lennard-Jones, involving a hard wall, containing an ordinary nonexchange repulsive force acting at small distances along with the attractive Yukawa force. For simplicity, we shall not include spin terms; their role in saturation has been elucidated previously.[2–4] Without trying at present to fix the precise form of the two terms of the potential, which is obviously not yet essential, we must emphasize that the explanation of a fairly considerable number of experimental facts about the scattering of nucleons in all probability demands that, along with the attractive forces at distances $r > r_c$, there also be present a strong repulsion at small distances $r < r_c$.[6,7] We shall show that a special law of this type, of a form that can be generalized without difficulty (for example by adding our repulsive term to the "best" known potential from pseudoscalar mesodynamics) can explain the absence of a collapsed state in nuclei, just as is found for liquids and solids. Despite the fact that the presence of some repulsive part in nuclear forces at the smallest distances can evidently be inferred also on the basis of theoretical considerations[6,7] (though not completely conclusive ones) and also from certain hypothetical models of the structure of nucleons, one still must not forget the preliminary nature of both the empirical and the theoretical arguments in favor of such forces: precise choice of the shape of the potential remains out of the question.

Accordingly we assume that repulsive forces act between two nucleons at distances $r > r_c$ along with fuses of attraction, and at distances $r < r_c$ there exists only a stronger repulsion ("hard core"):

$$U_{12}(r) = +\infty, \quad \text{if} \quad r < r_c;$$
$$U_{12}(r) = B\frac{e^{-r/a}}{(r/a)^2} - C\frac{e^{-r/a}}{r/a}, \quad \text{if} \quad r > r_c. \tag{1}$$

B and C are parameters to be determined later; $r \equiv r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$ is the distance between nucleons 1 and 2. The choice of the nucleon potential in the form (1) means that the nucleons behave like hard spheres of diameter r_c interacting through the nuclear potential.

In the case of an interaction potential with a repulsive core r_c the wave function Ψ of the ground state of a nucleus containing A particles can be written as a product of a Slater determinant and a symmetric function $\Theta(r_1, r_2, \ldots, r_A)$ of the space coordinates of the A nucleons:[5]

$$\Psi = \left(\frac{1}{\sqrt{A!}}\sum_P^{A!}(-1)^P \prod_{i=1}^{A} \psi_i(q_i)\right) \times \Theta(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_A), \tag{2}$$

where $\psi_i(q_i)$ is the wave function of the ith nucleon

$$\psi_i(q_i) = \psi_i(\mathbf{r}_i)\, a_i(s_i) = \Omega^{-1/2}\exp\{i\mathbf{k}_i\mathbf{r}_i\}\, a_i(s_i), \tag{3}$$

$a_i(s_i)$ is the spin function of the ith particle, and $\Omega = (4\pi/3)R^3$ is the volume of the nucleus.

The introduction of the spatial correlation between the A particles by means of the factor Θ assures the fulfillment of the boundary conditions requiring that the wave function (2) vanish when any two nucleons approach each other to a distance $r \le r_c$. For simplicity we take the function Θ in a form that takes into account only the correlation between pairs of nucleons:

$$\Theta(\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_A) = \sum_{i<j=1}^{A} g(r_{ij}) \tag{4}$$

(where $r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$), omitting for the present the obvious generalizations to take into account various configurations inside the nucleus (deuteron, α -particle, etc.) which could be stimulated by many-particle forces. For simplicity we assume $g(r_{ij})$ to be a simple step function

$$g(r_{ij})=\begin{cases}1, & r_{ij}>r_c\\ 0, & r_{ij}<r_c.\end{cases} \tag{5}$$

By means of Eqs. (1) to (4) we obtain from the equation defining the mean value of the potential energy operator

$$V=\int \Psi^* U \Psi_{12} d\mathbf{r}_1 \ldots d\mathbf{r}_A, \tag{6}$$

the following expression for the total average potential energy of a nucleus composed of A nucleons

$$V=V^0+V^a, \tag{7}$$

where

$$V^0=\frac{1}{2}B\iint (d\mathbf{r}_1)(d\mathbf{r}_2)\frac{e^{-r/a}}{(r/a)^2}\rho^2 g^2(r_{12}) \tag{8}$$

$$-\frac{1}{2}C\iint (d\mathbf{r}_1)(d\mathbf{r}_2)\frac{e^{-r/a}}{r/a}\rho^2 g^2(r_{12});$$

$$V^a=\frac{1}{4}C\iint (d\mathbf{r}_1)(d\mathbf{r}_2)\frac{e^{-r/a}}{r/a}|\rho(\mathbf{r}_1,\mathbf{r}_2)|^2 g^2(r_{12}) \tag{9}$$

$$-\frac{1}{4}B\iint (d\mathbf{r}_1)(d\mathbf{r}_2)\frac{e^{-r/a}}{(r/a)^2}|\rho(\mathbf{r}_1,\mathbf{r}_2)|^2 g^2(r_{12}).$$

For the ordinary and matrix densities of the nucleons $\rho(r)$ and $\rho(\mathbf{r}_1,\mathbf{r}_2)$, we have by Eq. (3)

$$\rho(\mathbf{r}_1,\mathbf{r}_2)=\sum_{i=1}^{A}\psi_i^*(\mathbf{r}_i)\psi_i(\mathbf{r}_2) \tag{10}$$

$$=3\rho(\mathbf{r})\frac{\sin Kr-Kr\cos Kr}{(Kr)^3},$$

$$\rho(\mathbf{r})=\rho(\mathbf{r}_1,\mathbf{r}_1) \tag{11}$$

$$=\sum_{i=1}^{A}\psi_i^*(\mathbf{r})\psi_i(\mathbf{r})=\frac{2}{3\pi^2}K^3,$$

where K is the maximum magnitude of the wave number $\mathbf{k}$ of a nucleon in the Fermi distribution. The density $\rho(r)$ must satisfy the condition

$$\int \rho\, d\tau=A. \tag{12}$$

Using Eqs. (5) and (10) and the relation

$$\int (d\mathbf{r}_1)(d\mathbf{r}_2)F(r_{12})g^2(r_{12}) \tag{13}$$

$$=\Omega\int_{r>r_c}(d\mathbf{r})F(r),$$

we obtain from Eq. (9), after integration, the exchange potential energy of the nucleus as a function of the density of the nucleon distribution in the form

$$V^a=Q_1\Omega F_1(b_0,\varkappa_0,\rho) \tag{14}$$

$$+\frac{Q_0}{5}\Omega F_1(b_0,\varkappa_0,\rho)-\frac{Q_0}{10}\Omega\Big[F_0(b_0,\varkappa_0,\rho)$$

$$-\frac{1}{12\varkappa_0^2}\rho^{2/3}(Z-Z_2)+\left(\frac{1}{8\varkappa_0^3}\rho+\frac{1}{24\varkappa_0^5}\rho^{5/3}\right)Z_1\Big].$$

Here

$$F_0(b_0,\varkappa_0,\rho)=e^{-b_0}\Big[\frac{1}{b_0^5}-\frac{\varphi_3(b_0)}{\varkappa_0^2}\rho^{2/3} \tag{15}$$

$$-\left(\frac{1}{b_0^5}-\frac{\varphi_2(b_0)}{\varkappa_0^2}\rho^{2/3}\right.$$

$$\left.+\frac{1}{24b_0\varkappa_0^4}\rho^{4/3}\right)\cos\frac{b_0}{\varkappa_0}\rho^{1/3}$$

$$-\left(\frac{1}{\varkappa_0 b_0^4}\rho^{1/3}+\frac{1}{24}\frac{b_0+1}{b_0^2\varkappa_0^3}\rho\right)\sin\frac{b_0}{\varkappa_0}\rho^{1/3}\Big];$$

$$F_1(b_0,\varkappa_0,\rho)=\frac{1}{8}e^{-b_0}\Big[\varphi(b_0)+\frac{1}{2b_0}\left(\frac{1}{b_0}-1\right)\frac{1}{\varkappa_0^2}\rho^{2/3}$$

$$+\left(-\varphi(b_0)+\frac{1}{3b_0\varkappa_0^2}\rho^{2/3}\right)\cos\frac{b_0}{\varkappa_0}\rho^{1/3}$$

$$+\left(-b_0\varphi(b_0)-\frac{1}{6}\right)\frac{\rho^{1/3}}{\varkappa_0}\sin\frac{b_0}{\varkappa_0}\rho^{1/3}\Big]$$

$$+\left(\frac{1}{48}+\frac{1}{16\varkappa_0^2}\rho^{2/3}\right)(Z-Z_2)-\frac{1}{24\varkappa_0^3}\rho Z_1;$$

$$Q_0=576\,\pi B a^3\varkappa_0^6;\quad Q_1=576\,\pi C a^3\varkappa_0^6;$$

$$b_0=r_c/a;\quad \varkappa_0=b_0/\alpha.$$

$$\varphi(b_0)=\frac{1}{b_0^4}-\frac{1}{3b_0^3}+\frac{1}{6b_0^2}-\frac{1}{6b_0};\quad Z=\int_{x_0}^{\infty}\frac{e^{-\alpha u}}{u}du;$$

$$\varphi_2(b_0) = \frac{1}{12}\left(\frac{1}{b_0^3} + \frac{1}{b_0^2} - \frac{1}{b_0}\right); \quad (16)$$

$$Z_1 = \int_{x_0}^{\infty} e^{-\alpha u} \sin u \frac{du}{u};$$

$$\varphi_3(b_0) = \frac{1}{12}\left(-\frac{5}{b_0^3} + \frac{1}{b_0^2} - \frac{1}{b_0}\right);$$

$$Z_2 = \int_{x_0}^{\infty} e^{-\alpha u} \cos u \frac{du}{u};$$

$$\alpha = \varkappa_0 \rho^{-1/3}; \quad \varkappa_0 = (1/2\,a)(2/3\,\pi^2)^{1/3}.$$

After integration we obtain from Eq. (8) the expression for the ordinary potential energy,

$$V^0 = {}^1/_2 B\rho^2 4\pi a^3 e^{-b_0}\Omega - {}^1/_2 C\rho^2 4\pi a^3 e^{-b_0}(1 + b_0)\Omega. \quad (17)$$

The expressions (14) and (17) give the potential energy of a nucleus of nucleons distributed with constant density ρ in the volume Ω. For $\rho=\rho_0$ = const for $r \leq R$ and $\rho = 0$ for $r > R$, we have from (12)

$$\rho = A/\Omega = 3A/4\pi R^3. \quad (18)$$

From this we get the potential energy of the nucleus as a function of its radius R and the parameters of the interaction potential

$$V = V^0 + V^a = \frac{3}{2} B e^{-b_0} a^3 \frac{A^2}{R^3} - \frac{3}{2} e^{-b_0}(1 + b_0) C a^3 \frac{A^2}{R^3} - \frac{8}{15\pi^2} B \Phi_1(b_0, a, R) + \frac{16}{15\pi^2}(B + 5C)\Phi_2(b_0, a, R); \quad (19)$$

$$\Phi_1(b_0, a, R) = \frac{R^3}{a^3} F_0(b_0, \varkappa_0, \rho); \quad (20)$$

$$\Phi_2 = \frac{R^3}{a^3} F_1(b_0, \varkappa_0, \rho)$$

with $\rho = 3A/4\pi R^3$.

In the case of nucleons regarded as impenetrable spheres of radius r_c, the kinetic energy of a gas consisting of A nucleons contains, in addition to the Fermi term, a supplementary term $\sim r_c$.[5]

$$T = T_F\left(1 + \frac{2{,}16\, r_c A^{1/3}}{R}\right) = \left(\frac{3}{\pi}\right)^{4/3} \frac{3}{160} \frac{h^2}{MR^2} A^{5/3}\left[1 + \frac{2{,}16\, r_c A^{1/3}}{R}\right]. \quad (21)$$

The total energy is

$$E = V + T = V^0 + V^a + T. \quad (22)$$

From the requirement that the energy E be a minimum it follows that the value of the parameter R corresponding to the equilibrium state of the nucleus must be proportional to $A^{1/3}$. To determine the values of the potential parameters that give the equilibrium state we make use of an empirical value of the radius R_s of the nucleus. For this purpose we set

$$R = R_s x = r_0 A^{1/3} x. \quad (23)$$

The dimensionless quantity x determines the variation of the radius of the nucleus around its equilibrium value $R_s = r_0 A^{1/3}$. For $x = 1$ we have the stable radius $R = R_s = r_0 A^{1/3}$, which corresponds to a state of the nucleus with normal density. For the empirical constant r_0 we can take from the experimental data one of the two values (cf. Refs. 5, 8, 9, 10)

$$r_0 \approx 1.4 \times 10^{-13} \text{ cm}, \quad (24)$$

$$r_0 \approx 1.2 \times 10^{-13} \text{ cm}. \quad (25)$$

Using Eq. (23), we get from Eqs. (19), (21) and (22) the expression for the total energy of the nucleus

$$E = T + V^0 + V^a = \left\{\left(\frac{a_2}{x^2} + \frac{b_2}{x^3}\right) + \frac{(a_0 - a_1)}{x^3} - (a_3 \Psi_1(b_0, \beta x) - a_4 \Psi_2(b_0, \beta x))\right\} A. \quad (26)$$

Here

$$a_0 = \frac{3e^{-b_0}}{2\beta^3} B, \quad a_1 = \frac{3e^{-b_0}(1+b_0)}{2\beta^3} C;$$

$$a_3 = \frac{8}{15\pi^2} B, \quad a_4 = \frac{16}{15\pi^2}(B+5C);$$

$$a_2 = \left(\frac{3}{\pi}\right)^{4/3} \frac{3h^2}{160\, Mr_0^2}, \quad b_2 = 2{,}16\, a_2 b,$$

$$\beta = \frac{r_0}{a}, \quad b = \frac{r_c}{r_0}, \quad b_0 = \frac{r_c}{a};$$

$$\Psi_1(b_0, \beta x) = \Phi_1(b_0, a, R),$$

$$\Psi_2(b_0, \beta x) = \Phi_2(b_0, a, R)$$

with $R = r_0 A^{1/3} x$.

From the stability conditions

$$(\partial E / \partial x)_{x=1} = 0, \quad (\partial^2 E / \partial x^2)_{x=1} > 0 \qquad (27)$$

and the empirical value of the binding energy, found without taking into account the Coulomb and surface energies,

$$(E(x))_{x=1} = -\alpha_0 A, \quad \alpha_0 \approx 14 \text{ mev} \qquad (28)$$

we find the parameters B and C as functions of the constants b_0, β and r_0 in the form

$$B = \frac{1}{P(b_0, \beta)} \Big\{ 3\alpha_0 + a_2 + 10\, n_0 (2a_2 + 3b_2) \qquad (29)$$

$$\times \frac{3\Psi_2(b_0, \beta) + \Psi_2'(b_0, \beta)}{n_2 + 10\, n_0 \Psi_2'(b_0, \beta)} \Big\}$$

$$= \chi_1(b_0, \beta, r_0);$$

$$C = \frac{2a_2 + 3b_2 + [n_1 + n_0\Psi_1'(b_0, \beta) - 2n_0\Psi_2'(b_0, \beta)]\, B}{n_2 + 10 n_0 \Psi_2'(b_0, \beta)}$$

$$= \chi_2(b_0, \beta, r_0);$$

$$P(b_0, \beta) = [3\Psi_1(b_0, \beta) + \Psi_1'(b_0, \beta)]\, n_0$$

$$- [3\Psi_2(b_0, \beta) + \Psi_2'(b_0, \beta)]\, 2n_0$$

$$- 10\, n_0 \frac{3\Psi_2(b_0, \beta) + \Psi_2'(b_0, \beta)}{n_2 + 10\, n_0 \Psi_2'(b_0, \beta)}$$

$$\times [n_1 + n_0 \Psi_1'(b_0, \beta) - 2n_0 \Psi_2'(b_0, \beta)],$$

$$n_0 = 8/15\,\pi^2; \quad n_1 = 9e^{-b_0}/2\beta^3;$$

$$n_2 = 9e^{-b_0}(1+b_0)/2\beta^3,$$

$\Psi_1(b_0, \beta)$, $\Psi_2'(b_0, \beta)$, etc. are the values of the functions $\Psi_1(b_0, \beta x)$ $\Psi_2(b_0, \beta x)$ and their derivatives at $x = 1$.

Substituting Eqs. (29) into the inequality of (27), we have

$$6a_2 + 12b_2 + 4\,(n_1\chi_1(b_0, \beta, r_0) \qquad (30)$$

$$- n_2\chi_2(b_0, \beta, r_0)) - n_0\Psi_1''(b_0, \beta)$$

$$+ 2n_0(\chi_1(b_0, \beta, r_0)$$

$$+ 5\chi_2(b_0, \beta, r_0))\,\Psi_2''(b_0, \beta) > 0.$$

Moreover, the expressions for $\Psi_1(b_0, \beta)$, $\Psi_2(b_0, \beta)$, and their derivatives involve the values of the integrals (16) and their derivatives for $x = 1$, the function

$$Z = -\operatorname{Ei}(-b_0) = \int_{b_0}^{\infty} \frac{e^{-w}\, dw}{w}$$

and so on.

2. For given values of the constants r_0 and a consistent with the experimental data the inequality (30) makes it possible to set a lower limit to the values of the parameter $b_0 = r_c / a$ for which the nucleus can be in equilibrium. We assume that the effective radius of action of the two-nucleon force (1) is equal to the Compton wavelength of the π-meson:

$$a = h / 2\pi m_\pi c = 1.4 \times 10^{-13} \text{ cm}. \qquad (31)$$

For the values (24) and (31) of the quantities r_0, a, the inequality (30), which corresponds to the possibility of nuclear equilibrium, can be satisfied if $b_0 = r_c / a > 0.357$, or $r_c > 0.357a = 0.5 \times 10^{-13}$ cm. Thus the existence of an equilibrium state of the system of nucleons imposes a lower limit on the value of the radius r_c of the repulsive core of the assumed interaction potential (1). With the values $b_0 = 0.38$ and $b_0 = 0.43$, corresponding to the values of r_c used also in Refs. 5,6 and 7, we finally obtain from Eqs. (29), (31), and (24) the following systems of values for the parameters of our chosen two-nucleon potential (1):

$$B = 395.56 \text{ mev}; \quad C = 278.64 \text{ mev}; . \qquad (32)$$

$$a = 1.4 \times 10^{-13} \text{ cm};$$

$$r_0 = 1.4 \times 10^{-13} \text{ cm}; \quad b_0 = r_c / a = 0.38;$$

$$r_c = 0.532 \times 10^{-13} \text{ cm};$$

and

$$B = 935.167 \text{ mev}; \quad C = 610.029 \text{ mev}; \qquad (33)$$
$$a = 1{,}4\times10^{-13} \text{ cm};$$
$$r_0 = 1.4\times10^{-13} \text{ cm}; \; b_0 = r_c/a = 0.43;$$
$$r_c = 0.6\times10^{-13} \text{ cm}.$$

The ordinary repulsive potential energy $V^0/A = (a_0 - a_1)/x^3$, the exchange attractive potential energy $V^a/A = -a_3 \Psi_1 + a_4 \Psi_2$, and the total energy per nucleon, $E/A = V^0/A + V^a/A + T/A$ are shown as functions of the nuclear radius $x = R/r_0 A^{1/3}$ in Figs. 1 and 2, for the values of the parameters given in Eqs. (32) and (33). From these diagrams it is seen that the total potential energy $(V^0/A) + (V^a/A)$ of the nucleons in the case of forces given by Eq. (1) has a minimum value

$$W/A = V^0/A + V^a/A \approx -105 \text{ mev}$$

for the case corresponding to Eq. (32), at a nuclear radius $R = R_0 \approx 0.5\, r_0 A^{1/3} = 0.7\times10^{-13} A^{1/3}$, and a minimum value $W/A \approx -51$ mev for the case corresponding to Eq. (33), at a nuclear radius $R = R_0 \approx 0.8\, r_0 A^{1/3} = 1.12\times10^{-13} A^{1/3}$. Finally, for values of the constants r_0 and a from Eqs. (25) and (31) and $b_0 = 0.38$, we have from (29)

$$B = 321{,}580 \text{ mev}, \; C = 228{,}970 \text{ mev},$$
$$a = 1.4\times10^{-13} \text{ cm}, \qquad (34)$$
$$r_0 = 1{,}2\times10^{-13} \text{ cm}, \; b_0 = r_c/a = 0.38,$$
$$r_c = 0.532\times10^{-13} \text{ cm}.$$

For the parameter values (34) the total potential energy $W/A = V^0/A + V^a/A$ has a minimum value of ~ -128.2 mev at nuclear radius $R = R_0 = 0.5 r_0 A^{1/3} = 0.6\times10^{-13}$ cm. Because of the presence of the short-range repulsive force ($\sim Be^{-x}/x^2$) in the expression for the two-nucleon potential energy, Eq. (1), the potential energy W of the nucleus is positive (repulsion) for small radii, and thus the collapsed state of the nucleus is not stable. Owing to the presence of kinetic energy the total energy E/A takes a minimum value equal to the empirical value ~ -14 mev at a nuclear radius $R = R_s = r_0 A^{1/3}$ which corresponds to a state of the nucleus with normal density. In all the cases we are considering the behavior of E/A as a function of the nuclear radius shows that the state with normal density ($x = 1$) is stable with respect to both

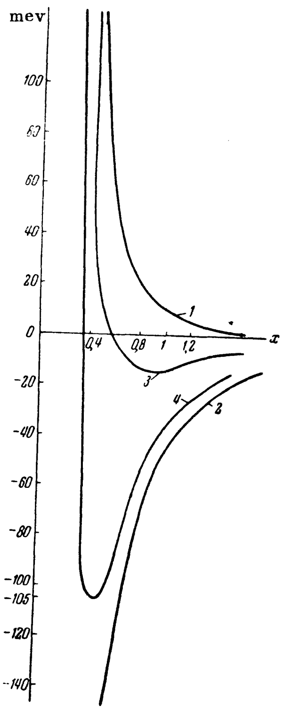

FIG. 1. Dependence on nuclear radius R of 1) the ordinary repulsive potential energy V^0/A, 2) the exchange attractive potential energy V^a/A, 3) the total energy per nucleon E/A, and 4) the sum $(V^0/A) + (V^a/A)$, for the parameter values (32).

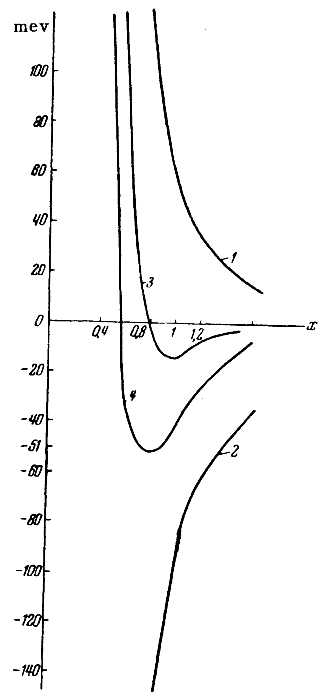

FIG. 2. Dependence on the nuclear radius R of 1) V^0/A, 2) V^a/A, 3) E/A, and 4) $(V^0/A) + (V^a/A)$ for the parameter values (33).

increase and decrease of the radius of the nucleus and that the binding energy is proportional to the mass number A, and not to A^2. In the case $B = 0$ the two-nucleon potential (1) goes over into the ordinary attractive potential of Yukawa supplemented by a repulsion of the Jastrow type at small distances:

$$U_{12} = \begin{cases} \infty & \text{for } r < r_c, \\ -C\dfrac{e^{-r/a}}{(r/a)} & \text{for } r > r_c. \end{cases} \tag{35}$$

According to Eq. (26) the binding energy of the nucleus for the interaction law (35) is

$$E = T + V^0 + V^a \tag{36}$$

$$= \left\{\frac{a_2}{x^2} + \frac{b_2}{x^3} - \frac{b_3}{x^3} + b_4\Psi_2(b_0, \beta x)\right\} A.$$

Here

$$b_3 = 3e^{-b_0}(1 + b_0)\,C / 2\beta^3; \quad b_4 = 16C / 3\pi^2.$$

It can easily be shown that for the energy expression (36) the requirements (27) and (28) are not compatible, i.e., equilibrium of the nucleus at the normal nuclear density cannot be secured. Thus when the statistical model is used the two-nucleon attractive Yukawa potential with a repulsive core r_c at small distances in the strict sense of the Jastrow model does not permit the establishment of nuclear stability. On the other hand we have shown that our proposed two-nucleon potential of the Lennard-Jones intermolecular type, which in our opinion represents in a semiphenomenological way the results of mesodynamics with suitable parameters, actually leads to saturation of the binding energy at a normal density of nucleons corresponding to the equilibrium nuclear radius. For the two systems of parameter values given in Eqs. (32) and (33) we present a graphical representation of the attractive and repulsive potential energies, and also of the total interaction potential energy between two nucleons, as functions of the distance between the two nucleons (Fig. 3). It is seen from the diagram that the total interaction force between two nucleons at large distances ($r \geq r_m$) is attractive, and at small distances ($r < r_m$) it is a repulsive force. The distance $r = r_m$ between the nucleons corresponds to the deepest point in the potential well U_m.

It must be remarked that the introduction of a correlation function of exponential form, as is used, for example, in the theory of liquids, must lead to a further weakening of the connection between the nucleons, in general agreement with the requirements of the shell model.

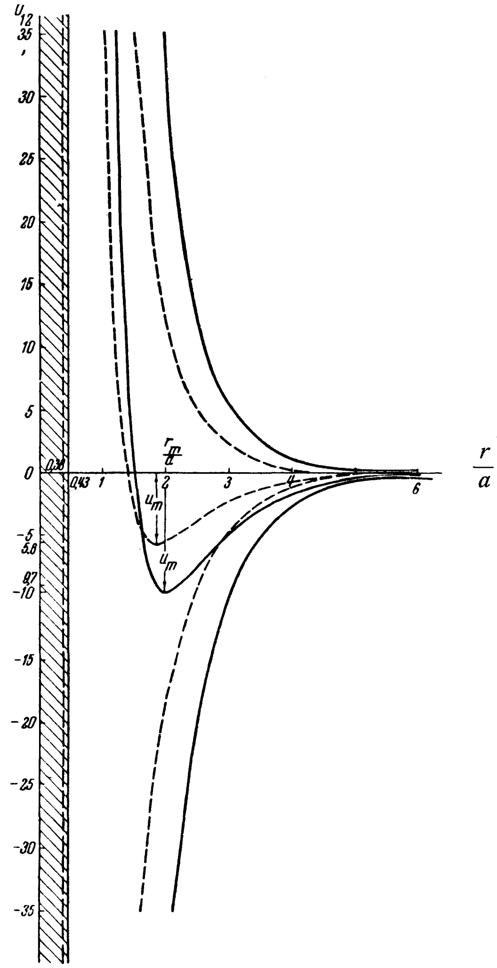

FIG. 3. Graph of the dependences of $-C\dfrac{e^{-r/a}}{(r/a)}$, $B\dfrac{e^{-r/a}}{(r/a)^2}$, and the total interaction potential energy $U_{12}(r)$ between two nuclei on the distance r between them; dashed curves are for the parameter values (32), solid curves for the values (33).

In conclusion we express our gratitude to L. I. Morozovskaia for carrying out our numerous calculations.

Note added in proof. Brueckner and others[11] have come to the conclusion that the two-nucleon potential of pseudoscalar mesodynamics without the "pair term" gives the required saturation of the energy at the radius $R_s = 1.15 \times 10^{-13} A^{1/3}$, if in addition to a repulsive core (r_c), taken different for the singlet and triplet states, one includes repulsion in the odd P-state.

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Theory of Localized Electron States in an Isotropic Homopolar Crystal

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The behavior of an electron localized near a defect in a nonmetallic homopolar crystal is examined, taking into account the "condenson" interaction of the electron with the crystal. We have calculated the energy levels of the system and the energy of thermal dissociation of the electron, considering the motion of the electron quantum mechanically, and the motion of the atoms in the lattice either classically or quantum mechanically. It is shown that the condenson interaction leads to a difference between the energies of thermal and of photo-dissociation of an electron. We have determined the shape of the absorption band due to a localized electron (the position of the maximum, the halfwidth and its temperature dependence). As an illustration we give numerical calculations in the case of a Coulomb potential of the defect (for instance, an impurity atom with a valence electron).

1. INTRODUCTION

JUST as the presence of defects accompanied by localized electron states leads to the occurrence of a number of peculiarities in the optical, magnetic, photoelectric and other properties in ionic crystals, the presence of defects in homopolar crystals can also essentially change their properties.

It is well known that any attempt at a quantitative consideration of the energy level scheme of the electrons of an impurity atom leads to the calculation of the motion of a valence electron in the field of the ionized impurity in a medium characterized by a dielectric constant ϵ. We must at once remark that such a calculation which does not consider the interaction of the electron with the vibrations of the lattice is unable to consider quantitatively the width of the absorption band of impurity atoms, its temperature dependence, the difference between the energies of thermal and photodissociation, the difference between the energies of thermal and photo dissociation of impurity atoms, and so on.

In one of the papers[1] by the author and Pekar we investigated the question of the states of conduction electrons in a perfect homopolar crystal. It turned out that in a homopolar crystal also the interaction of the "extra" electron with the dielectric can partially be of an internal character. If as a result of an elastic deformation there occurs a region of increased density and thus a higher dielectric constant in some parts of the crystal, the electron must, according to macroscopic electrostatic theory, drift to those regions. Therefore, a region of higher density presents a potential trap to a conduction electron, and because of the inertia of the displacement of the atoms, it will not follow