The author is indebted to L. N. Rosentsveig for discussion of the results of this investigation.

* It is assumed that the electron spin is not flipped upon reflection. Let us note that Eq. (1) was obtained in Ref. 4; the operator $(\partial f/\partial t)_{sp}$, however, was not included here. Boundary conditions (13) of that reference are apparently not realizable in practice.

** The slow damping of **M** is linked to the fact that when $\delta \ll \delta_{eff}$ there appears an "anomalous skin-effect" for the magnetic moment: an integral relation appears between M and H_1. This does not take place out of resonance as $M \sim 10^{-6} H_1$ (while at resonance $M \sim 10^{-2} H_1$).

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"Repulsion" of Nuclear Levels

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1. DATA on the position and parameters of nuclear levels obtained by methods of neutron spectroscopy at excitation energies of the order of the neutron binding energy[1,2] allow the investigation of the empirical regularities of level behavior with the aim of checking the predictions of existing nuclear theories and their improvement. Interesting work has been done, for example, on the systematics of neutron widths[3] and on the systematics of radiation widths[4] *. The problem of regularities in the distribution of nuclear levels and in the fluctuations of adjacent level spacing has not been discussed in the literature.

For a purely random distribution of the distance between levels ϵ about its average value D, the distribution function must be of the form:

$$W(\varepsilon)\, d\varepsilon = \exp\{-\varepsilon / D\}\, d\varepsilon / D. \qquad (1)$$

It would be more to the point to examine the data for levels in the same spin state, i.e., for target nuclei of spin 0 (even-even nuclei). However, such nuclei have few levels in the range where the resolution of contemporary methods of nuclear spectroscopy suffices. Thus it is impossible to exclude the data obtained from nuclei with odd atomic weight, and consequently with two sets of nuclear levels, corresponding to $i - 1/2$ and $i + 1/2$ (where i is the spin of the target nucleus). It should be kept in mind that the presence of two sets of levels corresponding to various spins of the intermediate nucleus makes the correlation of various levels positions less obvious.

If the distribution function (1) holds for each set of levels, then the resulting distribution function will have the same form with $D = d_1 d_2 / (d_1 + d_2)$, (where $d_{1,2}$ are the distances between levels in each set).

We made use of experimental data on the level distribution for: In^{113}, In^{115}, Cs^{133}, Tb^{159}, Ho^{165}, Tm^{169}, Hf^{177}, Hf^{179}, Ta^{181} [1], U^{235} **, U^{238} [1].

In order to eliminate mistakes in the determination of ϵ due to ignoring levels because of insufficient experimental resolution, a curve was constructed for each element of the increase of the number of discovered levels with increase of neutron energy. Levels within a suitable energy limit were used, so that this increase was approximately linear.

To increase the statistical certainty of the experimental distribution of levels for each isotope, the quantity $x_i = \epsilon_i / D$ was calculated, and the distribution of levels as a function of x_i for all the enumerated nuclei was plotted (Fig. 1). The total number of cases $N = 134$. The curve is the distribution (1) normalized to the area of the histogram.

The level distributions for U^{238} (an even-even nucleus with eleven known levels) and for U^{235} (for which D is comparable to the entire level width) are shown separately.

Comparison of the curve and the histogram allows the qualitative confirmation of the relatively small number of closely spaced levels, which may be interpreted as the result of a "repulsion" of levels.

To eliminate the possibility that the relatively small number of closely spaced levels is due to insufficient resolving power of the experiments, we subjected the data to a stricter selection. The following were excluded from consideration: a) csses in which there mi ght be some doubt as to which isotope a given level belongs, b) levels for which the full width of the resolution function ΔE does not satisfy the condition $\Delta E < 0.2D$. The total number of available cases was thus reduced from 134 to 63, of which 19 belong to U^{235} (assuming that the resolution of the Brookhaven National Laboratory experiments were 0.07 μ sec/meter), 7 to U^{238}, and 37 to Cs^{133}, Ho^{165}, Tm^{169}, Hf^{177}, Hf^{179}, Ta^{181}.

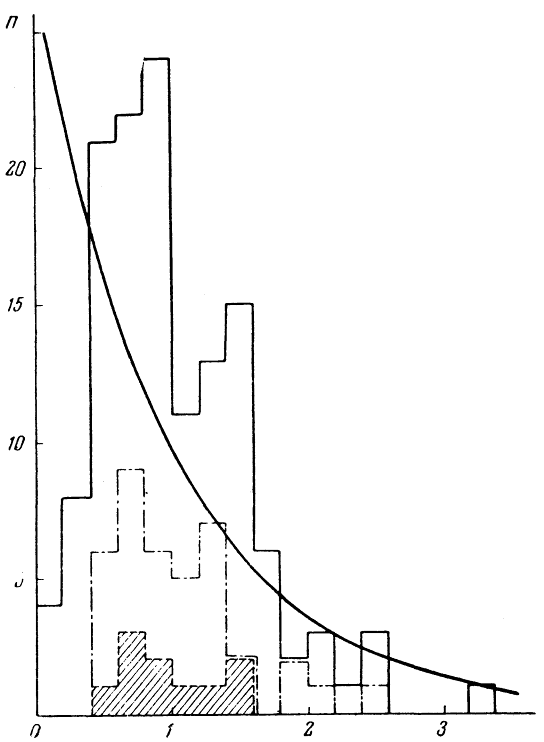

FIG. 1. Distribution of levels according to $x = \epsilon/D$. The shaded histogram $-U^{238}$; —.—corresponds to U^{235}; — to the sum. The curve corresponds to distribution (1) normalized to the area of the sum histogram.

Figure 2 shows the respective histograms. A general view of the histograms is still characterized by a relatively small number of cases with low x.

In order to determine whether the discrepancy between the experimental results and the distribution (1) is not coincidental, χ^2 was calculated for 63 cases, the distribution being divided into 9 groups. The magnitude of χ^2 was 27.5, and the corresponding probability of a coincidental discrepancy $P(\chi) < 0.001$.

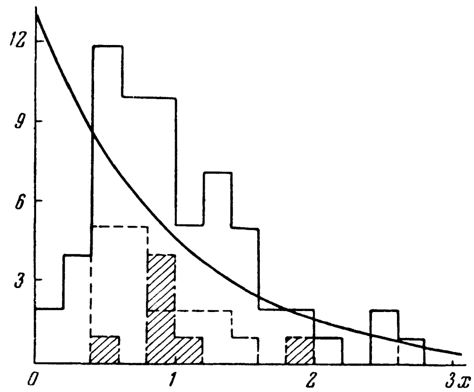

FIG. 2. The same as Figure 1, after additional selection.

The occurrence of level "repulsion" is seen also in the magnitude of the mean square fluctuation of the level spacing $\overline{\Delta x^2} = \overline{x^2} - \overline{x}^2$, ($\overline{\Delta\epsilon^2} = D^2 \overline{\Delta x^2}$). While $\overline{\Delta x^2} = 1$ for distribution (1), in our case $\overline{\Delta x^2} = 0.31$ (for all 63 cases).

The interaction of nuclear levels which we have examined are similar in their physical nature to the phenomenon of electronic term crossings in the spectra of diatomic molecules. A quantum mechanical view of the molecular term behavior[5] shows that their crossing, i.e., the superposition of two terms, cannot take place in the general case.

As can be seen from the above analysis, the interaction of nucleons in the nucleus leads to a distribution function of the nuclear levels which is closer to equidistant than would be predicted from statistical theory.[6]

2. An overwhelming part of the experimental data on the distance between levels is for two systems of levels of the same parity and various spin. In the interpretation of such data, two limiting assumptions can be made: a) the interactions between levels of the two systems are the same as between levels of a single system; b) levels of the two systems do not interact and are randomly distributed (we regard the second assumption as more probable).

In the case that assumption b) is valid, if the level density of one system is much greater than that of the other system, then the experimental distribution of the level distances [as also for assumption a)] will be practically that of one of the level systems. Consequently, if enough statistical material is obtained, the distribution function $\omega(\epsilon)$ for the particular system can be obtained.

The second situation for b), when the level densities of the two systems do not differ appreciably, can be analyzed in the following manner. Let $\omega_1(\epsilon)$ and $\omega_2(\epsilon)$ be the distribution functions for the distance between levels in each of the two systems. $[\omega_i = d_i^{-1} f(\epsilon/d_i)]$. The probability of finding a level of the second system at a distance ϵ from a level of the first system is given by the expression

$$\varphi_i(\varepsilon) = \frac{1}{d_i}\int_{\varepsilon}^{\infty} w_i(\eta)\, d\eta.$$

It is easy to show that the distribution function for the distance between levels for the entire system (levels are not distinguished) will be

$$W(\varepsilon) = Dd^2\Psi(\varepsilon)/d\varepsilon^2, \qquad (2)$$

where

$$\Psi(\varepsilon) = \psi_1(\varepsilon)\psi_2(\varepsilon),$$

$$\psi_i(\varepsilon) = \int_{\varepsilon}^{\infty} \varphi_i(\eta)\, d\eta = \frac{1}{d_i}\int_{\varepsilon}^{\infty} (\eta - \varepsilon)\, w_i(\eta)\, d\eta, \qquad (3)$$

D is the average distance between levels of the entire system $[\psi_i(0) = 1]$.

If we assume that the repulsion of levels for each system must make $\omega_{1,2}(0)$ go to zero, it follows from Eq. (2):

$$W(0) = \frac{2D}{d_1 d_2} = \frac{1}{D}\frac{2\alpha}{(1+\alpha)^2}, \qquad (4)$$

where $\alpha = d_1/d_2$ the ratio of the level distances of the two systems.

The presently available statistical material is not sufficient to allow detailed analysis along the above lines.

In conclusion we thank S. T. Beliaiev and V. M, Galitskii for fruitful discussions about two systems of related levels.

* The work of Levin and Hughes[4] was sent to the USSR before its publication, for which we thank the authors.

** Unpublished data from Brookhaven National Laboratory. We thank V. V. Vladimirskii for communicating these results, obtained from Prof. D.H. Hughes.

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The Polarization of Neutrons from the $D(\gamma, n)$ Reaction

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THE derivation of equations which give the cross section for photodisintegration of the deuteron for $E_\gamma \lesssim 10$ mev can be found in any textbook of nuclear physics (for example, Ref. 1), but there is no mention in the literature that the products of this process are polarized. As is shown below, this effect is caused by an interference of electric and magnetic transitions.

The polarization of the neutrons $\zeta = \langle \sigma_n \rangle$ from the $D(\gamma, n)$ reaction is

$$\zeta = \mathrm{Sp}\,(\sigma_n Q_\lambda Q_{\lambda'}^{+}) \qquad (1)$$
$$\times\ (\delta_{\lambda\lambda'} + \xi\Omega_{\lambda\lambda'})/\mathrm{Sp}\,(Q_\lambda Q_{\lambda'}^{+})\,(\delta_{\lambda\lambda'} + \xi\Omega_{\lambda\lambda'}),$$

where Ω has the components

$$\Omega_x = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\quad \Omega_y = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\quad \Omega_z = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}, \qquad (2)$$

ξ_x, ξ_y, ξ_z are parameters characterizing the polarization of the incident bundle of photons (Ref. 2), and Q_λ is the matrix element for the photodisintegration of the deuteron by a γ-quantum of momentum $k = \omega\varkappa$ ($|\varkappa| = 1$)* and polarization vector $e_\lambda (\lambda = 1,2)$

In the approximation of central $n-p$ forces with zero interaction radius and using the usual notation

$$Q_\lambda \sim \delta_{S1}\delta_{mm_0}\frac{(\mathbf{p}\cdot\mathbf{e}_\lambda)}{\alpha_1^2 + p^2} \qquad (3)$$

$$+\ \delta_{S0}\delta_{m0}\, i\,(\mu_p - \mu_n)\,(\chi_0, ([\varkappa\cdot\mathbf{e}_\lambda],$$

$$\sigma_p - \sigma_n)\,\chi_1^{m_0})\,\frac{\alpha_1 - \alpha_0}{4M(\alpha_0 + ip)}.$$