where φ^n is the wave function of the θ^0-meson and $n = M - m$ is the quantum number of the projection of its orbital moment. Substituting from Eq. (9) in Eq. (1), and taking account of the fact that functions φ^n with different n are orthogonal, we obtain an expression of the type Eq. (2), but with the off-diagonal elements of $\rho_{m_1 m_2}$ equal to zero. According to the preceding, correlation cannot be observed in this case, which contradicts experiment. Consequently, the spin of the θ^0-meson is different from zero. This result was obtained earlier[4] in a different way.

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Renormalization in the Equations of the New Tamm-Dancoff Method

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1. IT is well known (see, for example, Ref. 1) that the equations of the new Tamm-Dancoff method for scattering of a meson by a nucleon, including only three virtual particles, contain divergent self-energy kernels of the meson and the nucleon and finite kernels corresponding to scattering with the meson being first absorbed ("graph with absorption") and scattering with the meson first emitted ("graph with emission"). We shall not consider renormalization of the self-energy kernels of the meson and nucleon and the difficulties arising with this[1], but shall assume that these kernels are corrections to the propagator of the system, which can be neglected in first approximation. We omit also all two-particle amplitudes containing antiparticles, assuming that in first approximation their influence on the amplitude " +meson, +nucleon" is small.

As a result, in first approximation an equation for one amplitude, " +meson, +nucleon" without self-energy kernel remains. In the solution of this equation, infinities of a vertex and self-energy type occur because of the kernel corresponding to the "graph with absorption." The renormalization of these divergences was studied recently by Dalitz and Dyson[2]; they came to the conclusion that the removal of these divergences was connected with the introduction of two renormalized charges into the theory (respectively, for the $s_{1/2}$ and $p_{1/2}$ states with isotopic spin $T = 1/2$) in addition to the fundamental charge g^2. Renormalization of these divergences was studied independently by us by a method used in Ref. 3* and it was shown that, contrary to the results of Dalitz and Dyson, the removal of divergences is connected with a finite charge renormalization and does not lead to any new constants.

In this note we give briefly our results and clarify the contradiction between them and the results of Ref. 2. For brevity of exposition and convenience in making a comparison, the notation of Ref. 2 will be used where possible.

2. The general solution of the integral equation for the amplitude "+meson, +nucleon" in state α (state α is characterized by angular momentum j and the parity of the system) has the form**

$$c_\alpha(p,l) = g_\alpha(p,l) + \frac{Qg^2}{8\pi^2}\sqrt{\frac{(E_p-\eta m)(E_l-\eta m)}{E_p\omega_p E_l\omega_l}}\,\frac{V_\alpha(p,\varepsilon)\,V_\alpha(l,\varepsilon)}{\varepsilon-\eta m-QS_\alpha(\varepsilon)} \tag{1}$$

and is connected with the phase δ_α by the formula

$$\mathrm{tg}\,\delta_\alpha = -(\pi E_l\omega_l l \quad \varepsilon)\, c_\alpha(l,l).$$

In Eq. (1) $E_p = \sqrt{p^2+m^2}$, $\omega_p = \sqrt{p^2+\mu^2}$, $\epsilon = E_l + \omega_l$, $\eta = 1$ or -1, respectively, for even and odd states, $Q = 3$ or 0, respectively, for $T = 1/2$ and $3/2$; $g_\alpha(p,l)$ is the solution corresponding to the "graph"with emission." The integral equation for $g_\alpha(p,l)$ is obtained from the integral equation for $c_\alpha(p,l)$ if the term corresponding to the "graph with absorption" is omitted in the latter. Thus $g_\alpha(p,l)$ does not contain divergences. All divergences due to the "graph with absorption" are contained in the vertex function $V_\alpha(p,\epsilon)$ and the self-energy of the nucleon $S_\alpha(\epsilon)$. The divergence contained in the vertex function can be separated in the form of a factor Z_α, such that $V_\alpha(p,\epsilon) = Z_\alpha(V^R_\alpha(p,\epsilon)$, where $V^R_\epsilon(p,\epsilon)$ is a finite function, defined by some integral equation and falls off with large p as p^λ, where $\lambda < 0$. The self-energy of the nucleon $S_\alpha(\epsilon)$ can be written in the form

$$S_\alpha(\varepsilon) = \frac{Z_\alpha^2 g^2}{8\pi^2}\left[\int_0^\infty \frac{P_\varepsilon(k)\,|\,v_\alpha(k)\,|^2 k^2 dk}{E_k\omega_k(E_k+m)} - \frac{1}{2}\frac{Q'g^2}{8\pi^2} \times \int_0^\infty\int_0^\infty \frac{q^2dqp^2dp}{E_q\omega_qE_p\omega_p} v_\alpha(p)P_\varepsilon(p)U_\alpha(\varepsilon,p,q)P_\varepsilon(q)v_\alpha(q)\right], \tag{2}$$

where

$$v_s(p) = (E_p+m)V_s^R(p,\varepsilon), \quad v_p(p) = pV_p^R(p,\varepsilon)$$

(for the other notation, see Ref. 2). It is not difficult to show that both integrals in the brackets fall off more slowly than linearly; thus the factor Z^2_α does not yet separate all divergences in $S_\alpha(\epsilon)$. Therefore, we consider the derivative $dS_\alpha(\epsilon)/d\epsilon = Z^2_\alpha\Lambda_\alpha(\epsilon)$. The function $\Lambda_\alpha(\epsilon)$ is given in Ref. 2 by Eq. (93). The authors of Ref. 2 assert that the integrals defining this function diverge. In fact, Eq. (93) is in error and

$$\Lambda_\alpha(\varepsilon) = \frac{g^2}{8\pi^2}\left[\left(\frac{d}{d\varepsilon}\int_0^\infty P_\varepsilon(k)\right)\frac{|\,v_\alpha(k)\,|^2 k^2dk}{E_k\omega_k(E_k+m)} + \frac{1}{2}\frac{Q'g^2}{8\pi^4}\int_0^\infty\int_0^\infty \frac{p^2dpq^2dq}{E_p\omega_pE_q\omega_q} v_\alpha(p)P_\varepsilon(p)\frac{dU_\alpha}{d\varepsilon}P_\varepsilon(q)v_\alpha(q)\right] \tag{3}$$

This formula differs from Eq. (93) in that in the denominator of the first term the factor m is changed to E_k and before the second term ½ occurs. It is easy to see that the first term in $\Lambda_\alpha(\epsilon)$ converges; thus $V^R_\alpha(k) \sim k^\lambda$, $\lambda < 0$ for $k \to \infty$. Investigation shows that the second integral also converges, rather than diverging as asserted in Ref. 2. In fact, we consider the integrals

$$J_1 = \int_{c'}^\infty dp\int_0^c dq\frac{p^2q^2}{E_p\omega_pE_q\omega_q} v_\alpha(p)P_\varepsilon(p)\frac{dU_\alpha}{d\varepsilon}P_\varepsilon(q)v_\alpha(q),$$

$$J_2 = \int_c^\infty\int_c^\infty \frac{p^2dpq^2dq}{E_p\omega_pE_q\omega_q} v_\alpha(p)P_\varepsilon(p)\frac{dU_\alpha}{d\varepsilon}P_\varepsilon(q)v_\alpha(q),$$

where $c' \gg c \gg m$. It is possible to show that for $p \gg q, m$; $dU_\alpha/d\epsilon \sim (2p^2)^{-1}$; for $p, q \gg m$: $dU_\alpha/d\epsilon \sim (pq)^{-1}\left[1 + 2x - \frac{(1+x)^2}{x}\ln(1+x)\right]$, where x is the smaller of the ratios q/p, p/q. Substituting these expressions in J_1 and J_2, respectively, and using also the fact that for large p, $V^R_\alpha(p,\epsilon) \sim p^\lambda$, $\lambda < 0$, we find that J_1 and J_2 converge. Insofar as the integrand in the second integral in Eq. (3) is symmetrical with respect to p and q, the convergence of the second integral in Eq. (3) follows from the convergence of J_1 and J_2.

Thus $\Lambda_\alpha(\epsilon)$ is finite, and not infinite as in Ref. 2. This means that the entire divergence in $S_\alpha(\epsilon)$ can be put into the first term of the expansion

$$S_\alpha(\varepsilon) = S_\alpha(\eta m) + Z_\alpha^2\int_{\eta m}^{\varepsilon}\Lambda_\alpha(x)\,dx = S_\alpha(\eta m) + (\varepsilon-\eta m)Z_\alpha^2\Lambda_\alpha(\eta m) + Z_\alpha^2\frac{g^2}{8\pi^2}S_\alpha^R(\varepsilon) \tag{4}$$

and the factor Z^2_α. We substitute Eq. (4) and $V^R_\alpha(p,\epsilon) = Z^2_\alpha V^R_\alpha(p,\epsilon)$ into Eq. (1) and drop $S_\alpha(\eta m)$ in connection with the mass renormalization. Then, insofar as $Z^2_\alpha = \infty$, we find that the second term in Eq. (1) has the form

$$\frac{g^2}{8\pi^2}\sqrt{\frac{(E_p-\eta m)(E_l-\eta m)}{E_p\omega_pE_l\omega_l}}\frac{V_\alpha^R(p,\varepsilon)V_\alpha^R(l,\varepsilon)}{-\int_{\eta m}^{\varepsilon}\Lambda_\alpha(x)\,dx} = Q\frac{g_\alpha^2}{8\pi^2}\sqrt{\frac{(E_p-\eta m)(E_l-\eta m)}{E_p\omega_pE_l\omega_l}} \times\frac{V_\alpha^R(p)V_\alpha^R(l)}{\varepsilon-\eta m-(Qg_\alpha^2/8\pi^2)S_\alpha^R(\varepsilon)}, \tag{5}$$

where

$$g_\alpha^2 = Z_\alpha^2 g^2/(1-Z_\alpha^2 Q\Lambda_\alpha(\eta m)) = -g^2/Q\Lambda_\alpha(\eta m),$$

because $Z^2_\alpha = \infty$, and the remaining quantities are finite (it is possible to whos that $\Lambda_\alpha(\eta m) < 0$, so that $g^2_\alpha > 0$). Thus, by virtue of a finite renormalization of the charge g^2_α, a completely determined function g^2 results and no new constants arise in the removal of divergences. In particular, for small g^2: $g^2_\alpha = 2/3\, g^2$.

We mention by the way that the finite expressions Eqs. (96a) and (96b) in Ref. 2 are erroneous because of an incorrect sign in front of Q in Eqs. (55), (64), (67), (70) and (95), and in front of the second term in Eqs. (76) and (92), and also because of incorrect factors in front of $V_{\alpha}^{R}(p)$ and $V_{\alpha}^{R+}(l)$.

3. The integral equations for $g_{\alpha}(p, l)$ and $V_{\alpha}^{R}(p, \epsilon)$ have an infinite upper limit of integration. The solution of these equations should be taken as limits of the solutions of these equations with a finite upper limit of integration, p_{max}, as $p_{max} \to \infty$. Then further investigation shows that the equations for $g_{\alpha}(p, l)$ and $V_{\alpha}^{R}(p, \epsilon)$ have finite solutions, behaving for large p as p^{ν} and p^{λ}, respectively, where ν and λ are those roots of the equations

$$-Q'g^2/16\pi^2 = F_j(\nu + 1/2), \qquad -Q'g^2/16\pi^2 = F_j(\lambda),$$

$$F_{1/2}^{-1}(\lambda) = \pi[\lambda(\lambda+1)\sin\pi\lambda]^{-1} - \lambda^{-2} - (\lambda+1)^{-2} + (\lambda^2-1)^{-1} + (\lambda^2+2\lambda)^{-1},$$

which for $g^2 \to 0$ go over into the exponent of the first argument in the asymptotic kernel. For $T = j = 1/2$ with $g^2/4\pi > 3\pi(3\pi - 8)^{-1} \approx 6.61$, ν and λ are complex, and it is possible to show that in this case the equations for $g_{\alpha}(p, l)$ and $V_{\alpha}^{R}(o, \epsilon)$ do not have solutions in the sense that the solutions of the equations with a finite upper limit do not have a well-defined limit for $p_{max} \to \infty$. Thus $g^2/4\pi$ should be less than 6.61. This limit on $g^2/4\pi$ is not connected with the condition of normalization of the full solution, as taken in Ref. 2. In fact, from the integrability of the wave function in **x**-space for **x** = 0, it follows that the wave function in **p**-space for $p \to \infty$ should fall off faster than $p^{-3/2}$ However, the wave functions in p-space are the functions $\langle b_{-pu}\, a_{p\alpha}\rangle$, and not

$$\psi(p) = \sqrt{\omega_p}\sum_{u+} u\langle b_{-pu} a_{p\alpha}\rangle,$$

as taken in Ref. 2. Therefore, the functions $f(p)$ and $g(p)$ in Ref. 2 should fall off faster than const and not faster than $p^{-1/2}$. But this condition does not limit g^2.

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Note added in proof: It is necessary to note that the removal of divergences from the "graph with absorption" are connected with a finite charge renormalization not only in the equations of the new Tamm-Dancoff method, but also in covariant equations in the approximation considered here. For the finite renormalization it is necessary that the renormalized vertex function falls off with a negative power of p for large momenta.

* This method is essentially the same as that of Ref. 2.

** This equation is obtained from the initial equation for the amplitude "+meson, +nucleon" by decomposition of angle variables using spherical harmonics (see Ref. 1).

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Two τ-Mesons Detected in Photographic Emulsions

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A STACK, consisting of 126 layers of unbacked electron--sensitive emulsions of type *R*, thickness 450μ and diameter 10 cm, were exposed for 7 hours at a height of 27 km. Two τ-mesons were found in the scanning; the characteristics of these are given in the table.

One of the τ-mesons was observed by following the track of a π^+-meson which had come to rest. All of the prongs of the star in which the τ-meson was produced were followed either to their end or to the point where they left the stack. No further heavy particle decay was found among the prongs coming to their end.

Two π-mesons (π^+ and π^-) coming from the decay of the τ-meson stopped; the third left the stack. The tracks of these π-mesons lay in a plane; coplanarity was established to an accuracy of 3°. The decay energy was $Q = (74.2 \pm 2)$ mev, and the mass, determined from the decay scheme, $m_\tau = (965 \pm 4)\, m_e$. The second τ-meson was found by area-scanning. A microprojection of it is shown on the picture. All black and grey tracks from the primary star end in the stack; additional heavy unstable particles did not appear among them.

All of the π-mesons produced in the decay also stopped in the stack; two of them were positively