values α and R have to be determined from experiment. Substituting the function a_l into Eq. (3) we obtain:

$$\sigma_c = \frac{\pi\alpha^2}{2} R^2\left(\frac{4}{\alpha} - 1\right); \qquad \sigma_s = \frac{\pi\alpha^2}{2} R^2. \tag{5}$$

The values of α and R corresponding to the experimental results lie within the following limits: from $R = 0.86 \times 10^{-13}$ cm and $\alpha = 0.74$ to $R = 0.74 \times 10^{-13}$ and $\alpha = 0.99$. The values, corresponding to the mean values of σ_s and σ_c, are $R = 0.8 \times 10^{-13}$ and $\alpha = 0.85$. Furthermore, substituting the function a_l into Eq. (4), we can calculate the function $f(\vartheta)$:

$$|f(\vartheta)| = \lambda\!\!\!^{-} \int\limits_0^\infty \alpha J_0(l\vartheta)\, e^{-\lambda\!\!\!^{-2} l^2/R^2}\, l\,dl \tag{6}$$

$$= \alpha \frac{R^2}{\lambda\!\!\!^{-}} \int\limits_0^\infty e^{-y^2} J_0\left(\vartheta \frac{R}{\lambda\!\!\!^{-}} y\right) y\,dy =$$

$$= (\alpha R^2/2\lambda\!\!\!^{-}) \exp\{-\vartheta^2 R^2 / 4\lambda\!\!\!^{-2}\}.$$

The angular distribution $d\sigma/d\omega$ is determined by the function $|f(\vartheta)|^2$. The angular distribution calculated according to the formula (6) using the values $\lambda\!\!\!^{-} = 2.7 \times 10^{-14}$ cm, $R = 0.8 \times 10^{-13}$ cm and $\alpha = 0.9$, which correspond to $\sigma_c = 28$ mbn and $\sigma_s = 81$ mbn, is given in Fig. 1. The experimental data fit satisfactorily the theoretical curve.

The value $Z_l = 1 - |\beta_l|^2$, the physical meaning of which is the ' sticking probability " of particle, is of interest:

$$Z_l = \alpha e^{-\rho^2/R^2}\left(2 - \alpha e^{-\rho^2/R^2}\right), \tag{7}$$

where $\rho = \lambda\!\!\!^{-} l$. We shall note that for $\rho = 0$ the value of Z is close to unity for a wide interval of values of d. Thus, even for $\alpha = 0.73$ $Z_{\rho=0} = 0.93$. The mean square value of the impact parameter [averaged over the function (7)] is:

$$\rho_{cp} = (\overline{\rho^2})^{1/2} = \sqrt{\frac{1-\alpha/8}{1-\alpha/4}}\, R. \tag{8}$$

The values $\overline{\rho}$, satisfying the experimental data are contained in the interval $(0.8 - 0.9) \times 10^{-13}$ cm. We shall note that, contrary to what is maintained in Ref. 3, the fact that the nucleon cannot be regarded as a black body does not imply that the statistical theory of multiple production is not applicable.

The function (7) gives the probability of inelastic scattering taking place for the impact parameter ρ. The inelastic scattering can be treated on the basis of the statistical theory, too[8]. It is of interest to perform a similar analysis for nucleon-nucleon collisions.

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Radiative Disintegration of Λ^0-Particle

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BESIDES the normal decay scheme of the Λ^0-particle

$$\Lambda^0 \to p + \pi^-, \tag{1}$$

the following decay scheme is possible:

$$\Lambda^0 \to p + \pi^- + \gamma. \tag{2}$$

It is known from the experimental data that the spin of the π^--meson is zero and therefore the spin of the Λ^0-particle must be half-integral. In the present work the influence of the spin of Λ^0-particle upon the shape of the γ-quanta spectrum is investigated.

SPIN OF Λ^0 EQUAL TO ½

We shall use the notation of Ref. 1 and a system of units in which $h = c = 1$. The density of the interaction energy is given by the following expression:

$$U = ieN(\overline{\psi}_p \gamma_\mu \psi_p) A_\mu - ie\left(\varphi \frac{\partial \varphi^*}{\partial x_\mu} - \varphi^* \frac{\partial \varphi}{\partial x_\mu}\right) A_\mu + V, \tag{3}$$

where ψ_p and φ are the wave functions of the proton and of the meson, respectively, A_μ is the electromagnetic potential, V is the interaction energy of Λ^0, p and the π^--meson, of the following form:

$$V = gN(\psi_p \gamma_5 \psi_{\Lambda^0}) \varphi \quad (P\text{-coupling}) \tag{4,P}$$

$$V = \frac{f}{M+\Lambda} N(\overline{\psi}_p \gamma_5 \gamma_\mu \psi_{\Lambda^0}) \frac{\partial \varphi}{\partial x_\mu} \quad (A\text{-coupling}) \tag{4,A}$$

M, Λ and m are the masses of the proton, Λ^0-particle and the meson. (4, P) and (4, A) represent the pseudoscalar P-coupling and the pseudovector A-coupling. The normalization of the constant of (4,A) is chosen so that (4, P) and (4, A) give the same result for the probability of decay (1) in the first approximation of the perturbation theory.

The matrix element of the decay (2) is of the following form in the first non-vanishing approximation of the perturbation theory:

$$M = \frac{ieg(2\pi)^4}{2\sqrt{E_\pi k}} \overline{u}_p \left(\frac{2p_e + \hat{e}\hat{k}}{M^2\varkappa} + \frac{2q_e}{m^2\varkappa_1}\right) \times \gamma_5 u_\Lambda \delta(p_\Lambda - p - q - k), \tag{5,P}$$

$$M = \frac{ief(2\pi)^4}{2\sqrt{E_\pi k}} \overline{u}_p \left(\frac{2p_e + \hat{e}\hat{k}}{M^2\varkappa} + \frac{2q_e}{m^2\varkappa_1} - i\frac{\hat{e}}{\Delta}\right) \times \gamma_5 u_\Lambda \delta(p_\Lambda - p - q - k). \tag{5,A}$$

Here $\overline{u}_p$, u_Λ are the Dirac spinors of the proton and of the Λ^0-particle; $\hat{e} = e_\nu \gamma_\nu$, $p_e = e_\nu p_\nu$, $\Delta = M + \Lambda$, $M^2\varkappa = 2(pk)$, $m^2\varkappa_1 = 2(qk)$ and p, q, k are the 4-momenta of the proton, of the meson and of the γ-quantum, respectively. According to general rules, we obtain from (5) the probability of the decay (2), given by

$$dw = \frac{e^2 f^2}{8E_\pi E_p k}\Bigg\{\frac{1}{(pk)^2}\Bigg[\left(\mathbf{q}^2 - \frac{1}{k^2}(\mathbf{qk})^2\right) \times\left(E_p + k - M + 2\frac{(pk)}{\Delta}\right) - k(pk) + 2(E_p + M)\frac{(pk)^2}{\Delta^2} + 2\left(kM - (pk)\frac{(pk)}{\Delta}\right)\Bigg] + \frac{1}{(qk)^2}\left(\mathbf{q}^2 - \frac{1}{k^2}(\mathbf{qk})^2\right)(E_p - M) - \frac{1}{(pk)(qk)}\left(\mathbf{q}^2 - \frac{1}{k^2}(\mathbf{qk})^2\right)\left(k + 2E_p - 2M + 2\frac{(pk)}{\Delta}\right)\Bigg\} \times \frac{d^3k\, d^3q}{(2\pi)^5}\delta(E_\Lambda - E_p - E_\pi - k). \tag{6,A}$$

The probability for pseudoscalar interaction will be obtained from (6,A) dropping the terms in $1/\Delta$ and putting g instead of f.

2. SPIN OF Λ^0 EQUAL TO 3/2

The equations for the case of particle with spin 3/2 were studied in Ref. 2. The wave function of a particle with spin 3/2 has mixed transformation properties of a usual spinor and of a 4-vector, each component of which satisfies the equation

$$(i\hat{p} + M)\psi_\mu = 0, \tag{7}$$

with the auxiliary conditions:

$$\partial\psi_\mu / \partial x_\mu = 0, \quad \gamma_\mu \psi_\mu = 0.$$

For a particle at rest the wave function is[3]:

$$\vec{\psi}^{(1)} = \mathbf{e}_1 u^{(1)}, \quad \vec{\psi}^{(2)} = \mathbf{e}_2 u^{(2)}, \tag{8}$$

$$\vec{\psi}^{(3)} = \frac{\mathbf{e}_2}{\sqrt{3}} u^{(1)} + \sqrt{\frac{2}{3}}\mathbf{e}_3 u^{(2)},$$

$$\vec{\psi}^{(4)} = \frac{\mathbf{e}_1}{\sqrt{3}} u^{(2)} - \sqrt{\frac{2}{3}}\mathbf{e}_3 u^{(1)},$$

$$\psi_0^{(i)} = 0; \quad \mathbf{e}_1 = 2^{-1/2}(1, i, 0),$$

$$\mathbf{e}_2 = 2^{-1/2}(1, -i, 0), \quad \mathbf{e}_3 = (0, 0, 1);$$

$u^{(1)}$ and $u^{(2)}$ are spinors with polarization ±½.

The condition $\gamma_\mu \psi_\mu = 0$ implies that it is possible to form an invariant combination of the wave functions ψ, $\vec{\psi}$ only with the help of vector q_ν:

$$V = \frac{f}{m} N(\bar{\psi}_p \gamma \psi_\mu) \frac{\partial \varphi}{\partial x_\mu}, \tag{9}$$

where $\gamma = \gamma_5$ when $\bar{\psi}_p \psi_\mu q_\mu$ is a scalar and $\gamma = 1$ when it is pseudoscalar. The matrix element for the decay (2) can be written on the basis of Eq. (5) byputting $\hat{q} \to q_\nu$, $u_\lambda \to u_\nu$ therein. The expression for the decay proabability for the case of spin $3/2$ is:

$$dw = \frac{e^2 f^2}{24 m^2 E_p E_\pi k} \left\{ \left(\mathbf{q}^2 - \frac{1}{k^2} (\mathbf{qk})^2 \right) \right. \tag{10}$$

$$\times \left[\frac{\mathbf{q}^2}{(pk)^2} (E_p + k \pm M) + \frac{\mathbf{p}^2}{(qk)^2} (E_p \pm M) \right] -$$

$$\times - \frac{1}{(pk)(qk)} \left[2 \left(\mathbf{q}^2 + (\mathbf{qk}) + \frac{1}{4} |\mathbf{q}| k \right) \cdot \right.$$

$$\left. (E_p \pm M) + k(\mathbf{q}^2 + (\mathbf{qk})) \right] - \left. \frac{k \mathbf{q}^2}{(pk)^2} \right\}$$

$$\times \frac{d^3 k \, d^3 q}{(2\pi)^5} \delta (E_\lambda - E_p - E_\pi - k).$$

The signs (+) and (–) correspond to the cases $\gamma = 1$ and $\gamma = \gamma_5$, respectively. For small photon frequency ($k \ll 1$) we obtain from (6) and (10)

$$\frac{dw}{dw_e} = \frac{\alpha}{2\pi} \left(\mathbf{q}^2 - \frac{1}{k^2} (\mathbf{qk})^2 \right) \tag{11}$$

$$\left(\frac{1}{(pk)} - \frac{1}{(qk)} \right)^2 k \, dk \sin \theta \, d\theta,$$

where dw_e is the probability of decay mode (1) and $\alpha = 1/137$.

When the particles can be regarded as nonrelativistic, Eq. (11) simplifies to the well-known relation[4]

$$dw/dw_e = (2\alpha/3\pi)(q^2/E_\pi^2) \, dk/k. \tag{12}$$

For small photon frequencies the spin of the Λ^0-particle has, therefore, no influence upon the decay probability. In theTable below, values are given for the ratio of the total probability w_2 of decay with emission of a photon with energy $\geq E$ to the probability w_1 of decay without emission of radiation. This ratio is calculated for spin $3/2$ ($\gamma = 1$) and for spin ½ (P-interaction). The following values are taken for the masses of the particles: $M = 1837\, m_e$, $m = 270\, m_e$, $\Lambda = 2180\, m_e$.

E (mev)	1	5	10	Spin
$10^4\, w_2/w_1$	13	4,6	2,8	1/2
	10	3,0	0,97	3/2

It is evident from the above that the spin of Λ^0-particle has an influence upon the shape of the spectrum. This difference in forms of spectra is very substantial at high energies. Comparing theory with experiment we can try to determine the spin of the Λ^0-particle and the type of coupling as well.

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