It should be noted, as can be seen from Eq. (5), $P(\theta)$ depends only on σ_t and $d\sigma/do$. Since calculations made with our choice of parameters are in good agreement with experimental values of σ_t and $d\sigma/do$, our calculated polarization does not depend on a particular model, and should approach the correct value.

In conclusion, I would like to thank Ia. A. Smorodinskii for his interest in this work.

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Parapositronium Annihilation Probability, with Account of the First Radiative Corrections

A. I. ALEKSEEV
Moscow Engineering and Physics Institute

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IT is known that parapositronium in the ground state, as a system of even parity, cannot decay into three photons. Therefore it is interesting to calculate the first radiative corrections to the probability for two photon annihilation,[1] since in the next approximation the lifetime of parapositronium is determined just by these radiative corrections and does not depend on the four photon annihilation.

The positronium annihilation probability W for any order of radiative correction is connected with the annihilation probability $W_{\rm Free}$ of the free particles with zero relative velocity by the equation[2]

$$W = [(\bar{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0))\;/\;(\bar{\psi}^{\mathcal{E}\sigma}_{\rm Free}(0)\,\psi^{\mathcal{E}\sigma}_{\rm Free}(0))]\,W_{\rm Free} \quad (1)$$

where $\psi^{E\sigma}(x)$ is the wave function (in relative coordinates) of the positronium in the ground state, and which satisfies a Bethe-Salpeter equation with the possible annihilation of the particles (Ref. 2; see also Ref. 3). $\psi^{\mathcal{E}\sigma}_{\rm Free}(x)$ is the wave function of the free particles which turns out to have the same implicit set of functions as $\psi^{E\sigma}(x)$ (but with energy $\mathcal{E} = E + \epsilon$, where $\epsilon > 0$ the binding energy). The sign of σ determines the spin state. In Eq. (1), the quantity $[\bar{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)] = \mathrm{Sp}\,[\bar{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)]$ should be calculated with the same accuracy as is obtained in the calculation of $W_{\rm Free}$.

If we limit ourselves to the first radiative corrections, it is not hard to calculate $[\bar{\psi}^{E\sigma}(0)\psi^{E\sigma}(0)]$ with the required accuracy to terms including the order e^2 ($\hbar = c = 1$) using the nonrelativistic approximation to the wave function $\psi^{E\sigma}(0)$. In this process we can drop the small terms of the wave functions in $(\bar{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)]$ since they are of order $v^2_{\rm rel}$ ($\sim e^4$). It remains to find the corrections of the order e^2 to the large components of $\psi^{E\sigma}(x)$. This can be done with the aid of ordinary perturbation theory[4] if we use the second approximation[5] to the Hamiltonian for the large components of the wave function. Simple calculations show that the first correction of the nonrelativistic value $\psi_{\rm NR}(0)$ is of the order e^4, and therefore does not need to be taken into account. To the desired degree of approximation we thus have

$$(\bar{\psi}^{E\sigma}(0)\,\psi^{E\sigma}(0)) = |\,\psi_{\rm NR}(0)\,|^2. \quad (2)$$

The wave function $\psi^{\mathcal{E}\sigma}_{\rm Free}(x_1, x_2)$ of the free particles used in $W_{\rm Free}$ is an implicit function of the full energy and momentum operators, and the full spin (since the orbital angular momentum is zero) and its projections. Since the spin operator does not include the spatial coordinates of the particle, the spatial and spin variables in $\psi^{\mathcal{E}\sigma}_{\rm Free}$ separate, and the wave function has the form:*

$$\psi^{\mathcal{E}\sigma}_{\rm Free}(x_1 x_2) = \Phi^{\sigma}(K)\, e^{-iK(x_1+x_2)/2}, \quad (3)$$

where the main component of the total particle momentum K is $K_0 = \mathcal{E} = 2m$, and $K \to 0$ in the center-of-mass system. Therefore, $[\bar{\psi}^{\mathcal{E}\sigma}_{\rm Free}(0)\,\psi^{\mathcal{E}\sigma}_{\rm Free}(0)]$ in Eq. (1) is a square of the spin function $\Phi^{\sigma}(K)$, which can always be made equal to unity.

The value of W does not change if we sum in Eq. (1) over all four spin states, since $W_{\rm Free} = 0$ in all spin states with a total spin $S = 1$. In $W_{\rm Free}$ (in the summation over all spin states) let us replace the summation over the complete set of spin functions ofthe entire spin operator by a summation over the complete set of spin functions which are themselves implicit spin operators of the spin of each particle individually. Then W_{Free} will be the probability, together with the first radiative corrections, for the two photon annihilation of the free particles with momenta $p_1 = p_2 \to m$. Using the results of Ref. 6, which were calculated for the Compton effect, then W_{Free} is written

$$W_{\text{Free}} = W_0 \left[1 - \frac{e^2}{\pi} \frac{\mathrm{Re}(U^{(1)})}{U}\right]. \quad (4)$$

Here W_0 is the first term of the zeroth approximation, and

$$U(k, \tau) = 4(\tau^{-1} - k^{-1})^2 - 4(\tau^{-1} - k^{-1}) + (k/\tau + \tau/k), \quad (5)$$

$$U^{(1)} = P(-k, \tau) + P(\tau, -k), \quad (6)$$

where

$$P(a, b) = (1 - 2y \operatorname{cth} 2y) \ln\lambda \cdot U(-a, b) - 2y \operatorname{cth} 2y\, [2h(y) - h(2y)\, U](-a, b)$$
$$+ [-4y \operatorname{sh} 2y \cdot (ab)^{-1}(2 - \operatorname{ch} 2y) + 2y \operatorname{cth} y]\, h(y) +$$
$$+ \ln a \left\{4y \operatorname{cth} 2y \left[\frac{4}{ab}\operatorname{ch}^2 y + \frac{a-6}{2b}\frac{1}{\operatorname{ch} 2y} + \frac{4}{a^2} - \frac{1}{a} - \frac{b}{2a} - \frac{a}{b} - 1\right]\right.$$
$$\left. + \frac{3b}{2a^2} + \frac{3b}{2a} + \frac{3}{b} + 1 - \frac{7}{ab} + \frac{8}{a} - \frac{8}{a^2} + \frac{2a - b^2 - a^2 b}{2a^2 b(a-1)} - \frac{1}{2b}\frac{2a^2 + b}{(a-1)^2}\right\}$$
$$+ y^2 \frac{1}{\operatorname{sh}^2 y}\left[\frac{2}{a} - \frac{7}{4}a - \frac{3}{4}\frac{b^2}{a}\right] - 4y \operatorname{th} y \left(\frac{1}{2} - \frac{1}{a}\right) + 4\left(\frac{1}{b} + \frac{1}{a}\right)^2$$
$$- \frac{12}{a} - \frac{3}{2}\frac{a}{b} - 2\frac{a}{b^2} + \frac{1}{a-1}\left(\frac{1}{2} + \frac{a}{b}\right) + G_0(a)\left[\frac{a^2}{b} + \frac{b}{a^2} + \frac{a}{b} + a\right.$$
$$\left. + \frac{1}{2}b + \frac{2}{a} - \frac{3}{b} - 1\right]. \quad (7)$$

With this

$$4 \operatorname{sh}^2 y = -(a+b), \quad h(y) = y^{-1}\int_0^y u \operatorname{cth} u\, du,$$

$$G_0(a) = -2a^{-1}\int_{1-a}^{1} \ln(1-u)\frac{du}{u};$$

$$m^2 k = 2p_1 q_1 = 2p_2 q_2; \quad m^2\tau = 2p_1 q_2 = 2p_2 q_1,$$

where p_1 and p_2 are the electron and positron momenta, and q_1 and q_2 the photon momenta.

The quantity λ, appearing in $U^{(1)}$, represents the photon "mass". For $\lambda \to 0$ the radiative correction to W_0 diverges logarithmically (the infrared catastrophe). We must calculate W_{Free} for a relative particle velocity of zero. As can be seen from Eqs. (5) and (6), the infrared divergence disappears in that case. Actually if $p_1 \to p_2$, then $k \to \tau \to 0$. Therefore, the quantity y and with it the entire coefficient of $\ln\lambda$ in Eq. (6) goes to zero.

We finally obtain (in the center-of-mass system) the following expression for the probability of two photon annihilation of parapositronium with account of the first radiative corrections:

$$W = W_0\left[1 - \frac{e^2}{\hbar c}\left(\frac{4}{3\pi} - \frac{\pi}{8} - \frac{2}{9\pi}\ln 2 - \frac{3}{2\pi}G_0(-2)\right)\right]$$
$$= \frac{1}{2}\left(\frac{e^2}{\hbar c}\right)^5 \frac{mc^2}{\hbar}\left[1 + 0.25\,\frac{e^2}{\hbar c}\right].$$

In conclusion, we thank A. D. Galanin for discussion of the results.

*We use the following summation convention:
$$ab = a_0 b_0 - a_1 b_1 - a_2 b_2 - a_3 b_3.$$

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