two neutrons the complete wave function $\Psi_{3/2\, M}$ must be antisymmetric in a single interchange of the space and spin coordinates of the neutrons. Since $\chi_{3/2\, M}$ is symmetric, the space wave function must be antisymmetric in an interchange of the space coordinates of the neutrons. The transformation $\xi \to \eta$, $\eta \to \xi$, $\mu \to \mu$ corresponds to an interchange of the neutrons. Therefore the antisymmetry of the function $\varphi_{3/2}(\xi, \eta, \mu)$ is expressed by the simple equation

$$\varphi_{3/2}(\xi, \eta, \mu) = -\varphi_{3/2}(\eta, \xi, \mu). \qquad (1)$$

From (1) it follows immediately that $\varphi_{3/2} = 0$ for $\xi = \eta$. Thus, in the quartet state, events in which the neutron being scattered falls inside the deuteron are impossible. In other words, in the quartet state the n–d interaction is described by some effective potential of repulsion whose range of influence coincides with the dimensions of the deuteron. In this connection, the scattering length a_4 must be positive and exceed the "radius" of the deuteron, that is $a_4 \geq 4.3 \times 10^{-13}$ cm. Of the two possible pairs α and β of the experimental values for the scattering lengths only the values α satisfy the condition $a_4 \geq 4.3 \times 10^{-13}$ cm. Hence it is necessary to take $a_4 = 6.2 \times 10^{-13}$ cm and $a_2 = 0.8 \times 10^{-13}$ cm.

[1] D. Hurst and N. Alcock, Canad. J. Phys. **29**, 36 (1951).

[2] Wollan, Shull and Koehler, Phys. Rev. **83**, 700 (1951).

[3] M. Gordon, Phys. Rev. **80**, 1111 (1950).

[4] A. Troesch and M. Verde, Helv. Phys. Acta **24**, 39 (1951).

[5] L. Motz and J. Schwinger, Phys. Rev. **58**, 26 (1940).

[6] R. Christian and J. Gammel, Phys. Rev. **91**, 100 (1953).

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Contribution to the Theory of π-Meson Disintegration

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LET us consider the problem of the decay of the π-meson according to the scheme

$$\pi \to \mu + \gamma + \nu,$$

assuming that the μ-meson has an anomalous magnetic moment $\mu_a = \mu' + e/2M$, where M is the μ-meson mass. We consider the meson interaction as scalar (a pseudoscalar interaction yields the same results[1]):

$$H_{\pi,\,\mu\nu} = g(\bar{\varphi}_\nu \varphi_\mu)\Psi_\pi + \text{compl. conj.}$$

The interaction of a μ-meson with a γ-ray is given by the expression

$$H_{\mu,\gamma} = -ie\hat{A} - \tfrac{1}{2} i\mu'\gamma_i\gamma_k F_{ik},$$

$$F_{ik} = \partial A_k / \partial x_i - \partial A_i / \partial x_k.$$

The matrix element of the process is given by

$$M = \frac{2\pi e g}{\sqrt{E_\pi |k|}} \bar{u}_\mu \times \left[\hat{e} - \frac{i\mu'}{2e}(\hat{k}\hat{e} - \hat{e}\hat{k})\right](i\hat{p} + i\hat{k} - M)^{-1} u_\nu$$

where u_μ, u_ν are unitary bispinors of the wave functions of the μ-meson, and of the neutrino; $k = (\mathbf{k}, |k|)$ is the 4-momentum of the photon; $p = (\mathbf{p}, M)$ is the 4-momentum of the μ-meson and e is the unit polarization vector of the photon.

Averaging over polarizations and spins, we get the decay probability

$$dw = \frac{1}{16\pi^3} \frac{e^2 g^2}{E_\pi E E_\nu k} \Big\{\Big[-2(\mathbf{pk} - Ek) \times (-\mathbf{pk} - E_\pi k + Ek) - \frac{[\mathbf{pk}]}{k^2}(E_\pi^2 - [M - m]^2)$$

$$+ 8\left[2\left(\frac{\mu'}{2e}\right)^2 (\mathbf{pk} - Ek)^2 (-\mathbf{pk} + Ek)\right.$$

$$+ 2\left(\frac{\mu'}{2e}\right)(\mathbf{pk} - Ek)^2 (M^2 - EE_\pi - mM)$$

$$+ \left(-\frac{\mu'}{2e}\right)(\mathbf{pk} - Ek)^2 (m - M)$$

$$\left.+ \left(-\frac{\mu'}{2e}\right)(\mathbf{pk} - Ek)(-ME_\pi k)\right]\Big\} \frac{d\mathbf{p}\, d\mathbf{k}}{(Ek - \mathbf{pk})^2},$$

where m is the mass of the neutrino and E_π is the energy of the decaying meson.

Integrating over the directions of the photon, we get the probability of a decay with emission of a μ-meson with momentum p (we take $m = 0$):

$$dw = \frac{e^2g^2}{2\pi}\frac{p^2dp}{EE_\pi}\Big\{\frac{(E_\pi - E)^2 - p^2}{pE_\pi}$$

$$\times\ln\frac{(E+p)(E_\pi - E + p)}{(E-p)(E_\pi - E - p)} + \frac{4}{p}\frac{E_\pi^2 - M^2}{(E_\pi - E)^2 - p^2}\Big(-2p$$

$$+E\ln\frac{E+p}{E-p}\Big) - 4\Big(-\frac{\mu'}{2e}\Big)$$

$$\times M\frac{(E_\pi - E)^2 - p^2}{pE_\pi}\ln\frac{(E+p)(E_\pi - E + p)}{(E-p)(E_\pi - E - p)}$$

$$-8\Big(\frac{\mu'}{2e}\Big)^2 M^2\Big(1 - \frac{EE_\pi}{M^2}\Big)\Big\}.$$

In the non-relativistic case, $p >> M, E_\pi$; $E = M + p^2/2M$. Assuming [2] that the mean free path R of the μ-meson is proportional to p^4 we get for the number of μ-mesons with mean free paths less than R

$$w = \int\limits_0^{p=p_0(R/R_0)^{1/4}} dw = [1+\tau]\,w_{\mu+\nu+\gamma}$$

$$+\frac{e^2g^2}{2\pi}\frac{M^2}{E_\pi}\int\limits_0^{(p_0/M)(R/R_0)^{1/4}}\Big[\frac{\tau^2}{2}\Big(\frac{E_\pi}{M} - 1\Big)x^2$$

$$+\frac{8\tau}{3}\Big(-\frac{M}{E_\pi} + \frac{E_\pi}{M}\Big)\frac{x^4}{(E_\pi/M - 1)^2 - x^2}\Big]dx.$$

Here p_0 and R_0 are the momentum and the mean free path of the meson in the decay $\pi \to \mu + \nu$; $\tau = \mu'/(e/2M)$; $w_{\mu+\nu+\gamma}$ is the decay probability for $\tau = 0$ derived by Ioffe and Rudik[1] ; $w_{\mu+\nu} = (g^2/2)(1 - M^2/E_\pi^2)\,P_0$ is the probability of the decay $\pi \to \mu + \nu$.

The comparison with the results of Ioffe and Rudik[1] shows that the μ-meson having an anomalous magnetic moment can lead to an increase of the number of mesons especially of those with short mean free paths. Similar results should be expected in the case of mesons with spin greater than ½.

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1 B. L. Ioffe and A. P. Rudik, Dokl. Akad. Nauk SSSR 82, 359 (1952).

2 B. Rossi and K. Greisen, *Interaction of cosmic rays with matter*, page 17, IIL (1948) (Russian translation).

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Production of π-Meson Pairs on Nuclei by High Energy γ-Quanta

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THE production of π-meson pairs on nuclei by high energy γ-quanta has been discussed by Pomeranchuk.[1,2] In the case of high energy γ-quanta, only small angles between the momenta of the π-mesons and the γ-quanta are important. The range of the process is found to be greater than the dimensions of the nucleus. Therefore, the knowledge of the wave function outside the nucleus is sufficient to determine the cross section of the process. In Refs. 1 and 2, the wave function was taken as a plane wave plus a wave scattered by a perfectly black sphere of radius R (radius of the nucleus). In this paper we take into account the influence of the Coulomb interaction between the π-mesons and the charge of the nucleus on the pair formation.

The matrix element of the process of formation of a π^+, π^- – pair is given by

$$M = -ie\sqrt{\frac{2\pi}{\omega}}\int[\psi_+^*\,\mathbf{j}\cdot\nabla\,\psi_-^* - \psi_-^*(\mathbf{j}\nabla)\,\psi_+^*]\,e^{i\mathbf{k}\cdot\mathbf{r}}d\mathbf{r}, \quad (1)$$

where $\mathbf{k}$, ω and $\mathbf{j}$ are the wave vector, the frequency and the polarization of the incoming quantum. The wave functions ψ_+ and ψ_- of the created mesons are the sums of plane and converging waves:

$$\psi_+ = \frac{1}{\sqrt{2E_+}}\Big\{e^{i\mathbf{p}_+\mathbf{r}} \quad (2)$$

$$+\frac{p_+}{2\pi i}\int\frac{\exp\{-ip_+|\mathbf{r}-\boldsymbol{\rho}|\}}{|\mathbf{r}-\boldsymbol{\rho}|}\{1 - \Omega^*(\rho)\}\,d\boldsymbol{\rho}\Big\},$$

where ρ is the radius in a plane orthogonal to the momentum $\mathbf{p}_+$ of the created meson and passing through the center of the nucleus:

$$\Omega(\rho) = \begin{cases} 0 & \rho \leqslant R, \\ e^{2i\eta_+(\rho)}, & \rho > R; \end{cases}$$

$y_+(\rho) \approx n_+ \log p + \rho$ is the Coulomb scattering phase, with $n_+ = ze^2 E_+/p_+$ and E_+ is the energy of the π^+-meson. We break the wave function into three parts: