Bipolar Diffusion of Current Carriers in the Presence of Deep Traps

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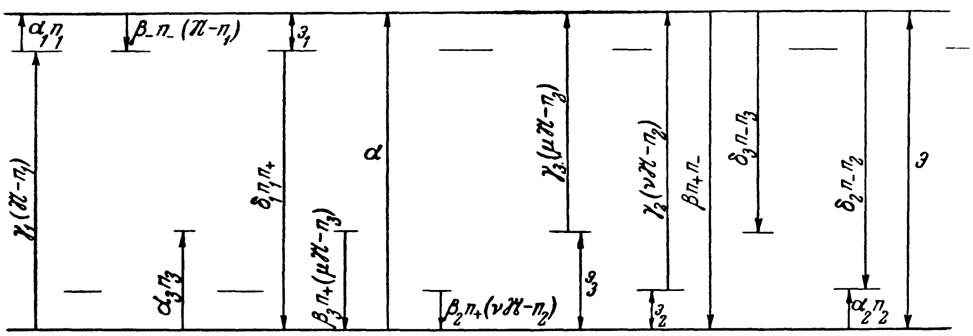
We have investigated bipolar diffusion in the presence of traps, taking into consideration the dependence of minority carrier lifetime on the population of the traps, and have established criteria for the validity of the linear recombination law. An equation was derived for the concentration distribution of minority carriers which obey a nonlinear recombination law

IN papers on the bipolar diffusion of current carriers in semiconductors it is usually assumed that when the concentration of minority carriers p is considerably lower than the equilibrium concentration of majority carriers n_0, a linear recombination law with a definite lifetime τ holds true. However, the linear recombination law can possibly be violated even when $p \ll n_0$. This will occur in the presence of hole-trapping levels which are widely separated from the valence band (for definiteness the semiconductor will hereafter be assumed to be n-type), in which case the holes will have a long lifetime on these levels before recombination. Thus the concentration of trapped holes and consequently, the number of recombinations with electrons will no longer be a linear function of the number of free holes. Lashkarev[1] has found a criterion for the validity of a linear recombination law in this case. However, he did not in the same paper consider bipolar diffusion in the case of nonlinear recombination.



Semiconductor energy scheme: n_1 = concentration of electrons on donors; n_2 and n_3 = concentrations of holes on acceptors and in traps. Arrows indicate the directions of electron transitions. Transition probabilities are indicated alongside the arrows.

Additional holes can be created either by irradiation of the semiconductor or by injection through a p — n junction. Our further discussion will concern radiation—induced "photoholes" without limiting its generality.

The figure shows schematically the location of the usual donor levels *1*, the acceptor levels *2* and the deep traps *3*. Inscriptions alongside the arrows show the number of corresponding transitions per unit time. In Ref. 2 an expression has been derived for the divergence of the electronic current, ej_-, given by the difference between the numbers of excitations and recombinations of conduction electrons, when only acceptor and donor levels are present. A more general expression can be obtained similarly when deep traps are present. Just as in Ref. 2, where it is assumed that there is a relatively rapid exchange between donor levels and the (electron conduction band, and between acceptor levels and the valence band, it can be assumed that the recombination of captured holes with conduction electrons goes through a "bottleneck", but that capture and liberation of holes take place much more rapidly. Then instead of Eq. (8) of Ref. 2, we obtain

$$\operatorname{div} j_- \quad (1)$$

$$= -\mathfrak{N}^2\left[\beta + \frac{\delta_1}{N+a_1} + \frac{\nu\delta_2}{z+a_2} + \frac{\mu\delta_3}{z+a_3}\right](Nz - b),$$

where $\mathfrak{N}$, $\nu\mathfrak{N}$ and $\mu\mathfrak{N}$ are the concentrations of donors, acceptors and deep traps; and $N\mathfrak{N}$ and $z\mathfrak{N}$ are the concentrations of free electrons and traps. The meaning of the recombination coefficients β, δ_1, δ_2 and δ_3 is clear from the Figure. In addition, we have used the notation

$$a_1 = (B_-/\mathfrak{N})\,e^{-\vartheta_1/kT};\quad a_{2,3} = (B_+/\mathfrak{N})\,e^{-\vartheta_{2,3}/kT};$$
$$b = (B_+B_-/\mathfrak{N}^2)\,e^{-\vartheta/kT};\quad B_\pm = 2h^{-3}(2\pi m_\pm kT)^{3/2}, \quad (2)$$

assuming that $\mathfrak{N} << B\pm$ and that ϑ_1 and ϑ_2 are not too large compared with kT, we neglect z and N compared with a_3. When to (1) are added the equations which define the electron current j_- and the hole current j_+ as well as Poisson's equation we obtain a complete set of equations; for the one-dimensional case and with dimensionless quantities, these are

$$dN/d\xi = Ny + \varphi, \quad (3a)$$

$$dz/d\xi = -zy - (\lambda - \varphi)/K, \quad (3b)$$

$$dy/d\xi = N - z - 1 + (\nu) - \mu z/(z+a_3) + (\mu), \quad (3c)$$

$$\frac{d\varphi}{d\xi} = \left[A + \frac{D}{z+a_3}\right](Nz - b), \quad (3d)$$

where

$$\varphi = -j_-/\mathfrak{N}\vartheta_-\varkappa;\ \lambda = I/e\mathfrak{N}\vartheta_-\varkappa;\ y = (e/\varkappa kT)\,dV/dx,$$
$$\varkappa = \sqrt{\frac{4\pi e^2\mathfrak{N}}{\varepsilon kT}}; \quad (4)$$

$$A = \frac{\varepsilon}{4\pi e u_-}\left[\beta + \frac{\delta_1}{a_1} + \frac{\delta_2}{a_2}\right];\ D = \frac{\varepsilon\mu\delta_3}{4\pi e u_-}.$$

Here I is the current density, ϑ_- and u_- are the diffusion coefficient and mobility for electrons, V is the electric potential, $\xi = \varkappa x$ is a dimensionless coordinate, ϵ is the dielectric constant and $K = u_+/u_-$. The quantities ν and μ in brackets may be omitted when acceptors and hole traps containing electrons are neutral (we consider ionized donors to be positively charged).

When we confine ourselves to illumination under which the concentration of photocurrent carriers is considerably smaller than the concentration of dark carriers, the case under consideration of possible saturation of traps by photoholes can be realized in practice, clearly, only in an impurity semiconductor ($z_{\text{dark}} << N_{\text{dark}}$). Therefore we shall assume that everywhere $b << 1$ and $z << 1$.

In accordance with the usual experimental arrangement, when there is no current through the specimen (subject to a photoelectromotive force) $\lambda = 0$, while φ and z can be considered small first order quantities. Since in a large part of its volume the specimen can be considered almost exactly neutral, it follows from Eq. (3c) that

$$N = 1 + z - (\nu) - (\mu) + \mu z/(z+a_3). \quad (5)$$

Eliminating N and $dz/d\xi$ from (3a), (3b) and (5) we obtain the field y which we substitute in (2b):

$$\frac{dz}{d\xi} = \left[1 - \frac{\mu a_3 z(z+a_3)^{-2} + (1-K)z}{1 + 2z - (\nu) - (\mu) + \mu z(z+a_3)^{-1} + \mu a_3 z(z+a_3)^{-2}}\right] \times \frac{\varphi}{K}. \quad (6)$$

Since $za_3/(z+a_3)^2 \leq 1/4$, when $\mu < 1 - (\mu) - (\nu)$, the fraction within the square brackets can be omitted as an approximation by comparison with $1 - (\nu) - (\mu)$.
The above inequality denotes that the trap concentration is smaller than the conduction electron for $z = 0$, as can be seen from (5). Following the indicated simplification we obtain instead of (6)

$$dz/d\xi = \varphi/K. \quad (7)$$

Eliminating the electronic current φ from (7) and (3d), we have

$$\frac{d^2z}{d\xi^2} = \frac{1}{K}\left(A + \frac{D}{z+a_3}\right) \times \left[z\left(1 - (\nu) - (\mu) + z + \frac{\mu z}{z+a_3}\right) - b\right]. \quad (8)$$

We determine the dark concentration of holes $\bar{z}$ from the condition that the right-hand side of Eq. (8) vanishes and we set $z = \bar{z} + \zeta$. For ζ we obtain the equation

$$\frac{d^2\zeta}{d\xi^2} = \frac{1}{K}\left(A + \frac{D}{\bar{z} + a_3 + \zeta}\right)\left(1 - (\nu) - (\mu) + 2\bar{z} + \zeta + \frac{\mu \bar{z} a_3}{(\bar{z} + a_3)(\bar{z} + a_3 + \zeta)} + \frac{\mu(\bar{z} + \zeta)}{\bar{z} + a_3 + \zeta}\right)\zeta, \quad (9)$$

where $\bar{z}$ is defined by the relation

$$\bar{z}\left(1 - (\nu) - (\mu) + \bar{z} + \frac{\mu \bar{z}}{\bar{z} + a_3}\right) = b. \quad (10)$$

The next-to-last term in the second bracket of (9) can be omitted by comparison with $1 - (\nu) - (\mu)$, just as was done in Eq. (6). Following such simplification the equation can be reduced to quadrature. With the boundary conditions $\zeta = \zeta = 0$ for $\xi = \infty$ and $\varphi = \varphi_0$ for $\xi = 0$, the solution is

$$\xi = -\int_{\zeta_0}^{\zeta} d\zeta / \sqrt{F(\zeta)}, \quad (11)$$

$$F(\zeta) = \beta^2\zeta^2 + 2\beta^2\gamma\zeta + 2\left[\beta^2\gamma a_3' + \frac{\mu D}{K}(\bar{z} - 3a_3')\right]\ln\left(\frac{a_3' + \zeta}{a_3'}\right) + 2\frac{\mu D}{K}\frac{(a_3' - \bar{z})}{a_3' + \zeta}\zeta + \frac{2A}{3K}\zeta^3. \quad (12)$$

Here the following notation has been introduced:

$$a_3' = a_3 + \bar{z};\ \beta = [(A/K)(1 - (\nu) - (\mu) + \mu + 2\bar{z}) + D/K]^{1/2};$$
$$\gamma = (1/K\beta^2)[A\mu(\bar{z} - a_3') + D(1 - (\nu) - (\mu) + \mu - \mu_3' + 2\bar{z})]. \quad (13)$$

The constant of integration ζ_0 is determined from Eqs. (7) and (11) for the point $\xi = 0$ in terms of the hole current density $-\varphi_0$ which enters an unilluminated semiconducting region from the illuminated region $\xi < 0$. In particular, when illumination is produced by a narrow luminous probe whose width is considerably smaller than the diffusion length, $-\varphi_0$ is eqal to half the number of holes created by light in unit time in a unit area which is perpendicular to the ξ axis. This condition is expressed by

$$\sqrt{F(\zeta_0)} = -\varphi_0/K. \quad (14)$$

The integral in (11) can be calculated in the two limiting cases.

1. The case of small photohole densities ($\zeta \ll a_3'$). Then, expanding $F(\zeta)$ in powers of ζ / a_3' and stopping at the quadratic terms, we obtain after integration

$$\zeta = Ce^{-\alpha\xi},\ \alpha = \left[\frac{1}{K}\left(A + \frac{D}{a_3'}\right)\left(1 - (\nu) - (\mu) + 2\bar{z} + \frac{\mu\bar{z}}{a_3'}\right)\right]^{1/2}; \quad (15)$$

$1/\alpha$ is the dimensionless diffusion length.

2. The case of high photohole densities ($\zeta \gg a_3'$). In this case only the first two terms of Eq. (12) can be retained. Integration then gives

$$\zeta = \gamma\left[\left(\frac{\zeta_0}{\gamma} + 1\right)\operatorname{ch}\beta\xi - \sqrt{\frac{\zeta_0^2}{\gamma^2} + 2\frac{\zeta_0}{\gamma}}\operatorname{sh}\beta\xi - 1\right]. \quad (16)$$

According to (14)

$$\beta\sqrt{\zeta_0^2 + 2\gamma\zeta_0} = -\varphi_0/K. \quad (17)$$

When $\zeta_0 \gg a_3'$ it is possible to use (16) approximately in the region $\zeta > a_3'$ and (15) for $\zeta < a_3'$, and to determine the constant C in (15) from the continuity of ζ at the point $\zeta = a_3'$, where both criteria are in agreement. When $\zeta_0 \ll a_3'$ Eq. (15) is valid everywhere and

$$C = \zeta_0 = -\varphi_0/K\alpha. \quad (18)$$

As can be seen from (16), ζ is a diminishing function of ξ as far as the point $\xi_{\lim}$ which is defined by the equation

$$\operatorname{ch}\beta\xi_{\lim} = (\zeta_0/\gamma) + 1; \quad (19)$$

at this point ζ and $d\zeta / d\xi$ vanish simultaneously. The solution has physical meaning only in this case and can be connected continuously with the solution in (15).

From a comparison of (15) and (16) it can be seen that for $\zeta < a'_3$ the concentration decreases exponentially with the exponent $\alpha\xi$, while for $\zeta < a'_3$ it decreases more slowly whan an exponential with the exponent $\beta\xi$. At the same time we have from the definitions of α and β:

$$\alpha^2 = \beta^2 + \frac{D}{Ka'_3}\left[1 - (\nu) - (\mu) + 2\bar{z} + \frac{\mu\bar{z}}{a'_3} - a'_3\right] - \frac{\mu A}{K}\frac{a'_3 - \bar{z}}{a'_3}. \quad (20)$$

When a'_3 is small α can be very much larger than β. Thus, when $a'_3 \ll 1$, in the region $a'_3 < \zeta \ll 1$, which is usually considered linear, the linear recombination law can be violated seriously and the mean hole lifetime can increase. This must evidently occur frequently in semiconductors. Thus, for example, for $\mathfrak{N} = 10^{16}$ cm^{-3}, $\mathcal{J}_3 = 0.3$ ev and at room temperature, $a_3 = 0.014\,(m_+/m_0)^{3/2}$.

Consequently, in investigations of photoconduction and the photoelectromotive force even in the "linear region", where the concentration of light-induced carriers comprises only a small percentage of the concentration of dark carriers, the condition $\zeta \ll a'_3$ can be violated, at least near the illuminated region. To fulfill the linear recombination law throughout the crystal it follows from (15) and (18) that the following condition must be satisfied:

$$\frac{J\gamma}{2\hbar\omega}\frac{l}{d}\frac{L_+}{\vartheta_+} \ll B_+\left(e^{-\mathcal{J}_+/kT} + \frac{B_-}{\mathfrak{N}}e^{-\mathcal{J}/kT}\right); \quad (21)$$

where J and ω are the intensity and frequency of the incident light, γ is the quantum yield of the photoconductive effect, l is the width of the illuminated region along the x-axis and d is the thickness of the specimen.

When the trapping levels *3* are located close to the middle of the forbidden zone and the probability of hole capture is of the same order as the probability of recombination between a captured hole and an electron, instead of the last term in the square bracket of Eq. (1), a calculation gives

$$\frac{\mu\delta_3}{(b_3 + N)(\delta_3/\beta_3) + a_3 + z}, \quad (22)$$

where

$$b_3 = (B_-/\mathfrak{N})\,e^{-(\mathcal{J}-\mathcal{J}_+)/kT}. \quad (23)$$

Since $N \sim 1$, for $\delta_3 \sim \beta_3$ it is always possible to neglect z in the denominator within the linear region ($z \ll 1$), and there will be no effects associated with trap saturation.

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[1] V. E. Lashkarov, Izv. Akad. Nauk SSSR, Ser. Fiz. 16, 186 (1952); K. B. Tolpygo and I. G. Zaslavskaia, J. Tech. Phys. (U.S.S.R.) 25, 955 (1955).

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