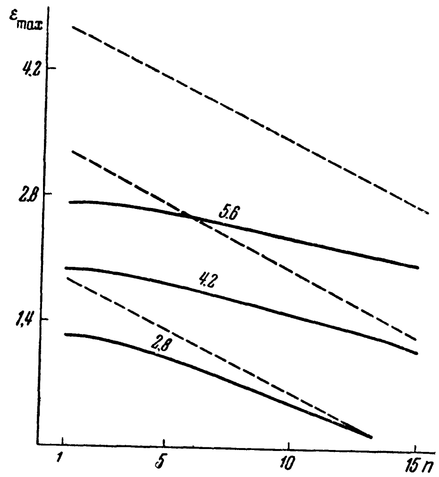

FIG. 1. Maximum energy of a π — meson formed as a result of a $\pi - N$ collision.

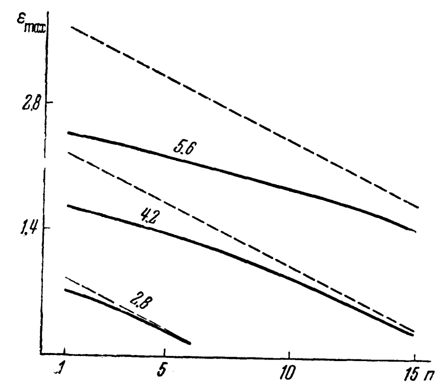

FIG. 2. Maximum energy of a π — meson formed as a result of an $N - N$ collision.

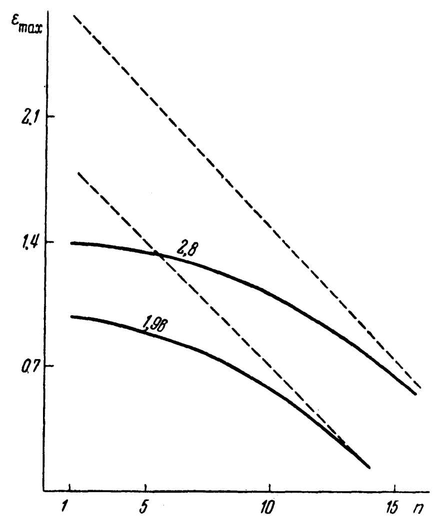

FIG. 3. Maximum energy of a π — meson formed as a result of the annihilation of a nucleon and an antinucleon.

(θ_{max} is the angle in the laboratory coordinate system, v_c the velocity of the system of the center-of-mass in the laboratory system). The author proposes to use θ_{max} as a criterion for identifying particles. Evidently the existence of a maximum angle for nucleon recoil should be taken into account during integration over the angle θ.

In Figs. 1-3 the solid curves represent maximum meson energies in certain reactions. The number of π — mesons produced is plotted along the horizontal axis; the numbers labeling the curves give the total energy of the system; the dotted curves represent results which take into account only the law of conservation of energy which, measured in the system of center-of-mass, is equal to 10^9 ev.

I express my thanks to Prof. D. Ivanenko for discussions.

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Polarization of Neutrons Scattered by Carbon Taking Nuclear Volume into Account

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EXPERIMENTS on the double scattering of high energy nucleons by nuclei indicate a considerable polarization of the primary scattered beam[1]. A comparison of the data for light elements (H, Be, He, C) shows that the polarization is independent of the kind of nucleus and shows that the angular distribution is practically unchanged for energies in the range 100-300 mev. One may assume that the scattered particle interacts with the nucleus as a whole. In order to explain polarization accompanying scattering by nuclei with spin 0 it is useful to take into account spin-orbit forces of the form

$$U_{s.o.} = (a / r)(dU_1 / dr)\,\sigma \mathbf{l}. \quad (1)$$

The complete interaction potential between the nucleon and the nucleus may in such a case be written in the form

$$U(r) = U_1(r) + iU_2(r) + U_{s.o.}(r) \quad (2)$$

Calculations in which the interaction is represented in the form of a complex rectangular well have led, both in the quasi-classical approximation and also in the exact method of phases (in particular, for nucleons of energy 290 mev scattered by C^{12}), to the appearance in the angular distribution of the polarization coefficient $P(\vartheta)$ of a region in which the sign changes and which corresponds to the region of the first diffraction minimum in the angular distribution of the differential scattering cross section [see Ref. 2 in which the relation between $P(\vartheta)$ and $\sigma(\vartheta)$ is shown]. However, the measurements of polarization do not confirm the presence of a region in which the sign changes. It is, of course, possible that the region in which the sign of the polarization changes is so narrow that it is not accessible to observation at the present time. The measurements of the polarization for a wide group of elements (He, Be, C, Cu, Al) at energies of 100-300 mev have led to the conclusion that the possibly existing region in which the sign changes has been smoothed out. Some authors suppose that the discrepancy between the calculations and the experimental data may be explained by the form of the potential utilized. Because of the factor $(a/r)dU_1/dr$ in the spin-orbit term $\boldsymbol{\sigma}\cdot\mathbf{l}$ the polarization is predominantly an edge effect and the region in which the sign changes is very sensitive to the shape of $U_1(r)$. Apparently the rectangular potential well which was used earlier turns out to be only a rough approximation from the point of view of taking into account the edge effect. One would expect that the smoothing out of its edge, being a better representation of the actual nuclear potential, should give for $P(\vartheta)$ a better agreement with experiment.

Below we present the results of the calculation of the polarization carried out, utilizing a more realistic way of taking into account the influence of the form of the nuclear potential. The static scalar meson field which is a good approximation for the given problem is defined by the equation

$$(\nabla^2 - k_0^2)\varphi = -4\pi\rho. \quad (3)$$

The distribution of the nucleons inside the nucleus is chosen in accordance with the result of Ref. 3

$$\rho = \begin{cases} \rho_0 & \text{for} \quad r \leqslant R_0, \\ \rho_0 (R_0/r)^2 e^{-\alpha(r-R_0)} & \text{for} \quad r \geqslant R_0. \end{cases} \quad (4)$$

Taking into account (4) we obtain from (3)

$$\varphi_1 = a_1(e^{-k_0 r} - e^{k_0 r})/r + c_1 \quad \text{for} \quad r \leqslant R_0, \quad (5)$$

$$\varphi_2 = a_2 r^{-1} e^{-k_0 r} + b_2 r^{-1} e^{k_0 r} Ei(-[\alpha + k_0] r) + d_2 r^{-1} e^{-k_0 r} Ei(-[\alpha - k_0] r) \quad \text{for} \quad r \geqslant R_0,$$

where a_1, c_1, a_2, b_2, d_2 are constants depending on R_0, α, ρ, k_0, g. We assume the spin-orbit forces to be of the form

$$(a/r)(d\varphi_1/dr)\,\boldsymbol{\sigma}\mathbf{l} \quad \text{for} \quad r \leqslant R_0, \qquad (a/r)(d\varphi_2/dr)\,\boldsymbol{\sigma}\mathbf{l} \quad \text{for} \quad r \geqslant R_0, \quad (6)$$

where $a = \frac{1}{2}\lambda(\hbar/mc)^2$, $\lambda = 15$ as given by Heisenberg[4].

The functions φ_1 and φ_2 can be approximated with sufficient accuracy by the corresponding branches of the parabolas

$$y_1 = -0.248g + 0.029gr^2 \quad \text{for} \quad 0 \leqslant r \leqslant R_0, \quad (7)$$

$$y_2 = -0.41g + 0.183gr - 0.021gr^2 \quad \text{for} \quad R_0 \leqslant r \leqslant 6r_0,$$

where $R_0 = 1.602 \times 10^{-13}$ cm, $\alpha = 0.68 \times 10^{-13}$ cm and $k_0 = 0.675 \times 10^{13}$ cm^{-1}, $r_0 = 10^{-13}$ cm. The radial factor for the spin-orbit forces is replaced correspondingly by $0.058g$ and $-0.032g + 0.143\, g/r$. The coupling constant g varies from $3e$ to $5e$. Thus the total interaction potential between the neutron and the C^{12} carbon nucleus will be of the form

$$U_I(r) = gy_1(1 + i\varepsilon) + 0.058ag^2 \quad \text{for} \quad 0 \leqslant r \leqslant R_0, \quad (8)$$

$$U_{II}(r) = gy_2(1 + i\varepsilon) - g^2(0.032 - 0.143r^{-1}) \quad \text{for} \quad R_0 \leqslant r \leqslant 6r_0,$$

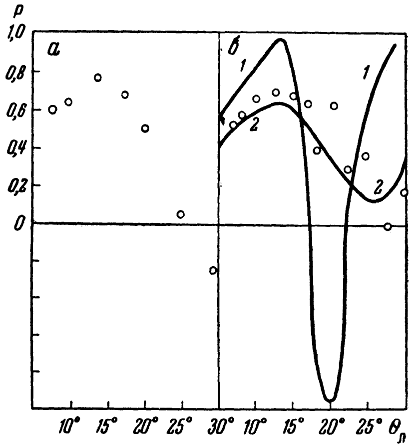
$\epsilon = 1$ (for energies of 300 mev).

The polarization is defined by the formula

$$P(\vartheta) = \{|f - ig| - |f + ig|\} / \{|f - ig| + |f + ig|\}; \quad (9)$$

$$f(\vartheta) = -\sum_{l=0}^{L} [(l+1)A_l^+ + lA_l^-] P_l^0(\cos\vartheta); \quad (10)$$

$$g(\vartheta) = -\sum_{l=0}^{L} (A_l^+ - A_l^-) P_l^1(\cos\vartheta),$$



a – polarization of protons on He (E = 315 mev), *b* – the same on C (E = 290 mev). *1* – results of calculation with a rectangular well, *2* – the same, taking into account the volume distribution.

$$A_l^{\pm} = (\exp\{2i\delta_l^{\pm}\} - 1)/2i, \qquad (11)$$

$\delta_l^{\pm}$ is the phase shift for the partial wave with $J = l \pm 1/2$, which we calculate in the WKB approximation; P_l^0, P_l^1 are the Legendre polynomials. The results of the calculation of $P(\vartheta)$ are given by the graph, and show that, because of the more accurate description of the behavior of the potential at the edge of the nucleus by means of introducing a certain nucleon distribution inside it, we have succeeded in obtaining better agreement with experiment. This is explained by the inclusion of higher phases. The polarization remains approximately constant for g varying from $3e$ to $5e$ which can be understood from the nature of the phenomenon. A decrease in ϵ makes the agreement with experiment worse.

In conclusion, I express my thanks to Prof. D. Ivanenko for his continued interest in the work and for the discussion of results.

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Calculation of Elastic Scattering of Slow Electrons in Hydrogen by the Integral Equation Method

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THE problem of the elastic scattering of slow electrons by the hydrogen atom, taking exchange into account, was first solved by Morse and Allis[1] by means of a numerical integration of the corresponding integro-differential equation and later by Massey and Moiseiwitsch[2] by means of a variational principle. The object of the present calculation consisted of checking the effectiveness of the method of integral equations proposed by Drukarev[3].

We shall start with the same approximate representation of the wave function of the system "electron + hydrogen atom" which was used by Morse and Allis:

$$\Psi(r_1 r_2) = \psi(r_1) F^{\pm}(r_2) \pm \psi(r_2) F^{\pm}(r_1), \qquad (1)$$

where $\psi(r)$ is the function of the ground state of the hydrogen atom. $F^+(r)$ and $F^-(r)$ are considered to be spherically symmetric, which means that only s-scattering is taken into account.

With the choice of $\Psi(r_1 r_2)$ made above the functions $f^{\pm}(r) = rF^{\pm}(r)$ must satisfy the equation

$$\left(\frac{d^2}{dr^2} + k^2\right) f^{\pm}(r) = V(r) f^{\pm}(r) \pm 2u(r) \times \int_0^{\infty} u(r') f^{\pm}(r') \left\{\gamma(rr') + \varepsilon - \frac{k^2}{2}\right\} dr' \qquad (2)$$

and the boundary conditions

$$f^{\pm}(0) = 0, \qquad (3)$$

$$f^{\pm}(r) \sim \frac{1}{k}\sin kr + a^{\pm} e^{ikr}, \qquad (4)$$

where

$$V(r) = 2\left\{\int_0^{\infty} u^2(r')\gamma(rr')\,dr' - \frac{1}{r}\right\}, \qquad u(r) = r\psi(r),$$

$$\gamma(rr') = \begin{cases} 1/r & r \geqslant r' \\ 1/r' & r \leqslant r', \end{cases}$$