

decay is proportional to the square of the matrix element for the process with change in strangeness  $\Delta S = 1$ , i.e., proportional to  $g^2$ , where  $g$  is the coupling constant. At the same time, the difference in masses is proportional to the first power of the matrix element for the transition  $\theta \rightleftharpoons \bar{\theta}$ , with change in strangeness  $\Delta S = 2$ . Actually, if we write symbolically

$$-i\partial\theta/\partial t = E_0\theta + f\bar{\theta},$$

$$-i\partial\bar{\theta}/\partial t = E_0\bar{\theta} + f\theta,$$

we get  $E_s = E_0 + f$ ,  $E_a = E_0 - f$ ; since we are dealing with the excitation of a created system, then the  $E_0$  for  $\theta$  and  $\bar{\theta}$  are identically equal. According to considerations on the magnitude of  $\Delta S$  for the conversion  $\theta \rightarrow \bar{\theta}$ , we can expect that  $f \sim g^2$ , so that  $\Delta m \sim \hbar / \tau c^2$  (as was assumed by Pais and Piccioni), where  $\tau$  is the period of decay  $\sim 1.5 \times 10^{-10}$ ; numerically, we obtain  $\Delta m = 10^{-11} m_e$ , where  $m_e$  is the mass of the electron.

Another approach to the problem of the difference of the masses of  $\theta_s$  and  $\theta_a$  is based on the direct consideration of that coupling of the  $\theta$ -particles with other fields, which determines their decay. If we assume that the spin of  $\theta$  is zero, then the pair  $\pi^+$ ,  $\pi^-$  which are generated in the decay, is found in a state which is even relative to charge conjugation; only the decay  $\theta_s = \pi^+ + \pi^-$  is possible, not the decay of  $\theta_a$ . The decay of  $\theta_s$  gives information on the coupling of the field of  $\theta_s$  with the field of the pions\*. According to the usual formulas of perturbation theory, such a coupling must produce a displacement of the level, i.e., a change of the energy of  $\theta_s$ , along with the decay which produces a broadening of the level. We write down side by side the energy shift and the decay probability:

$$w = 2\pi M^2(E) \left. \frac{dN}{dE} \right|_{E=E_0}, \quad \Delta E = \int_0^\infty \frac{M^2(E)}{E_0 - E} \frac{dN}{dE} dE,$$

$M(E)$  is the matrix element of the transition from the state  $\theta_s$  into the state of continuous spectrum, i.e., into the pair  $\pi^+$ ,  $\pi^-$  with energy  $E$ ;  $dN/dE$  is the density of levels of the continuous spectrum. The integral in  $\Delta E$  is taken in the sense of the principal value; therefore the immediate neighborhood of  $E_0$  does not determine its values. In order that the integral converge, it is

necessary that the falling off of  $M(E)$  be sufficiently rapid for  $E \rightarrow \infty$ . To compute  $\Delta E$ , not knowing the properties of  $M(E)$  is impossible. From the expressions that have been given, it is evident only that  $\Delta E$  is of the same order of magnitude as  $w$ ; dimensional quantities — the coupling constants, etc. — enter into  $\Delta E$  and  $w$  in the same degrees.

\* Decay into muons, which is less probable, is not considered here.

<sup>1</sup>M. Gell-Mann and A. Pais, Phys. Rev. 97, 1387 (1955).

<sup>2</sup>A. Pais and O. Piccioni, Phys. Rev. 100, 1487 (1955).

<sup>3</sup>Ia. B. Zel'dovich, Dokl. Akad. Nauk SSSR 86, 505 (1952).

<sup>4</sup>M. Gell-Mann, Phys. Rev. 92, 833 (1953).

<sup>5</sup>Balandin, Balashov, Pontecorvo and Selivanov, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 265 (1955); Soviet. Phys. JETP 2, 98 (1956).

R. W. Thompson, Prog. Cosmic Ray Physics III, cited by G. Costa and N. Dallaporta, Nuovo Cimento 2, 519 (1955).

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### Angular Correlation in Cascade Decay of hyperons

V. B. BERESTETSKII AND V. P. IGNATENKO  
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**S**TUDY of the angular distribution of the decay products of hyperons can give evidence on the spin of the latter. The distribution of pions in the cascade decay  $\Xi \rightarrow \lambda \rightarrow p$  was considered in Ref. 1. Here we consider the cascade decay  $\Sigma^0 \rightarrow \Lambda^0 + \gamma \rightarrow p + \pi^- + \gamma$ . The wave function pertaining to the motion of a proton and a  $\pi^-$  particle has the following form:

$$\psi_{jm_j}(\mathbf{p}, \sigma) = C_{lm'l_2\sigma}^{jm_j} Y_{lm} \left( \frac{\mathbf{p}}{p} \right) \chi_\sigma,$$

where  $\mathbf{p} = \mathbf{p}_\pi - \mathbf{p}_p$  is the momentum of the relative motion,  $j$  is the spin of the  $\Lambda$ -particle,  $m_j =$  its projection,  $\psi_\sigma =$  the spin wave function of the proton,  $Y_{lm}$  is the spherical harmonic,  $C_{\dots}$  are the coefficients of vector addition. Since the spin of the proton is  $1/2$  and the internal parity of the pion is odd, then  $l$  is uniquely determined by the spin and parity of the  $\Lambda$ -particle  $g$ :  $l = j \pm 1/2$ ;  $(-1)^l = (-1)^{g+1}$ .  $m = m_j - \sigma$  is determined in the same fashion.

The wave function of the entire system in the final state is

$$\Psi_{JM_J}(\mathbf{P}, \mathbf{p}, \sigma, m_j, \mu) = \sum_{L\lambda} \rho_{L\lambda} C_{LMjm_j}^{JM_J} \psi_{jm_j}(\mathbf{p}, \sigma) \Phi_{LM}^\lambda\left(\frac{\mathbf{P}}{P}, \mu\right),$$

where  $\mathbf{P} = \mathbf{p}_\lambda - \mathbf{p}_\Lambda$  is the momentum of the relative motion of the photon and the  $\Lambda$ -particle (in the system in which the  $\Sigma$ -particle  $\mathbf{P} = 2\mathbf{p}_\gamma$  is at rest),  $J =$  spin of the  $\Sigma$ -particle,  $M_J$  its projection,  $\mu =$  the spin (polarization) variable of the photon,  $\lambda, L, M$  are the parity, moment, and projection of the momentum relative to the motion of the photon and the  $\Lambda$ -particle. For given spins and parity of the  $\Sigma$ - and  $\Lambda$ -particle,  $L$  and  $\lambda$  can be different in the general case; the values of  $L$  for different  $\lambda$  ought to differ in parity. Since  $m_j$  and  $M_J$  are given, then  $M = M_J - m_j$ .  $\Phi_{LM}^\lambda$  is a vector spherical harmonic; its three values, corresponding to the three values of  $\mu$ , are appropriately combined in the vector  $Y_{LM}^{(\lambda)}$ ; for given  $L$  and  $M$ , two vector spherical harmonics are possible ( $\lambda = 0, 1$ ), which differ in parity. Similarly, we denote the wave function  $\Psi_{JM}(\dots, \mu) \rightarrow \Psi_{JM}(\dots)$ . In such a fashion,

$$\Psi_{JM}(\mathbf{P}, \mathbf{p}; \sigma, m_j) = \sum_{L\lambda} \rho_{L\lambda} C_{LMjm_j}^{JM_J} C_{lm_j \mu \sigma}^{jm_j} \chi_\sigma Y_{LM}^{(\lambda)}\left(\frac{\mathbf{P}}{P}\right) Y_{lm}\left(\frac{\mathbf{p}}{p}\right). \tag{1}$$

The angular distribution of the particles is determined by the function

$$I(\theta) = \sum_{\sigma m_j M_J} |\Psi_{JM}|^2.$$

Here  $\theta = \mathbf{P} \cdot \mathbf{p} / Pp$ ; the  $\Sigma$ -particle is assumed to be unpolarized, in view of which summation is carried out over the  $M_J$ . It is appropriate to direct the  $Z$  axis along  $P$ . Then it is easy to get (from the properties of the spherical harmonics)

$$Y_{LM}^{(0)} Y_{LM}^{(1)*} = 0, \quad Y_{LM}^{(0)} Y_{L'M}^{(0)} = Y_{LM}^{(1)} Y_{L'M}^{(1)*} = \frac{1}{4\pi} \sqrt{(2L+1)(2L'+1)} \delta_{M, \pm 1}.$$

In this way the spherical vectors with different  $\lambda$  do not interfere, and in squaring (1), the terms which contain the index  $L$  will be multiplied by terms containing  $L' + 2k$ , where  $k$  is an integer. Making use of this, we find

$$I(\theta) = \sum_L \sum_{L'=L+2n} \sum_{M_j m_j \sigma} V_{L\lambda} V_{L'\lambda'} \times C_{LMjm_j}^{JM_J} C_{L'M_j m_j}^{JM_J} (C_{lm_j \mu \sigma}^{jm_j})^2 |Y_{lm}(\theta)|^2.$$

Making use of the well-known formulas for the decomposition of the square of the spherical harmonics in Legendre polynomials  $P_n$  and the properties of the sum of the coefficients of spherical harmonics<sup>2,3</sup>, we obtain

$$I(\theta) = \sum_{n=0}^N A_n P_{2n}(\cos \theta), \tag{2}$$

$$A_n = C_{l_0 l_0}^{2n 0} W(jjll; 2n 1/2)$$

$$\times \sum_L \sum_{L'=L+2k} V_{L\lambda} V_{L'\lambda'} C_{L'L-1}^{2n 0} W(jjLL'; 2nJ),$$

$$N = j - 1/2; \quad V_{L\lambda} = \sqrt{(2L+1)/4\pi} \rho_{L\lambda},$$

$W =$  Racah coefficients.

Below are given the angular distribution  $I(\theta)$  for  $j = 3/2$  (for  $j = 3/2$ , the distribution will be spherically symmetric) for different values of  $J$  (the coefficients  $a, \alpha$ , etc. are determined by the decay mechanism and are expressed by  $V_{L\lambda}$ . If only the smallest  $L$  plays a role, then the two first terms remain in the formula):

$$J = 1/2: I = 1 - 0,6 \cos^2 \theta + a (1 + \cos^2 \theta);$$

$$J = 3/2: I = 1 + 0,75 \cos^2 \theta$$

$$+ \alpha (0,4 - 1,2 \cos^2 \theta) + \alpha^2 (0,37 + 0,48 \cos^2 \theta) + b;$$

$$I = 5/2: I = 1 - 0,45 \cos^2 \theta$$

$$+ \beta (0,4 - 1,2 \cos^2 \theta) + \beta^2 (0,33 + 0,43 \cos^2 \theta)$$

$$+ c [(1 - 0,14 \cos^2 \theta) + \gamma (0,5 - 1,5 \cos^2 \theta)$$

$$+ \gamma^2 (0,44 - 0,1 \cos^2 \theta)];$$

$$J = 7/2: I = 1 - 0,6 \cos^2 \theta$$

$$+ \delta (0,5 - 1,36 \cos^2 \theta) + \delta^2 (0,26 + 0,48 \cos^2 \theta)$$

$$+ d [1 + 0,23 \cos^2 \theta + \epsilon (0,7 - 2,1 \cos^2 \theta)$$

$$+ \epsilon^2 (0,5 + 0,01 \cos^2 \theta)]$$

*Note added in proof.* The problem of the correlations in the decay of a  $\Sigma$ -particle have also been considered in the recent publication of Gatto.<sup>4</sup>

<sup>1</sup>R. Gatto, *Nuovo Cimento* 2, 841 (1955).

<sup>2</sup>A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics*, GFTI, 1953).

<sup>3</sup>Biedenharn, Blatt and Rose, *Rev. Mod. Phys.* 24, 249 (1952).

<sup>4</sup>R. Gatto, *Nuovo Cimento* 3, 665 (1956).

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### Gamma Resonances in Reactions of Proton Capture by Silicon Isotopes

S. P. TSYTKO AND I. P. ANTUF'EV

*Physico-technical Institute, Academy of Sciences, Ukrainian SSR*

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THE reactions  $\text{Si}(p, \gamma)P$  were first studied<sup>1</sup> by means of the yield of  $\gamma$ -radiation from thick targets for the proton energy interval from 0.3 to 0.55 mev. Somewhat later,<sup>2</sup> Tangen reported the results of more detailed investigations of this reaction in the same proton energy interval. He found resonances for  $E_p = 326$  and 414 kev, which he attributed to the reaction  $\text{Si}^{29}(p, \gamma)P^{30}$ , since he observed them by the activity of  $P^{30}$ , and also resonances for  $E_p = 367$  and 499 kev, which he attributed to the reaction  $\text{Si}^{30}(p, \gamma)P^{31}$ . Recently, Milani, Cooper and Harris observed<sup>3</sup>  $\gamma$ -resonances in the reaction  $\text{Si}^{29}(p, \gamma)P^{30}$  at approximately  $E_p = 696, 727,$

917, 956 kev. They carried out their investigations both on thin and thick targets of  $\text{Si}^{29}$ .

The integral excitation function of the  $\text{Si}(p, \gamma)P$  reactions was measured on the 4 mev electrostatic generator of the Physico-technical Institute of the Academy of Sciences, USSR, in the proton energy interval from 500 to 2600 kev. A thick target with the natural mixture of the isotopes of silicon ( $\text{Si}^{28}$ , 92.28%;  $\text{Si}^{29}$ , 4.67%;  $\text{Si}^{30}$ , 3.05%) was prepared from a pure (99.98%) single crystal of silicon, which has been obtained by vacuum distillation. To test the various impurities in the silicon crystal, investigations were carried out on thick and thin targets, prepared from silicon which has served as the initial material in the preparation of the single crystal. The tests showed that Al, Fe and Pb, which appeared as impurities in the original material, had been removed. The energy of the accelerated protons was measured by an electrostatic analyzer with accuracy to within 0.05%; the inhomogeneity in the energy of the proton beam amounted to 0.8 kev for the entire interval of proton energies, the  $\gamma$ -rays were detected by a copper counter. The current at the target was measured by a current integrator of the Watt type. During the measurements, the target temperature was maintained at the level 300-500° to avoid weakening of the carbon film; vapors of the oil of the diffusion pumps were carefully frozen out with a liquid nitrogen trap. In the measurements of the integral excitation function, the position and width of 26 new  $\gamma$ -resonances were determined. New  $\gamma$ -resonances were found for  $E_p = 619.5, 717, 753, 775, 800, 831, 895, 940, 980, 1520, 1618, 1635, 1647, 1663, 1680, 1699, 1774, 1810, 1849, 1879, 2520, 2543, 2553, 2557.5, 2570$  and 2575 kev. The experimental widths were observed to lie within the limits 0.8 to 8 kev.

For identification of the reactions corresponding to the resonances, the differential excitation function was measured on thin targets by the yield of positron activity of  $P^{29}$  and  $P^{30}$ . At present, this work has been carried out only to proton energies of  $E_p = 1000$  kev. Not a single resonance was found for  $P^{29}$ . Also, no new resonances have been found for  $P^{30}$  up to 1000 kev. The positions of the two  $\gamma$ -resonances mentioned earlier<sup>3</sup> for the reaction  $\text{Si}^{29}(p, \gamma)P^{30}$  for  $E_p = 917$  and 956 kev were determined more accurately by us; according to our measurements,  $E_p = 916.5 \pm 0.5$  and  $956 \pm 1$  kev. The lower accuracy of the determination of the resonance for 956 kev was caused by its weak