

Statistics of Electrons and Holes in a Homopolar Semiconductor Taking into Account their Interaction with the Lattice Vibrations

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The interaction between electrons and phonons in a homopolar semiconductor is taken into account in the examination of the stationary electron distribution, and is not considered as being merely the cause of transitions between stationary states of "free" electrons and phonons. The limiting cases of high and of low temperatures are investigated. It is shown that because of the interaction between electrons and phonons a perfectly pure semiconductor has a conductivity different from zero at the temperature of absolute zero.

IN the existing theory of homopolar semiconductors the interaction of the current carriers with the vibrations of the crystalline lattice is taken into account only in the consideration of non-equilibrium processes: it is considered that it only induces transitions between the stationary states of electrons in a rigid lattice, determining for example Joule heat losses, etc. However, such a procedure is not entirely logical, since actually the interaction with the lattice vibrations always takes place; consequently, in taking it into account as the cause which induces the transitions we—in the same approximation—destroy the consistency of the concept of the states between which these transitions take place. The above means that strictly speaking the interaction between electrons and phonons should be taken into account even in the determination of the energy spectrum and of the density matrix for the electrons in the lattice, i.e., in the solution of the equilibrium problem. It should be emphasized that the necessity to do so arises logically not only in the case of strong coupling between the electrons and the phonons,^{1,2} but also in the case of weak coupling which may be taken into account by means of some form of perturbation theory. As will be seen below, in this last case the interaction of electrons and holes with the lattice vibrations leads to changes in the density matrix which are not large quantitatively, but which are (at low temperatures) of considerable significance in principle.

Unfortunately the formulation of the problem in a general form suitable for arbitrary temperatures is very complicated. Therefore we shall restrict ourselves here only to the limiting cases of high and of low temperatures (strictly speaking in the second case we will be concerned with a semiconductor at absolute zero). We shall consider a semiconductor having intrinsic conductivity

and with spherical surfaces of constant energy; we will take into account only the interaction with the acoustic lattice vibrations and, for the sake of simplicity, we shall take the coupling constant to be the same both for the conduction electrons and for the holes. Naturally such a model is not too suitable for exact calculations (in particular the zero point optical vibrations should, without any doubt, be taken into account); however, the principle of the fundamental result which we shall obtain will not be affected by these approximations.

Under the above approximations there exists no interaction with transverse waves and the Lagrangian for the interaction between the electrons and the longitudinal acoustic lattice vibrations has the form³ (we use the interaction representation and we set $\hbar = c=1$, where c is the velocity of sound):

$$L_{\text{int}} = \int L(x) dx \quad (1)$$

$$L(x) = \left\{ g \sum_s \bar{\psi}_s(x) \psi_s(x) + \rho(x) \right\} \varphi; \quad (2)$$

$$\varphi = \partial A / \partial x_0.$$

Here $\psi(x)$ and $A(x)$ are the quantized wave functions of the "noninteracting" electron and phonon fields, $\rho(x)$ is the "external charge density", $x = \{x, x_0\}$, x_0 is the time, s is the spin index, g is the coupling constant (which in the system of units adopted has the dimensions of length). In the usual units g has the form:

$$g = \frac{2}{3} \pi^{1/2} C c^{-2} \sqrt{V_0/M}, \quad (3)$$

where V is the volume of the elementary cell, M is the mass of the atom of the lattice, C is a constant which occurs in the theory of electrical

conductivity.⁴

Inasmuch as all the operators in which we will be interested will turn out to be diagonal in the spin indices the latter will not be explicitly indicated in what follows.

2. SEMICONDUCTOR AT ABSOLUTE ZERO

The "single particle" density matrix for the electrons and holes at absolute zero can be found most simply by calculating Green's function for the electron field interacting with phonons. The appropriate methods were developed in an earlier paper of the author.⁵ As was shown there,⁵ the Green's function for the electrons

$$G(x, y) = \frac{i}{\langle S \rangle_0} \langle T \{ \psi(x) \bar{\psi}(y) S \} \rangle_0$$

is determined by the equation

$$\begin{aligned} G(x, y) &= iK(x, y) & (4) \\ &- ig \int dz K(x, z) G(z, y) a(z) \\ &- i \int dz dx' K(x, z) \Delta E(z, x') G(x', y), \end{aligned}$$

where

$$\Delta E(x, z') \quad (5)$$

$$\begin{aligned} &= ig \int dz' dz'' \frac{\delta a(z')}{\delta \rho(z)} G(z, z'') \frac{\delta G^{-1}(z'', z')}{\delta a(z')}, \\ a(x) &= -i \delta \ln \langle S \rangle_0 / \delta \rho(x), \quad (6) \end{aligned}$$

in which S is the S -matrix, T is the symbol representing a T -product, $\langle \dots \rangle_0$ is an average over the "ground state in the absence of interaction."

The expansion of ΔE in a series in powers of the coupling constant leads in the second order in g to the following linear equation for the Green's function [for $\rho(x)=0$]:

$$\begin{aligned} G(x, y) &= iK(x, y) & (7) \\ &- g^2 \int dz dz' K(x, z) F(z', z) K(z', z) G(z', y), \\ K(x, y) &= \langle T \{ \psi(x) \bar{\psi}(y) \} \rangle_0, \\ F(x, y) &= \langle T \{ \varphi(x) \varphi(y) \} \rangle_0. \end{aligned}$$

In the present case we have

$$\begin{aligned} F(x, y) & & (8) \\ &= \frac{1}{2\pi^3} \int df f \exp \{ i(f, x - y) - if | x_0 - y_0 | \} \end{aligned}$$

(f is the quasimomentum of the phonon, $f \leq f_0$, where f_0 is the Debye upper limit);

$$\psi(x) = (2\pi)^{-3/2} \quad (9)$$

$$\begin{aligned} &\times \int d\mathbf{p} \{ \exp \{ i\mathbf{p}\mathbf{x} - iW_c(\mathbf{p}) x_0 \} a(\mathbf{p}) \\ &+ b^*(\mathbf{p}) \exp \{ i\mathbf{p}\mathbf{x} - iW_v(\mathbf{p}) x_0 \} \}, \end{aligned}$$

a^* , a , b^* , b are the Fermi operators for the creation and annihilation of conduction electrons and of holes in the valence band;

$$W_c(\mathbf{p}) = W_0 + \mathbf{p}^2 / 2m^*, \quad (10)$$

$$W_v(\mathbf{p}) = -\mathbf{p}^2 / 2m^{**},$$

W_0 is the width of the forbidden band (calculated in the absence of interaction with phonons), m^* and m^{**} are respectively the effective masses of the conduction electron and of the hole in the valence band (in a rigid lattice).

Noting that

$$\langle a^*(\mathbf{p}) a(\mathbf{p}) \rangle_0 = \langle b^*(\mathbf{p}) b(\mathbf{p}) \rangle_0 = 0, \quad (11)$$

we obtain

$$\begin{aligned} K(x, y) &= \lim_{\varepsilon \rightarrow 0} \frac{i}{(2\pi)^4} \int d\mathbf{p} e^{i(\mathbf{p}, \mathbf{x} - y)} & (12) \\ &\times \int_{-\infty}^{+\infty} d\rho_0 e^{-i\rho_0(x_0 - y_0)} K_\varepsilon(\mathbf{p}, \rho_0); \end{aligned}$$

$$\begin{aligned} K_\varepsilon(\mathbf{p}, \rho_0) &= (2\rho_0 - W_c - W_v) / (\rho_0 - W_c + i\varepsilon) \\ &\times (\rho_0 - W_v - i\varepsilon). \end{aligned}$$

Equation (7) can be solved easily. Setting

$$G(x, y) = \lim_{\varepsilon \rightarrow 0} \frac{i}{(2\pi)^4} \int d\mathbf{p} d\rho_0 \quad (13)$$

$$\times \exp \{ i(\mathbf{p}, \mathbf{x} - y) - i\rho_0(x_0 - y_0) \} G_\varepsilon(\mathbf{p}, \rho_0),$$

we obtain

$$G_\epsilon(\mathbf{p}, p_0) = \frac{i(2p_0 - W_c - W_v)}{(p_0 - W_c + i\epsilon)(p_0 - W_v - i\epsilon) + ig^2(2p_0 - W_c - W_v)f(\mathbf{p}, p_0)}, \quad (14)$$

$$\begin{aligned} f(\mathbf{p}, p_0) &= \int F(-x)K(x)e^{-ipx+ip_0x} dx dx_0 \\ &= \frac{im^*}{\pi^2 p} \left\{ -\frac{16}{3}(1+\gamma^2)m^*f_0p - \frac{2}{3}f_0^2p(1+\gamma) \right. \\ &\quad + \frac{f_0^3}{3} \ln \left| \frac{(f_0-p-m^*)^2-a^2}{(f_0+p-m^*)^2-b^2} \right| + \gamma \frac{f_0^3}{3} \ln \left| \frac{(f_0-p-m^{**})-c^2}{(f_0+p-m^{**})-d^2} \right| \\ &\quad + a[a^2+(p+m^*)^2] \ln \left| \frac{f_0-p-m^*+a}{f_0-p-m^*-a} \frac{p+m^*+a}{p+m^*-a} \right| \\ &\quad + \gamma c[c^2+(p+m^{**})^2] \ln \left| \frac{f_0-p-m^{**}+c}{f_0-p-m^{**}-c} \frac{p+m^{**}+c}{p+m^{**}-c} \right| \\ &\quad + \left[\frac{(p+m^*)^3}{3} - a^2(p+m^*) \right] \ln \left| \frac{(f_0-p-m^*)^2-a^2}{(p+m^*)^2-a^2} \right| \\ &\quad + \gamma \left[\frac{(p+m^{**})^3}{3} - c^2(p+m^{**}) \right] \ln \left| \frac{(f_0-p-m^{**})^2-c^2}{(p+m^{**})^2-c^2} \right| \\ &\quad - \left[\frac{(-p+m^*)^3}{3} - b^2(-p+m^*) \right] \ln \left| \frac{(f_0+p-m^*)^2-b^2}{(-p+m^*)-b^2} \right| \\ &\quad - \gamma \left[\frac{(-p+m^{**})^3}{3} - c^2(-p+m^{**}) \right] \ln \left| \frac{(f_0+p-m^{**})^2-d^2}{(-p+m^{**})^2-d^2} \right| \\ &\quad - b[b^2+(-p+m^*)^2] \ln \left| \frac{f_0+p-m^*+b}{f_0+p-m^*-b} \frac{p-m^*-b}{p-m^*+b} \right| \\ &\quad \left. - \gamma d[d^2+(-p+m^{**})^2] \ln \left| \frac{f_0+p-m^{**}+d}{f_0+p-m^{**}-d} \frac{p-m^{**}-d}{p-m^{**}+d} \right| \right\}. \end{aligned} \quad (15)$$

To reduce the length of the above expression, the following notation has been introduced

$$\begin{aligned} a^2 &= 2m^* \left(p_0 + p + \frac{m^*}{2} - W_0 + i\epsilon \right); \quad (16) \\ b^2 &= 2m^* \left(p_0 - p + \frac{m^*}{2} - W_0 + i\epsilon \right); \\ c^2 &= 2m^{**} \left(p_0 + p + \frac{m^{**}}{2} - i\epsilon \right); \\ d^2 &= 2m^{**} \left(p_0 - p + \frac{m^{**}}{2} - i\epsilon \right); \\ \gamma &= m^{**}/m^*; \quad p = |\mathbf{p}|. \end{aligned}$$

The "single electron" and the "single hole" density matrices are obtained from (13) by means of formula (2.5) of the author's earlier paper.⁵

We note that in order of magnitude we have (if m^* , m^{**} are of the order of an electron mass, and the width of the forbidden band is approximately 1 eV): $f_0 \sim 10^8 \text{ cm}^{-1}$; m^* , $m^{**} \sim 10^5 \text{ cm}^{-1}$; $W_0 \sim 10^{10} \text{ cm}^{-1}$. Therefore, approximately,⁺⁺

⁺⁺Since this expression diverges as $f_0 \rightarrow \infty$, it is clear that high frequency phonons play the principal role, and that therefore all our results have only a highly qualitative meaning.

$$f(\mathbf{p}, p_0) \approx -2im^*f_0^2(1+\gamma)/\pi^2. \quad (17)$$

The integral over p_0 in (13) can easily be evaluated, and we obtain for the "single electron" density matrix in the p -representation the expression

$$\begin{aligned} \rho(\mathbf{p}, \mathbf{p}') &= \delta(\mathbf{p} - \mathbf{p}') 2\beta^2 \left\{ \left[p^2 \frac{m^* + m^{**}}{2m^*m^{**}} + W_0 \right]^2 + 4\beta^4 \right\}^{-1/2}; \\ \beta^2 &= 2\pi^{-2}g^2m^*f_0^2(\gamma + 1). \end{aligned} \quad (18)$$

The poles of the function (14), W_1 and W_2 , determine (for $\epsilon \rightarrow 0$) the spectrum of the elementary excitations of the system. We have

$$\begin{aligned} W_1 &= 1/2 \{ W_c + W_v - 2\beta^2 \\ &\quad + \sqrt{(W_c - W_v)^2 + 4\beta^4} \}, \\ W_2 &= 1/2 \{ W_c + W_v - 2\beta^2 \\ &\quad - \sqrt{(W_c - W_v)^2 + 4\beta^4} \}, \end{aligned} \quad (20)$$

As in the unperturbed problem we obtain here two allowed bands separated by a forbidden region of width

$$\Delta W = \sqrt{W_0^2 + 4\beta^4}. \quad (21)$$

Thus, as should be expected, the excitation spectrum has a typical "semiconductor" form. This to a certain extent justifies the qualitative conclusions of the band theory (as applied to the present case). However, it is essential to note that, as may be seen from (18), due to the interaction between electrons and phonons the density matrix does not have a "stepwise" character. As is well known,⁶ this suffices to produce the result that an arbitrarily small external electric field will give rise to a current. We emphasize that we are now discussing the ground state, and that consequently by definition the current carriers in the present case are not the elementary excitations which might arise by thermal means in an equilibrium system. (The latter naturally might also have served as current carriers, but at absolute zero they are absent.)

One may without difficulty compute the "effective concentration" n^* of the current carriers in the ground state of the semiconductor under consideration. In our approximation

$$n^* \approx (2/3\pi^2) p_m^3 (1 + W_0^2/4\beta^4)^{-1/2}, \quad (22)$$

where p_m is a certain maximum value of the absolute magnitude of the quasimomentum of the electron.

A precise evaluation of n^* is difficult, as it is very difficult to determine p_m . It is only clear that under the ordinary experimental conditions the quantity n^* is small compared to the density of the excitation of the semiconductor type and does not manifest itself in any noticeable way. However, at sufficiently low temperatures, when the conductivity determined by current carrying excitations (and which apparently is the only type that has been observed until now) practically disappears, the resistance of the semiconductor should not increase without limit: because of the presence of current carriers in the ground state the conductivity of the semiconductor must remain finite (though small). In other words, because of the interaction between the electrons and the zero point vibrations of the lattice the semiconductor becomes "metallized" at low temperatures. We emphasize that this refers to any arbitrary semiconductor, and is not connected

with the presence of impurities. Unfortunately we do not know of any experimental work the results of which might be unambiguously interpreted along the lines of the arguments presented above. In our opinion the carrying out of appropriate investigations might be of definite interest, although apparently this would not be a simple matter (single crystals should be used which do not have any impurities in order to avoid effects connected with "impurity bands" ⁷).

3. HOMOPOLAR SEMICONDUCTOR AT HIGH TEMPERATURES

At high temperatures, taking into account the interaction between the electrons and the phonons in a consistent way does not lead to such fundamentally significant results as at low temperatures: however, it is apparently necessary to take it into account for the investigation of kinetic processes in semiconductors without the use of the kinetic equation. Indeed, it is well known (see, for example Refs. 8,9) that the criterion for the applicability of the kinetic equation is fulfilled rather poorly in semiconductors. If we desire to construct a theory without making use of the kinetic equation it seems to us that we should first of all investigate the equilibrium properties of the system of conduction electrons (and holes) interacting with phonons. This section is devoted to the above topic.

Following Landau and Lifshitz¹⁰ we shall say that those temperatures are high for which kT appreciably exceeds the average energy of the interaction between electrons and phonons (evaluated per electron). In this case in order to investigate equilibrium systems of electrons interacting with lattice vibrations it is natural to use thermodynamic perturbation theory¹⁰ with the one difference that in this case it turns out to be more convenient to compute not the free energy F , but the thermodynamic potential $\Omega = F - \mu N$, where μ is the chemical potential and N is the average number of particles in the system. The expression for Ω (including terms up to second order) is obtained in the same way as the corresponding formula for F (see Ref. 10, Sec. 32):

$$\Omega = \Omega_0 + \frac{1}{2} \sum_{n, m} \frac{(n | H_{\text{int}} | m)^2}{E_n^{(0)} - E_m^{(0)}} \left\{ \exp \frac{\Omega_0 - E_n^{(0)}}{kT} - \exp \frac{\Omega_0 - E_m^{(0)}}{kT} \right\} + \frac{\bar{H}_{\text{int}}^2}{2kT}. \quad (23)$$

Here Ω_0 is the thermodynamic potential of

the system of "free" electrons and phonons; n, m are the sets of indices (in this case the occupation numbers of the electron and phonon states) which characterize the states of the unperturbed system with energies $E_n^{(0)}$ and $E_m^{(0)}$.

The Hamiltonian of the interaction H_{int} is obtained from (1) and (2), it being sufficient in the present case to consider separately only the conduction electrons or the holes.

Setting

$$\psi(x) = (2\pi)^{-3/2} \quad (24)$$

$$\times \int \exp\{ipx - iW_c(p)x_0\} a(p) dp,$$

we obtain

$$\Delta\Omega_1 = \frac{g^2}{4\pi^3} \int dp df \cdot f \frac{1 - \exp \frac{W_c(p+f) - W_c(p) + f}{kT}}{W_c(p+f) - W_c(p) + f} \frac{\exp \frac{W_c(p) - \mu}{kT}}{F(p, f)}, \quad (27)$$

$$\Delta\Omega_2 = \frac{g^2}{4\pi^3} \int dp df \cdot f \frac{1 - \exp \frac{W_c(p+f) - W_c(p) - f}{kT}}{W_c(p+f) - W_c(p) - f} \frac{\exp \frac{W_c(p) - \mu + f}{kT}}{F(p, f)}. \quad (28)$$

Here the function $F(p, f)$ is determined by the relation

$$F(p, f) = \left[\exp \frac{W_c(p+f) - \mu}{kT} + 1 \right] \quad (29)$$

$$\times \left[\exp \frac{W_c(p) - \mu}{kT} + 1 \right] [e^{f/kT} - 1].$$

A similar result is naturally obtained also for the phonon-hole gas.

Since $\Delta\Omega_1$ and $\Delta\Omega_2$ are obtained in the given approximation by adding together quantities corresponding to various electron states it is possible to introduce the thermodynamic potential $\Omega(p)$ for electrons having a momentum p and a momentum distribution function for the electrons $\Phi_c(p)$,

$$\Phi_c(p) = -\partial\Omega(p)/\partial\mu. \quad (30)$$

From (26)–(29) we obtain

$$H_{\text{int}} = ig (2\pi^3)^{-1/2} \quad (25)$$

$$\times \int dp df f^{1/2} [a^*(p) a(p+f) \xi^*(f) \exp\{-i[W_c(p) - W_c(p+f) - f]x_0\} - a^*(p) a(p-f) \xi(f) \exp\{-i[W_c(p) - W_c(p-f) + f]x_0\}],$$

where $f = |f|$, $\xi^*(f)$ and $\xi(f)$ are the Bose operators for creation and annihilation of a phonon with the quasimomentum f . The matrix elements (25) are well known, and carrying out the summation we obtain (evaluating Ω per unit volume):

$$\Omega = \Omega_0 + \Delta\Omega_1 + \Delta\Omega_2, \quad (26)$$

$$\Phi_c(p) = \left[\exp \frac{W_c(p) - \mu}{kT} + 1 \right]^{-1} \quad (31)$$

$$+ \frac{g^2}{4\pi^3} \frac{\partial}{\partial(-\mu)}$$

$$\times \int df \cdot f F^{-1}(p, f) \left\{ \frac{1 - \exp \frac{W_c(p+f) - W_c(p) + f}{kT}}{W_c(p+f) - W_c(p) + f} + \frac{1 - \exp \frac{W_c(p+f) - W_c(p) - f}{kT}}{W_c(p+f) - W_c(p) - f} \right\}$$

$$\equiv \Phi_c^{(0)}(p) + \Delta\Phi(p).$$

In the general case the evaluation of the integral leads to quite awkward expressions. If we restrict ourselves to only the nondegenerate case we may write in place of (29)

$$F^{-1}(p, f) \quad (32)$$

$$\approx \frac{\exp\{[2\mu - W_c(p+f) - W_c(p)]/kT\}}{\exp(f/kT) - 1}$$

and integrate the resulting expression approximately, having in mind that because of the appearance

of the factor $\exp \{ -f^2 / 2m^* kT \}$ only small values of f are of importance. We then obtain:

$$\Delta\Phi(p) = 8g^2m^*f_0 \exp \frac{\mu - W_c(p)}{kT} \left\{ \frac{3}{2} e^{-z_0^2} \operatorname{ch} \alpha z_0 \right. \quad (33)$$

$$- \left(\frac{5}{2} + z_0^2 + \frac{3}{4} \alpha^2 \right) e^{-z_0^2} \frac{\operatorname{sh} \alpha z_0}{\alpha z_0}$$

$$- e^{-\alpha^2/4} \frac{V\pi}{8} \left(1 + \frac{3}{2} \alpha^2 \right) \left[\Phi \left(z_0 - \frac{\alpha}{2} \right) \right. \\ \left. + \Phi \left(z_0 + \frac{\alpha}{2} \right) \right] \left. \right\},$$

where

$$\Phi(x) = \frac{2}{\pi} \int_0^x e^{-t^2} dt; \quad (34)$$

$$z_0 = f_0 / \sqrt{2m^*kT}; \quad \alpha = 2p / \sqrt{2m^*kT}.$$

In order of magnitude (at room temperature) $z_0 \sim 20$, and $\alpha \sim 1$ (for the most interesting values of p). Therefore (33) takes on the approximate form:

$$\Delta\Phi(p) \approx -2 \sqrt{\pi} g^2 m^* f_0 \left(1 + \frac{3p^2}{m^*kT} \right) \quad (35)$$

$$\times \exp \frac{\mu - W_0 - p^2 / 2m^*}{kT} \Phi(z_0).$$

Remembering that the principal term in the distribution function is in this case equal to $\exp \{ [\mu - W_c(p)] / kT \}$, we find the electron density in the conduction band (whose statistical weight is $2G$);

$$N_c \approx 2G (2\pi)^{-3} (2\pi m^* kT)^{3/2} e^{(\mu - W_0) / kT} \quad (36)$$

$$\times \{ 1 - 6.9 g^2 m^* f_0 \Phi(z_0) \}.$$

Thus, in the given approximation the influence of the phonon-electron interaction on the position of the level of the chemical potential reduces simply to a "renormalization" of the effective mass:

$$m^* \rightarrow m_r^* = m^* [1 - 6.9 g^2 m^* f_0 \Phi(z_0)]^{1/2}. \quad (37)$$

The latter therefore becomes dependent on the temperature, increasing with increasing temperature. However, we wish to emphasize that for the calculation of various statistical quantities which characterize the semiconductor (for example the magnetic susceptibility, etc.) the "renormalization" alone is not sufficient— one should also use the improved distribution function.

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¹⁰L. D. Landau and E. M. Lifshitz, *Statistical Physics*, Gostekhizdat, 1951.

Translated by G. M. Volkoff

Other Errata

Page	Column	Line	Reads	Should Read
Volume 4				
38	1	Eq. (3)	$\dots \frac{\pi r^2 \rho^2 \rho_n^2}{\rho_s^2},$	$\dots \frac{\pi r^2 \rho^2 \rho_n}{\rho_s^2},$
196		Date of submittal	May 7, 1956	May 7, 1955
377	1	Caption for Fig. 1	$\delta_{35} = \eta - 21 \cdot \eta^5$	$\delta_{35} = -21^2 \eta^5.$
377	2	Caption for Fig. 2	$\alpha_3 = 6.3^\circ \eta$	$\alpha_3 = -6.3^\circ \eta$
516	1	Eq. (29)	$s^2/c^2 \dots$	s/c
516	2	Eqs. (31) and (32)	Replace $A_1 s^2/c^2$ by A_1	
497		Date of submittal	July 26, 1956	July 26, 1955
900	1	Eq. (7)	$\dots \frac{i}{4\pi} \sum_{c, \alpha} \frac{\partial w_a(t, P)}{P^\alpha} \dots$	$\dots 2\pi^2 i \sum_{c, \alpha} \dots$
			(This causes a corresponding change in the numerical coefficients in the expressions that result from the calculation of the effects of the plasma particles on each other).	
804	2	Eq. (1)	$\dots \exp \{-(\bar{T} - V')\}$	$\dots \exp \{-(\bar{T} - V')\tau^{-1}\}$

Volume 5

59	1	Eq. (6)	$v_l (l \partial F_0 / \partial x) + \dots$ where E_l is the projection of the electric field E on the direction l	$\overline{(v \partial F_0 / \partial x)} + \dots$ where the bar indicates averaging over the angle θ and E_l is the projection of the electric field E along the direction l
91	2	Eq. (26)	$\Lambda = 0.84 (1 + 22/A)$	$\Lambda = 0.84 / (1 + 22/A)$
253		First line of summary	$T_1^{204, 206}$	$T_1^{203, 205}$
318	1	Figure caption	$\dots e^2 mc^2 = 2.8 \cdot 10^{-23} \text{ cm},$	$\dots e^2 / mc^2 = 2.8 \cdot 10^{-13} \text{ cm},$
398		Figure caption	\dots to a cubic relation. A series of points etc.	\dots to a cubic relation, and in the region 10–20°K to a quadratic relation. A series of points ●, coinciding with points ○, have been omitted in the region above 10°K.