The magnitude of the conversion coefficient of the observed 325 kev transition is of great interest. However, it is impossible to calculate a more or less reliable value from the data which were obtained in this experiment with spectrometers of poor resolution. A qualitative estimate of the magnitude of this conversion coefficient was carried out. To do this, the values of the conversion coefficients and the ratio N_K/N_L for In^{113} were assumed known. For the transition considered a ratio of $N_K/N_L \sim 0.3$ was obtained. The theoretical value of the conversion coefficient for a 320 kev, $E5$ transition in a nucleus with atomic number 50 is about unity. Therefore, the experimental and theoretical values of the conversion coefficients are near to each other.

In conclusion, I should like to use this opportunity to express my gratitude to Iu. V. Trebukhovskii, G. R. Kartashov and V. S. Kuryshev, who gave me the opportunity of using their apparatus for carrying out this experiment. The author is particularly thankful to V. V. Vladimirskii and N. P. Rudenko for valuable suggestions and advice.

* This transition is convenient to use for calibrating the β-spectrometer.

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Theory of Interaction of Excitons with the Phonon Field

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THE presence of excitons in an ionic crystal lattice is responsible for its polarization[1-3] which, in turn, reacts on the exciton. This interaction with the lattice is small in crystals with a weak inertial polarization, and also when the effective mass of the electron and hole are approximately equal. In this case the exciton is called "nonpolarizing". The object of this note is to clarify the influence of the inertial polarization of the lattice on several properties of nonpolarizing excitons. The polarization of the medium is calculated by macroscopic methods. The Hamiltonian is written in the effective mass approximation.

The Hamiltonian of the system consists of two terms $H = H_0 + H_i$, where H_0 is the Hamiltonian of the exciton and free phonon field

$$H_0 = \frac{P_R^2}{2m} + H_r + \Sigma_f E(f)\, a_f^+ a_f, \qquad (1)$$

and H_i is the interaction

$$H_i = \sum_{\mathbf{f}} A_f \left[e^{i\mathbf{f}\mathbf{R}} e_f(r)\, a_f + a_f^+ e_{-f}(r)\, e^{-i\mathbf{f}\mathbf{R}}\right]. \qquad (2)$$

Here we employ

$$H_r = \frac{P_r^2}{2\mu} - \frac{e^2}{xr}, \quad e_f(r) = e^{iy_1(\mathbf{f},\mathbf{r})} - e^{-iy_2(\mathbf{f},\mathbf{r})}; \qquad (3)$$

$$A_f = -(e/f)\sqrt{2\pi\hbar\omega c/V}, \; y_1 = \mu_2/m, \; y_2 = \mu_1/m,$$

where μ_1 and μ_2 are the effective masses of the electron and hole, respectively; m and R are the mass and coordinate of the center of mass of the electron and hole; μ and r, the reduced mass and relative coordinate of the electron and hole; f, $E(f) = \hbar\omega$, the wave vector and energy of the phonon; ω the limiting frequency of the longitudinal optical oscillations; $c = 1/x - 1/\epsilon$ with x the square of the index of refraction of the crystal and ϵ, the dielectric constant; a_f is the Bose operator. In zero-order approximation the coordinate R describes the free motion of a quasi-particle of mass m and momentum P_R. The energy spectrum of the system in this approximation consists of a continuum labelled by the quantum states of relative motion.

In calculating the interaction H_i it is convenient to introduce the total momentum of the system, which is a constant of motion, according to the following canonical transformation[4-6]

$$\eta_f = a_f e^{i(\mathbf{f},\mathbf{R})}; \quad \eta_f^+ = a_f^+ e^{-i(\mathbf{f},\mathbf{R})}; \tag{4}$$

$$P_R = P - \hbar \sum_{\mathbf{f}} \mathbf{f}\eta_f^+\eta_f; \quad \mathbf{P} = -i\hbar\,\partial\,\partial\mathbf{R}.$$

In the new variables the Hamiltonian of the system is

$$H = \frac{P^2}{2m} + H_r + \sum_{\mathbf{f}} \mathcal{E}(f)\,\eta_f^+\eta_f \tag{5}$$

$$+ \frac{\hbar^2}{2m}\sum_{\mathbf{f},\mathbf{g}} (\mathbf{f},\mathbf{g})\,\eta_f^+\eta_g^+\eta_f\eta_g \cdot$$

$$+ \sum_{\mathbf{f}} A_f(e_f\eta_f + \eta_f^+ e_{-f}),$$

$$\mathcal{E}(f) = E(f) + \hbar f^2 f^2/2m - \hbar\mathbf{f}\mathbf{P}/m. \tag{6}$$

To obtain corrections to the total energy of the system we use the method of successive diagonalization of the Hamiltonian by a unitary transformation[4,6,7].

$$H' = U^{-1}HU. \tag{7}$$

We find the operator U to be

$$U = e^S; \quad S = \sum_g (\alpha_g\eta_g - \eta_g^+\alpha_g^+). \tag{8}$$

The quantities α_g, which are of the same order as H_i, are assumed to depend on the operators H_r, r. For convenience, we expand the unitary operator U in powers of the small parameter of the theory

$$U = 1 + S + {}^1/_2 S^2 + \ldots \tag{9}$$

The transformed Hamiltonian will be diagonal to second order only if S, and consequently α_g, are defined from the condition $H_i = [S, H_0]$. Then H' is

$$H' = H_0 + {}^1/_2[H_i, S].$$

It is easy to show that α_f is defined by the condition

$$[\alpha_f, H_r] + \alpha_f\mathcal{E}(f) + (\hbar/m)\,\alpha_f\mathbf{P}_F\mathbf{f} = A_f e_f, \tag{10}$$

where $\mathbf{p}_F = \hbar\sum_{\mathbf{f}} \mathbf{f}\eta_f^+\eta_f$ is part of the total momentum belonging to the field.

If $\mathbf{P}_F$ is to be viewed as the mean value of the field momentum, then the operators α_f can be defined as matrices in the space of the eigenfunctions of the operator H_r:

$$(\alpha_f)_{n,m} = \frac{A_f(e_f)_{n,m}}{E_m - E_n + \mathcal{E}'(f)}, \tag{11}$$

$$(\alpha_f)^+_{n,m} = \frac{A_f(e_{-f})_{n,m}}{E_n - E_m + \mathcal{E}'(f)},$$

$$\mathcal{E}'(f) = E(f) + (\hbar^2 f^2/2m) - (\hbar/m)\,\mathbf{f}(\mathbf{P} - \mathbf{P}_F).$$

The diagonal matrix element of the Hamiltonian H' gives the energy of the system with inclusion of second-order corrections

$$(1, \ldots \bar{n}_f \ldots |H'| 1, \ldots \bar{n}_f \ldots) \tag{12}$$

$$= P^2/2m + E_1 + \sum \mathcal{E}(f)\bar{n}_f$$

$$- \sum_{\mathbf{f},m} (1 + \bar{n}_f)\frac{A_f^2|(e_f)_{1,m}|^2}{E_m - E_1 + \mathcal{E}'(f)}$$

$$+ \sum_{\mathbf{f},m} \bar{n}_f \frac{A_f^2|(e_{-f})_{1,m}|^2}{E_1 - E_m + \mathcal{E}'(f)} +$$

$$+ \frac{\hbar^2}{2m}\sum_{\mathbf{f},\mathbf{g}} (\mathbf{f},\mathbf{g})\,\bar{n}_g(\bar{n}_f - \delta_{f,g}),$$

where $\bar{n}_f$ is the mean Planck value, $(e_f)_{1,m}$ is the matrix element between hydrogen-like functions, m is the totality of the three hydrogen quantum numbers, E_m is the corresponding energy level. For zero phonon field and for small momenta P we obtain

$$E = \frac{P^2}{2m} + E_1 - \sum_{\mathbf{f},m}\frac{A_f^2|(e_f)_{1,m}|^2}{E_m - E_1 + E_1(f)} \tag{13}$$

$$- \frac{\hbar^2 g^2}{3m^2}\sum_{\mathbf{f},m}\frac{A_f^2 f^2|(e)_{1,m}|^2}{[E_m - E_1 + E_1(f)]^3},$$

where $E_1(f) = \hbar\omega + \hbar^2 f^2/2m$.

The discrete level of the exciton as a result of the interaction is lowered by an amount

$$E_1' = E_1 - \sum_{f,m} \frac{A_f^2 |(e_f)_{1,m}|^2}{E_m - E_1 + E_1(f)} \,. \tag{14}$$

For the same reason the altered exciton mass μ_{eff} is equal to

$$\frac{1}{\mu_{\text{eff}}} = \frac{1}{m}\left[1 - \frac{2}{3}\frac{\hbar^3}{m}\sum_{f,m}\frac{A_f^2 f^2 |(e_f)_{1,m}|^2}{[E_m - E_1 + E_1(f)]^3}\right]. \tag{15}$$

An approximate evaluation of the sum for the case $\mu_1 = \mu_2$ gives

$$E_1' = E_1\left(1 + \frac{5}{2} c x \frac{\hbar\omega}{|E_1|}\right); \tag{16}$$

$$\mu_{\text{eff}} = m\left(1 + \frac{2}{3} c x \frac{\hbar\omega}{|E_1|}\right),$$

where $E_1 = -\mu e^4 / 2\hbar^2 x^2$.

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Correlation between the Planes of Production and Decay of Λ^0-Particles

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1. RECENTLY, correlations between planes of production and decay of hyperons have been found in a series of experiments[1-3]. These correlations consist in the fact that the angle between these planes is usually small. A theoretical analysis of the effect of correlation was carried out[4] and conclusions were drawn about the large spin of the Λ^0-particle and about the spin of the θ^0-meson being different from zero. It is not without interest, we believe, to consider another approach to this problem in addition to the method of Ref. 4.

2. We consider the z-axis to be perpendicular to the plane of production of the Λ^0-particle, and the direction of motion of the Λ^0-particle is taken as the y-axis. Then the angle between the planes of production and decay will be equal to the azimuthal angle φ. For calculation we now go to the system where the Λ^0 is at rest. it is not difficult to see that the angle φ does not change in this transition.

Let $\Phi_J^M(x, q)$ be the wave function of the system $\Lambda^0 + \theta^0$ describing the state with total angular momentum J and component $\hbar M$ along the z-axis; x and q denote dynamical variables of the Λ^0 - and θ^0-particles, respectively.

The Λ^0-particle as a sub-system of the system $\Lambda^0 + \theta^0$ is characterized by the density matrix $\rho(x', x)$[5]:

$$\rho(x', x) = \int \Phi_J^{M*}(x', q)\, \Phi_J^M(x, q)\, dq. \tag{1}$$

The $\rho(x', x)$ can be expanded in the wave functions of the Λ^0-particle, which we denote by $\Lambda_m(x)$ (in which m can be considered the quantum number of the spin projection on the z-axis)

$$\rho(x', x) = \sum_{m_1, m_2} \rho_{m_1 m_2} \Lambda^*_{m_1}(x')\, \Lambda_{m_2}(x). \tag{2}$$

3. We now turn to the calculation of the distribution of angles φ between the planes of production and decay of the Λ^0-particle.

The total moment of the proton and π^--mesons which are formed in the decay at rest of the Λ^0-particle, coincides with its spin j. If we designate by $\Psi_j^m(\theta, \varphi)$ the angular (and spin) part of the wave function of the stationary state of the system $p + \pi^-$, then

$$\Psi_j^m(\theta, \varphi) = c_1^m \chi^{+1/2} P_l^{m-1/2}(\theta)\, e^{i(m-1/2)\varphi} + c_2^m \chi^{-1/2} P_l^{m+1/2}(\theta)\, e^{i(m+1/2)\varphi}, \tag{3}$$