direct support to the conclusion that both single positive pion formation in nucleon-nucleon collisions at an energy of 660 mev and pair formation of pions at energies of 1720 and 2300 mev are essentially due to strong meson-nucleon interaction in the intermediate $P_{3/2,3/2}$ state.

5. The ratio of yields of positive and negative pions increases with energy up to 160-180 mev in the center-of-mass system. The ratio of integral yeilds of positive and negative pions is noticeably lower than the values predicted by theory based on the assumption that formation and decay in the intermediate $P_{3/2,3/2}$ state are independent.

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Solution of the Fundamental Diffusion Equation for Cosmic Ray Particles Emitted by a Constant Energy Concentrated Pulsed Source

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The fundamental diffusion equation deduced by Terletskii[1] for cosmic ray protons emitted in magnetized interstellar space during a short period of time by a concentrated source of given energy is solved. It is shown that consideration of the particles which remain after collision of cosmic ray protons with protons of the interstellar gas leads to a power spectrum similar to that observed experimentally, if the source is assumed to be a supernova which appeared in the center of the galaxy over 10^8 years ago.

THE distribution function of cosmic ray protons in interstellar space $f(\mathbf{r}, E, t)$ can be found from the diffusion equation for cosmic ray particles.[2-4] In full form, with account taken of particles that remain after the collision of cosmic ray protons with the protons of an interstellar gas, this equation was found in Ref. 1 :

$$\frac{\partial f(E)}{\partial t} + \frac{f(E)}{T} - D\Delta_r f(E) + \frac{\partial}{\partial E}[\alpha E f(E)] - \frac{\partial^2}{\partial E^2}[\alpha E f(E)] - \frac{1}{T}\left[\frac{1}{a_1} f\left(\frac{E}{a_1}\right) + \frac{1}{a_2} f\left(\frac{E}{a_2}\right)\right] = Q, \quad (1)$$

where Δ_r is the Laplace operator, α is a coefficient characterizing the mean increase of energy per unit time ($d\bar{E}/dt = \alpha\bar{E}$), T is the mean lifetime of a particle up to its collision with an atom of interstellar gas, Q is the density of sources of particles, D is the amount of energy of the first particle that insures meson formation in a collision with motionless particles, a_1 and a_2 are the amounts of energy possessed by the two particles after collision ($a_0 + a_1 + a_2 = 1$).

To find the particle spectrum, definite assumptions must be made on the particle sources. We assume that the primary cosmic ray protons arise in outbursts of nova and supernova[5-7], are first accelerated in the expanding envelopes of the stars, and then are ejected into the interstellar medium. Acceleration in the envelopes of the stars is controlled by the action of the statistical mechanism of Fermi.[8] In this case, there do not arise the usual difficulties, associated with the fact that this mechanism begins to operate only for energies which exceed the threshold energy determined by ionization losses to the surrounding medium. Actually, for envelopes expanding at $\alpha = 10^{-9}$/sec (see Ref. 9), the threshold energy for protons is negligibly small and does not exceed the initial kinetic energy associated with particles in the explosion of a star.

Further acceleration to the limiting energies occurs upon multiple collisions with the turbulent pulsations of the interstellar gas,[4] i. e., by the Fermi mechanism.

It is evident that acceleration in the envelope of the star is possible up to a certain limiting energy E which is determined by the condition that the radius of curvature r of the trajectory of the particle in a magnetic field H of the envelope corresponding to the energy E ought to be many times smaller than the radius of the envelope ρ. The diffusion path length of the particles, $L = [\sqrt{6Dt}]^{1/2}$, where t is the acceleration time, ought also not to exceed ρ, i.e., the condition

$$\rho \gg r = E/300H, \quad \rho \gg L = \sqrt{6Dt}. \quad (2)$$

ought to be satisfied.

On the other hand, the energy E and the time t are connected in approximate fashion:

$$E \approx e^{\alpha t}. \quad (3)$$

Because of (2) and (3), the particle can be found within the limits of the envelope only during a finite time t_0.

According to Refs. 9, 10, taking

$$\rho \approx 10^{19}\text{ cm}, \quad H \approx 10^{-4}\text{ oe},$$

$$D \approx 10^{24}\text{ cm}^2/\text{sec}, \quad \alpha \approx 10^{-9}\text{ sec}^{-1},$$

we estimate [from (2) and (3)] the acceleration time of the particle in the envelope of the star up to the threshold energy to be $t_0 \sim 10^{10}$ sec or ~ 300 years.

Thus, we can assume that, at time $t_0 \sim 300$ years after eruption of the star, particles are generated from the resulting envelope and are accelerated in the medium to the energy E_0 ($E_0 \approx 10^{10}$ ev). Particles with such an energy can accelerate in interstellar space. Therefore we can represent the sources which generate particles in interstellar space in the form

$$Q = Q_0 \delta(t - t_0)\,\delta(\mathbf{r} - \mathbf{r}_0)\,\delta(E - E_0); \quad (4)$$

Q_0 = strength of source.

The further process of acceleration of the particle in interstellar space is described by Eq. (1) with Q from (4). For the solution of this equation we multiply both sides by E and carry out the following change of variables:

$$E = e^x, \quad u = Ef(E)$$

After substitution Eq. (1) is simplified:

$$\frac{\partial u}{\partial t} - D\Delta_r u - \alpha\frac{\partial^2 u}{\partial x^2} + \frac{1}{T}[u(x) - u(x + \Delta_1) - u(x + \Delta_2)] = Q_0\delta(\mathbf{r} - \mathbf{r}_0)\,\delta(x - x_0)\,\delta(t - t_0); \quad (5)$$

$$\Delta_1 = \ln(1/a_1), \quad \Delta_2 = \ln(1/a_2).$$

We look for a solution in the form of a Fourier integral

$$u(\mathbf{r}, x, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\mu x} F(\mu, \mathbf{r}, t)\,d\mu \quad (6)$$

and obtain for F the equation

$$\frac{\partial F}{\partial t} - D\Delta_r F + \alpha\mu^2 F + \frac{F}{T}(1 - e^{i\mu\Delta_1} - e^{i\mu\Delta_2}) = Q_0 e^{-i\mu x_0}\,\delta(\mathbf{r} - \mathbf{r}_0)\,\delta(t - t_0). \quad (7)$$

We set

$$F(\mathbf{r}, \mu, t) = \Phi_0(\mu, t) X(\mathbf{r}, \mu, t), \tag{8}$$

$$\Phi_0(\mu, t) = \exp\left\{-t\left[\alpha\mu^2 + \frac{1}{T}(1 - e^{i\mu\Delta_1} - e^{i\mu\Delta_2})\right]\right\}.$$

Substituting (8) in (7), we get for X the inhomogeneous diffusion equation, the solution of which has the form

$$X = \frac{Q_0 e^{-i\mu x_0}}{8\pi^{3/2}\Phi_0(t_0)} \tag{9}$$

$$\times \exp\left\{-\frac{(\mathbf{r}-\mathbf{r}_0)^2}{4D(t-t_0)}\right\}\frac{1}{[D(t-t_0)]^{3/2}}.$$

Thus, the solution of Eq. (5) has the form

$$u(\mu, \mathbf{r}, t) = \frac{\dot{Q}_0}{16\pi^{3/2}}\frac{\exp\{(\mathbf{r}-\mathbf{r}_0)^2/4D(t-t_0)\}}{[D(t-t_0)]^{3/2}} \tag{10}$$

$$\times \exp\left\{-\frac{t-t_0}{T}\right\} I,$$

where

$$I = \int_{-\infty}^{\infty} \exp\left\{i\mu(x - x_0) \right. \tag{11}$$

$$\left. - \alpha\mu^2(t-t_0) + \frac{t-t_0}{T}[e^{i\mu\Delta_1} + e^{i\mu\Delta_2}]\right\} d\mu.$$

Now everything is reduced to the calculation of the integral (11). Introducing the notation

$$x - x_0 = \ln\frac{E}{E_0} = \varkappa,$$

$$\frac{t-t_0}{T} = \beta', \quad \alpha' = \alpha(t - t_0),$$

$$f(\mu) = i\mu - \frac{\alpha'}{\varkappa}\mu^2 + \frac{\beta'}{\varkappa}(e^{i\mu\Delta_1} + e^{i\mu\Delta_2}),$$

we write the integral in the form

$$I = \int_{-\infty}^{\infty} e^{\varkappa f(\mu)} d\mu$$

and compute its value by the method of steepest descents, assuming that $\varkappa$ is a large parameter.

For the computation we must find the line of steepest descent

$$\operatorname{Im} f(\mu) = s - \frac{2\alpha'}{\varkappa} s\sigma + \frac{\beta'}{\varkappa} e^{-\Delta_1\sigma}\sin(\Delta_1 s) \tag{12}$$

$$+ \frac{\beta'}{\varkappa} e^{-\Delta_2\sigma}\sin(\Delta_2 s) = \text{const}$$

and the saddle point, which is determined from the equation

$$\left.\frac{\partial f}{\partial\mu}\right|_{\mu=\mu_0} = i\left[1 + 2\frac{\alpha'}{\varkappa}z + \frac{\Delta_1\beta'}{\varkappa}e^{\Delta_1 z} + \frac{\Delta_2\beta'}{\varkappa}e^{\Delta_2 z}\right] = 0, \tag{13}$$

where $z = i\mu_0$. It is then easy to show that saddle point $\mu_0 = -iz = i\sigma_0$ lies on the imaginary axis in the upper half plane:

$$s - \frac{2\alpha'}{\varkappa} s\sigma + \frac{\beta'}{\varkappa} e^{-\Delta_1\sigma}\sin(\Delta_1 s) \tag{14}$$

$$+ \frac{\beta'}{\varkappa} e^{-\Delta_2\sigma}\sin(\Delta_2 s) = 0,$$

where $\mu = s + i\sigma$. We now specify the contour determined by this equation. We show that σ is an even function of s. Denoting the left side of Eq. (14) by $F(s, \sigma(s))$, we have

$$F(s, \sigma(-s)) = F(-s', \sigma(s'))$$

$$= F(-s, \sigma(s)) = 0.$$

It then follows that $\sigma(s) = \sigma(-s)$ and the line of steepest descent is symmetrical relative to the imaginary axis.

We write Eq. (14) in the form

$$\frac{\varkappa}{\beta'}\left(1 - \frac{2\alpha'}{\varkappa}\sigma\right)e^{\Delta_2\sigma} \tag{15}$$

$$+ e^{(\Delta_2-\Delta_1)\sigma}\frac{\sin(\Delta_1 s)}{s} + \frac{\sin(\Delta_2 s)}{s} = 0$$

and construct a nomogram for the graphical investigation of this equation.

We erect vertical axes Y_1 and Y_2 on the plane, perpendicular to the X axis and a certain curve Y, the coordinates of a point of which we shall denote as (x_3, y_3). We connect the points with the coordinates y_1 and y_2 on the axes Y_1 and Y_2 by a straight line and find the point of intersection of this line with the curve Y (Fig. 1).

We then have the relation

$$-y_1 + \frac{y_3}{x_3} + y_2\left(1 - \frac{1}{x_3}\right) = 0. \tag{16}$$

Equation (16) coincides with Eq. (15) if

$$y_2 = \sin(\Delta_1 s)/s,$$

$$-y_1 = \sin(\Delta_2 s)/s, \quad 1/x_3 = 1 - e^{(\Delta_2-\Delta_1)\sigma},$$

$$\frac{y_3}{x_3} = \frac{\varkappa}{\beta'}\left(1 - \frac{2\alpha'}{\varkappa}\sigma\right)e^{\Delta_2\sigma}.$$

With increase in s, the points y_1 and y_2 undergo damped oscillations about the points $y_1 = y_2 = 0$. The point x_3, y_3 has a damped oscillation about the point $y_3(\sigma_0) = 0$, which corresponds to $\sigma_0 = \varkappa / 2\alpha'$. Therefore the contour of integration has the form shown in Fig. 2.

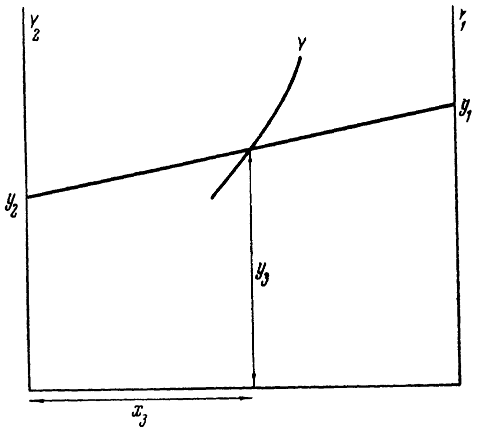

FIG. 1

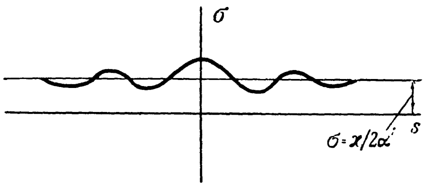

FIG. 2

The rest of the problem reduces to the integration over the contour C. Since $\operatorname{Im} f(\mu) = 0$ on C, then

$$I = \int_C \exp\{\varkappa \operatorname{Re} f(\mu)\}\, d\mu \tag{17}$$

$$= \int_C \exp\Big\{\varkappa\Big[-\sigma - \frac{\alpha'}{\varkappa}s^2 + \frac{\alpha'}{\varkappa}\sigma^2 + \frac{\beta'}{\varkappa}\cos(\Delta_1 s)\, e^{-\Delta_1\sigma} + \frac{\beta'}{\varkappa}\cos(\Delta_2 s)\, e^{-\Delta_2\sigma}\Big]\Big\} d(s + i\sigma).$$

The integral over $d\sigma$ vanishes.

The principal contribution to the integral comes from the cutting off of the contour of integration which runs to the saddle point.

We have

$$I = \exp\{-\varkappa\sigma_0 + \alpha'\sigma_0^2\} \int_{-\varepsilon}^{\varepsilon} \exp\{-\alpha' s^2 + \beta'\cos(\Delta_1 s)\, e^{-\Delta_1\sigma_0} + \beta'\cos(\Delta_2 s)\, e^{-\Delta_2\sigma_0}\}\, ds.$$

Setting $\epsilon = (\alpha')^{-2/5}$, expanding the integrand in a series in s and limiting ourselves to the first 3 terms of the expansion, we get

$$I = \exp\Big\{-\varkappa\sigma_0 + \alpha'\sigma_0^2 + \frac{\beta'}{e^{\Delta_1\sigma_0}} + \frac{\beta'}{e^{\Delta_2\sigma_0}}\Big\} \times \sqrt{\pi\Big/\Big(\alpha' + \frac{\Delta_1^2\beta'}{2e^{\Delta_1\sigma_0}} + \frac{\Delta_2^2\beta'}{2e^{\Delta_2\sigma_0}}\Big)}\,[1 + O(\alpha'^{-1/5})].$$

Thus, the desired fundamental solution of Eq. (1) is

$$f(\mathbf{r}, x, t) = \frac{Q_0}{16\pi^2 D^{3/2}(t - t_0)^2} \times \exp\Big\{-\sigma_0(x - x_0) - x - \frac{(\mathbf{r} - \mathbf{r}_0)^2}{4D(t - t_0)} + \frac{t - t_0}{T}(-1 + e^{-\Delta_1\sigma_0} + e^{-\Delta_2\sigma_0} + \alpha\sigma_0^2 T)\Big\} \times \Big[\alpha + \frac{\Delta_1^2}{2T}e^{-\Delta_1\sigma_0} + \frac{\Delta_2^2}{2T}e^{-\Delta_2\sigma_0}\Big]^{-1/2} \tag{18}$$

Hence it follows that

$$f(\mathbf{r}, E, t) = E^{-\gamma}\varphi(\mathbf{r}, t), \tag{19}$$

where $\gamma \approx 1 + \sigma_0$ and $\varphi(\mathbf{r}, t)$ does not depend on E. Consequently, the exponent γ of the power spectrum of the primary component of the cosmic rays is determined by the quantity $\sigma_0 = -z$ which is a root of the transcendental Eq. (13).

The exponent γ satisfies the equation

$$1 - \frac{2\alpha'}{\varkappa}(\gamma - 1) + \frac{\Delta_1\beta'}{\varkappa}e^{-\Delta_1(\gamma-1)} + \frac{\Delta_2\beta'}{\varkappa}e^{-\Delta_2(\gamma-1)} = 0.$$

Recalling the definitions of α', β, $\varkappa$, and assuming that for our galaxy the factor $\alpha T = 0.6$ (see Refs. 7,8), we obtain

$$\frac{0{,}6}{\alpha(t - t_0)}\ln\frac{E}{E_0} = 1{,}2(\gamma - 1) - \Delta_1 e^{-\Delta_1(\gamma-1)} - \Delta_2 e^{-\Delta_2(\gamma-1)}. \tag{20}$$

In the collision of two protons of high energy, one of the colliding protons carries away the principal amount of energy, and the energy of the second is less than 1/5 the initial energy of the colliding particles.* The estimates

*Private communication of N. L. Grigorov; see also Ref. 11.

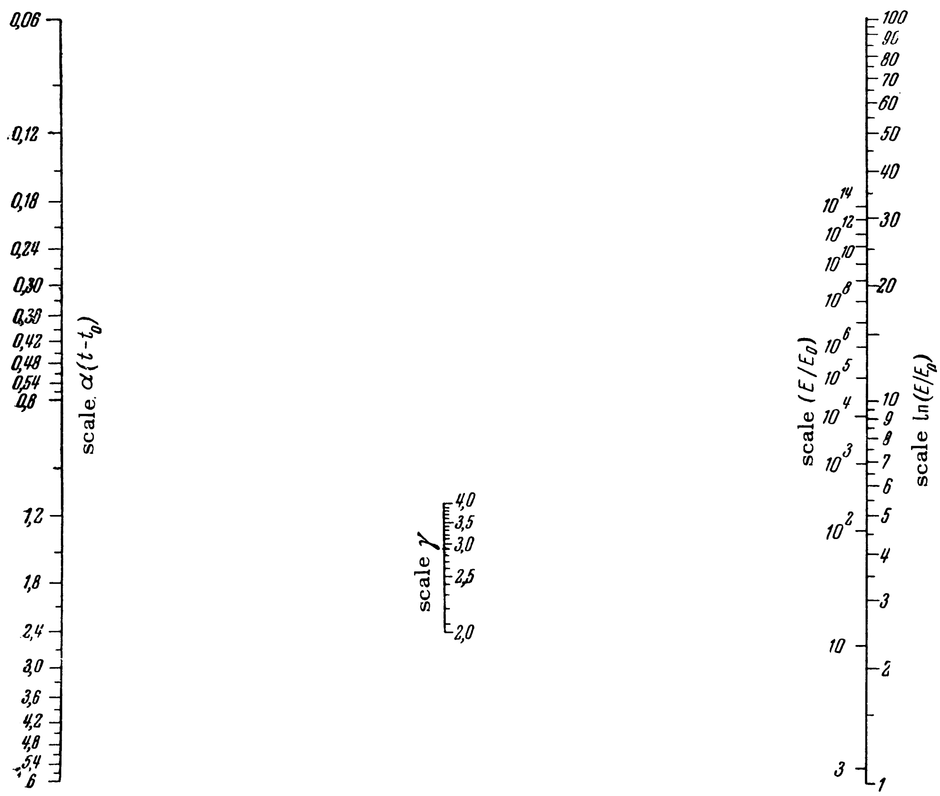

FIG. 3

$$a_1 = 0.1; \quad a_2 = 0.6; \quad \Delta_1 = 2.3 \text{ and } \Delta_2 = 0.5.$$

then follow.

A nomogram is shown in Fig. 3 for the solution of Eq. (20). It is evident from the nomogram that for $0.4 \leqq \alpha' \leqq 6$ and $3 \leqq E/E_0 \leqq 10^{10}$, the exponent is confined to the range $2 \leqq \gamma \leqq 3$.

In particular, in accord with the experimental data, for an initial energy of $E_0 = 5 \times 10^8$ ev, which is approximately equal to the energy of the protons, we obtain for the energy $E \sim 10^9 - 10^{10}$ ev in the case of $\alpha' = 0.6$, $\gamma = 2.2$.

It follows from the nomogram that α' must be greater than 0.6 in order to determine the spectrum in the range of very high energies.

For example,

$$\text{for } E \sim 5 \cdot 10^{11}\,\text{eV} \quad \gamma = 2.5 \quad \text{for } \alpha' = 2.75; \tag{21}$$

$$\text{for } E \sim 10^{15}\,\text{eV} \quad \gamma = 3 \quad \text{for } \alpha' = 4.$$

This means that the high energy primary particles are long-lived particles $[(t - t_0) > T]$ which experience a large number of collisions with the turbulent pulsations of the interstellar material and are accelerated to high energies, or are particles with a real increase in energy per unit time above the average increase, corresponding to $\alpha T = 0.6$.

It then follows that the source of particles which reach the earth with comparatively low energies ($E \sim 10^{10}$ ev) and consequently, which are accelerated in interstellar space for a comparatively short time must be Nova and Supernova which erupted about T years ago. The source of particles that are accelerated to the highest energies ($E \sim 10^{15} - 10^{16}$ ev), and which consequently remain in the interstellar fields a far longer time, must be stars that erupted $(1 - 10)\,T$ years ago.

Thus the spectrum is formed by particles which arise at different times as the result of the explosion of many stars, in which the protons of each of the explosions contribute to the power spectrum of the energy. Therefore it is quite apparent that the fundamental solution ought to be integrated over the variables $\mathbf{r}_0$, t_0 and E_0 of the separate explosions.

From the estimates (21) of the time $(t - t_0)$ it follows that the sources of the primary cosmic ray particles are located in the center of the galaxy at a distance of $r \sim 10^{23}$ cm from the earth.

Actually, taking the time $t - t_0$ as the time required for particles to travel a diffusion path from source to earth, we get, by Eq. (21):

$$(t - t_0) \sim (1 \div 10)\, T \sim (1 \div 10) \cdot 10^{16} \text{ sec}$$

and setting $l \sim 10^{19}$ cm, $v \sim 10^{10}$ cm/sec, we get

$$r \sim \sqrt{lv(t - t_0)} \sim 10^{23} \text{ cm.}$$

This analysis makes possible the assumption that the cosmic ray protons observed at present arose as the result of very many explosions of Nova and Supernova which took place at the center of the galaxy at different times (approximately $4 \times 10^8 - 3 \times 10^9$ years ago) and were then accelerated by the electromagnetic fields of interstellar space.

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