$$I_l = \int_0^\infty R_l(kr)\, g_l(pr)\, r^2 dr = 4\pi \left(\frac{p}{k}\right)^l \frac{1}{p^2 - k^2}.$$

For $\alpha \leq 1/p$, which is the case in this model, $I_l^q / I_l \leq 1/2\,(2l+1)$ and is small for large l. It is easy to see that if r_n is used as the upper limit, then $I_l^a / I_l = a^2/r_n^2 \ll 1$.

For large l, $I_{l-1} \ll I_{l+1}$; Substituting I_{l+1} in Eq. (2) and multiplying by the number of nucleons in the nucleus, we obtain

$$\frac{1}{\tau} = \frac{3(l+1)}{2(2l+1)} g^2 \frac{\mu^3 k}{p^3 M} \left(\frac{p}{k}\right)^{2l+2} \frac{1}{\tau_0}. \tag{4}$$

for $g^2 = 10$, $l = 5$, $\epsilon = 37$ mev, $k = \sqrt{2\mu\epsilon}$, $p = \sqrt{2M\mu}$, we get for the conversion coefficient $\tau_0/\tau \sim 5 \times 10^6$. For $p = \sqrt{M\mu}$, we get $\tau_0/\tau \sim 2 \times 10^5$.

If $r_n < l/p$, then we put I_{l+1} in place of I_{l+1}^a, with $a = r_n$. The value of the conversion coefficient in this case is also $\sim 10^6 - 10^5$. The large values of the conversion coefficient are explained by the large l, on the one hand, and on the other, by the fact that the momentum of the nucleon which absorbed the meson is larger than the momentum of the meson ($p/k \gg 1$), so that the centrifugal barrier is more transparent.

Thus the lifetime of the bound Λ-particle is approximately 10^6 times smaller than for the free Λ-particles, and the decay ought to be almost exclusively conversion, which contradicts experiment. Our conclusions do not depend on the character of the internal region, since only the small dimension of the Λ-particle is essential for it. It appears improbable that the contribution from the internal region could compensate the contribution of the external region, since such a compensation would have to be one of extraordinarily great exactness.

The problem of the spin of the Λ-particle is of interest also, aside from any dependence on the model considered by us. Suppose that the origins of the metastability of the Λ-particle are not due to a large angular momentum, but to some sort of forbidden principle[5,6] connected, for example, with the isotopic spin. In this case, the proof carried out above that there can be no large spin associated with the Λ-particle, is not valid. This is connected with the fact that now the effective dimensions of the Λ-particle can be of the order of $l \geq 4$, and in this case we cannot draw any conclusions on the magnitude of conversion in this interval. However, even in this case, it can be shown that the presence of a very large spin in the Λ-particle leads to an anomalously large "conversion coefficient". In fact, it follows from Eqs. (3) and (4) that the region of integration from $1/\mu$ to ∞, in the case of $l \geq 4$, gives a contribution which is of the order of magnitude of the integral from 0 to ∞ and, consequently, all our estimates maintain their force.

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[1] R. Arnowitt and S. Deser, Bull. Amer. Phys. Soc. **28**, 35 (1953).

[2] R. Arnowitt and S. Deser, Phys. Rev. **92**, 1061 (1953).

[3] S. F. Powell, Nature, **173**, 469 (1954).

[4] A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics*, Moscow, 1953.

[5] M. Gell-Mann and A. Pais, Proc. 1954 Glasgow Conference.

[6] M. Gell-Mann, Phys. Rev. **92**, 833 (1953).

Translated by R. T. Beyer
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Binding Energy of Hyper-Nuclei

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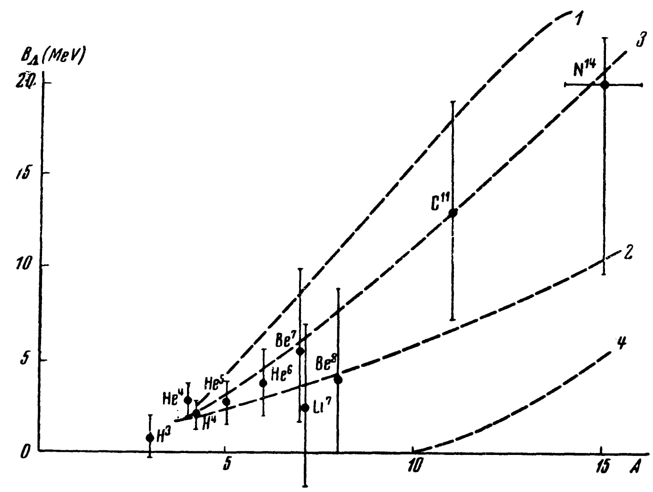
ONE of the fundamental discoveries of recent times has been the discovery of Danisz and Pinewski[1] of new types of nuclei--the so-called hyper- or Λ-nuclei, which contain a Λ^0-particle in addition to neutrons and protons. It has been shown that in many cases the binding energy of the Λ^0-particle to the nuclei is positive and that the comparatively short life of the Λ-nuclei ($\tau \sim 10^{-10}$ sec) is connected with the instability of the Λ^0-particle itself[1-12]. The character of the interaction of the Λ^0-particles with nucleons (N) has not yet been established[3,7,13]. Observations have permitted us to establish, although with low accuracy, the binding energy of a series of light nuclei[1-12] We have plotted the most trustworthy values for the binding energy (see Figure), which illustrates the dependence of the binding energy



of Λ^0-particles (to the nucleus B_Λ) on the number of nucleons A. As is evident from the graph, the fundamental regularities can be expressed in the following fashion: a) B_Λ increases with the number of nucleons in the nucleus: $B_\Lambda = -2.7 + 1.15A$ mev(A = mass number); b) the binding energy is independent of the charge and also of the spin of the nuclear part (Λ-nucleus minus Λ^0-particle); c) hyper-nuclei lighter than Λ-tritium (${}_\Lambda H^3$) are absent.

The absence of saturation of the binding energy, so clearly evident in ordinary nuclei, is rather astonishing. It creates the impression that the Λ^0-particle interacts on an equal basis with all the nucleons of the nucleus, in such a way that the mean binding energy of each nucleus is equal to $\sim$ 1 mev, i.e., considerably less energy per nucleon than in ordinary nuclei. We assume that the forces of interaction of Λ^0-particles and nucleons are identical to the nuclear forces N - N, in particular, they have, for example, the same radius of action[3-13]. We can then introduce a self-consistent potential for the Λ--nucleus. This potential must differ slightly from the potential of the bar nucleus, in view of the very weak interaction of the Λ^0-particle with the nucleus. A single hyperon, like particles of other types (such as protons or neutrons) will be found at the 1s level in the ground state of the Λ-nucleus. The energy of this level in the square well model with infinitely steep walls is given by

$$\varepsilon_{1s} = V_0 - \pi^2\hbar^2 / 2M_\Lambda R_0^2, \qquad (1)$$

where V_0 is the depth of the well, $R_0 = r_0 A^{1/3}$ is the radius of the well, M_Λ is the mass of the Λ^0-particle ($M_\Lambda = 2182\ m_e$). We take $R_0 = 1.4 \times 10^{-13} A^{1/3}$ as the radius of the well. This allows us to obtain the correct energy for the light nuclei. If we consider V_0 to be the same for all nuclei and equal to 43 mev, then the binding energy of Λ-nuclei increases rapidly with increase in A, as calculation shows (see the broken line *1* in the Figure). A similar calculation has also been carried out for the more realistic case of a rectangular well of finite depth. In this case[15,16]

$$\varepsilon_{1s} = V_0 - z_{1s}^2\hbar^2 / 2M_\Lambda r_0^2 A^{2/3}, \qquad (2)$$

where z is a quantity dependent on the product $V_0 R_0^2$. If we now use values of r_0 and V_0 in accord with the binding energy of the nuclei[15,16] ($r_0 = 1.38 \times 10^{-13}$ for all nuclei under consideration, with the exception of the very light, where r_0 increases slightly) and consider the depth of the well for the Λ^0-particle to be half that for nucleons (i.e., above 25 mev for all nuclei with the exception of the very light, where V_0 is somewhat smaller), then we get rather satisfactory agreement with the experimental data (see curve *3* in the drawing). Curve *2* corresponds to a well with V_0 = const = 15 mev and r_0 = const = 1.38×10^{-13}. Curve *3* should reach saturation for $A \sim$ 25-30, which would be interesting to verify experimentally. (A similar saturation sets in for curve *1* at somewhat smaller A, and for curve *2* at somewhat larger A.) The results of the calculations agree with the available data[3,13] in this regard, that the forces Λ^0 - N are weaker (although of the same order) than the forces N - N[3,13]. Curve *4* in the Figure portrays the calculation of the value of the binding energy of excited Λ-nuclei (for the same V_0 and r_0 as for curve *3*), when the Λ^0-particle is in the $1p$ state. In view of the large excitation energy, it is to be expected that the perturbed hyper-nuclei would undergo transitions to the ground state by means of the emission of a γ-quantum or would emit nucleons before the Λ^0-particle would decay. The other possibility of excitation of hyper-nuclei is connected with the transition of nucleons to higher levels.

The Λ-nucleus can exist if its decay to lighter hyper-nuclei (in particle, to the Λ^0-particle) and to nuclei (or to nucleons) is not energetically possible, i.e., if the mass of the initial hyper-nucleus is less than the sum of the masses of the possible products:

$$M(A,\ Z,\ \Lambda) - M(A;\ Z_1;\ \Lambda) - M(A - A_1;\ Z - Z_1) < 0,$$

where A is the total number of neutrons + protons + Λ^0-particles. If $M(A, Z, \Lambda) = ZM_p + NM_n + M_\Lambda - E(A, Z, \Lambda)$ [$E(A, Z, \Lambda)$ is the total binding energy of the given Λ-nucleus, wherein $E(A, Z, \Lambda) = E(A-1, Z) + B_\Lambda$], then, as is easy to work out, the condition for stability of the Λ-nucleus with regard to neutron emission will be (in mev): $E(A-1, Z) + 1.15 - E(A-2, Z) > 0$. The conditions for stability of the Λ-nucleus relative to decay with emission of a proton and Λ-particle will be, correspondingly:

$$E(A-1; Z) + 1{,}15 - E(A-2; Z-1) > 0$$

$$\text{and } E(A-1, Z) - 23{,}6 - E(A-5; Z-2) > 0.$$

Testing, by means of this formula, shows that the nuclei ${}_\Lambda\mathrm{HE}^4$, ${}_\Lambda\mathrm{He}^5$, ${}_\Lambda\mathrm{Be}^8$ are sufficiently stable. At the same time, ${}_\Lambda\mathrm{He}^6$ and especially ${}_\Lambda\mathrm{H}^5$ are slightly stable: the first relative to neutron emission, the second, to proton emission. If $E(\mathrm{H}^4) + 1.15 > E(\mathrm{H}^3)$, then ${}_\Lambda\mathrm{H}^5$ could be regarded as stable. ${}_\Lambda\mathrm{H}^3$, ${}_\Lambda n^3$ and $({}_\Lambda\mathrm{H}^3)^*$ evidently ought to be unstable and have actually not been discovered experimentally. However, there does exist the nucleus ${}_\Lambda\mathrm{H}^3$, in which, evidently, the spins of the neutron and proton are antiparallel as in the deuteron. From the viewpoint of isotopic spin, the existence of only one stable Λ-nucleus with mass number 3 is connected with the fact that the isotopic spin of ${}_\Lambda\mathrm{H}^3$ is equal to zero; the latter is possible if $T = 0$ for the Λ^0-particle itself. This also agrees with the empirical data on the absence of $\Lambda^{\pm}$-hyperons. From the viewpoint of meson theory, the interaction of the Λ^0-particle with a nucleon can occur[3] only be means of the emission of pions ($\Lambda^0 \to \Lambda^0 + \pi^+ + \pi^-$; $\Lambda^0 \to \Lambda^0 + \pi^0$), or by emission of K-mesons, $\Lambda^0 \to N^{0+}\ K^{0-}$, which is permitted by the rule of conservation of "strangeness"[6,14]. However, if $T = 0$ for the Λ^0-particle, then the possibility of exchange of π^0-mesons is excluded by the law of conservation of isotopic spin. Evidently this is associated with the very weak interaction of the Λ^0-particles with nucleons. In view of this, the existence of Λ-nuclei consisting of two particles (for example, ${}_\Lambda\mathrm{H}^2$ and ${}_\Lambda n^2$) is highly improbable, in agreement with the estimates based on the conclusion developed above on the approximately half strength interaction of Λ^0-particles with nucleons.

Note added in proof: A recent consideration of the idea that the binding energy of Λ-nuclei ought to saturate at sufficiently high mass numbers was reported in a research carried out simultaneously and just published[17].

1 M. Danisz and J. Pniewski, Phil. Mag. **44**, 348 (1953).

2 Fry, Schneps and Swami, Phys. Rev. **99**, 1561 (1955).

3 R. H.Dalitz, Phys. Rev. **99**, 1475 (1955).

4 M. Grilli and R. L. Setti, Nuovo Cimento Suppl. **12**, 466 (1954).

5 R. L. Setti, Nuovo Cimento Suppl. **2**, 263 (1955).

6 Proc. Pisa Conference, 1955.

7 Proc. 5th Rochester Conference, February 1955.

8 Baldo, Belliboni *et al.*, Nuovo Cimento **1**, 1180 (1955).

9 O. Haugeraud and S. O. Sörensen, Phys. Rev. **99**, 1046 (1955).

10 J. Hornbostel and E. O. Salant, Phys. Rev. **98**, 218 (1955).

11 Catagnoli, Cortini and Franzinetti, Nuovo Cimento **2**, 550 (1955).

12 M. Danisz, Nuovo Cimento **2**, 269 (1955).

13 R. Gatto, Nuovo Cimento **1**, 372 (1955); **2**, 373 (1955).

14 M. Gell-Mann, Phys. Rev. **92**, 833 (1953).

15 N. Kolesnikov, Dissertation, Moscow State Univ., 1955.

16 B. G. Jancovici, Phys. Rev. **95**, 389 (1954).

17 R. Gatto, Nuovo Cimento **3**, 499 (1956).

Translated by R. T. Beyer
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