The Effect of Noncentral Forces on Bremsstrahlung in Neutron-Proton Collisions

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The differential cross section for bremsstrahlung in the collision of high energy nucleons is calculated, including the effect of noncentral forces.

BREMSSTRAHLUNG during collision of high energy nucleons has been studied in a series of theoretical papers[1-5]. In addition, there are many experimental papers which deal with this problem[6]. It is of interest to consider the effect of noncentral forces on the bremsstrahlung during such collisions.

In calculating the differential cross section for bremsstrahlung during neutron-proton collisions, including noncentral forces, we shall, as in the earlier work, use the Born approximation and solve the problem for the nonrelativistic case. We choose the operator describing the nuclear interaction of the particles to be

$$V(r, \vec{\sigma}_p, \vec{\sigma}_n) = (\alpha + \beta\vec{\sigma}_p\cdot\vec{\sigma}_n + \gamma S_{pn}) P_M g_i e^{-\lambda r}/r; \quad (1)$$

here P_M is the Majorana operator, $\lambda^{-1} = 1.18 \times 10^{-13}$ cm, g_1 and g_3 are the depths of the potential well for single and triplet states, respectively[7]

$$g_1 = 0{,}280\,\hbar c, \quad g_3 = 0{,}404\,\hbar c, \quad (2)$$

$$\alpha = 1 - {}^1/_2 g, \quad \beta = g/2, \quad g = 0{,}07, \quad \gamma = 0{,}775,$$

$$S_{pn} = 6(\mathbf{S}\cdot\mathbf{r})^2/r^2 - 2\mathbf{S}^2. \quad (3)$$

In order to shorten the calculations we consider only exchange forces, as shown in earlier work[1]; the intensity of the radiation in neutron-proton collisions under the action of exchange forces is greater than the intensity of the radiation under the action of ordinary forces.

The operator describing the interaction of the proton with the radiation field is

$$H = -(e/Mc)\,\mathbf{p}\cdot\mathbf{A}, \quad (4)$$

where

$$\mathbf{A} = (2\pi)^{1/2}\hbar c \sum_{\varkappa} \frac{\vec{\varepsilon}_{\varkappa}}{\omega_{\varkappa}^{1/2}} \exp\{i\vec{\varkappa}\cdot\mathbf{r}\}(a_{\varkappa} + a_{-\varkappa}); \quad (5)$$

$a_{\varkappa}$ and $a_{-\varkappa}$ correspond to emission and absorption of a photon, respectively, $\omega_{\varkappa} = \hbar\nu_{\varkappa}$ is the photon energy. Thus we shall consider only electric radiation.

We shall calculate the matrix element H' which determines the probability of radiation of a photon:

$$H' = \frac{H_{AI}V_{IF}}{E_A - E_I} + \frac{V_{AII}H_{IIF}}{E_A - E_{II}}. \quad (6)$$

The wave functions for the neutron-proton system in Born approximation are:

$$\psi_A = \exp\left\{\frac{i}{\hbar}\mathbf{p}_0\cdot\mathbf{r}\right\}\chi_{m_{S^0}}^{S^0}; \quad \psi_I = \exp\left\{\frac{i}{\hbar}\mathbf{p}_I\cdot\mathbf{r}\right\}\chi_{m_{S'}}^{S'};$$

$$\psi_F = \exp\left\{\frac{i}{\hbar}\mathbf{p}\cdot\mathbf{r}\right\}\chi_{m_S}^{S};$$

$$\psi_{II} = \exp\left\{\frac{i}{\hbar}\left(\mathbf{p}_{II}\cdot\mathbf{r}\right)\right\}\chi_{m_{S''}}^{S''}.$$

The index A refers to the initial state, I indicates the intermediate state in which the photon has been emitted but the nuclear interaction has not yet occurred, II indicates the intermediate state in which the nuclear interaction has already occurred but the photon has not yet been emitted, while F refers to the final state of the system.

$\chi_{m_{S^0}}^{S^0}, \chi_{m_{S'}}^{S'}, \chi_{m_{S''}}^{S''}, \chi_{m_S}^{S}$ are spin functions characterizing, respectively, the initial, intermediate and final states of the spin and its projection on the axis of quantization. The superscript indicates the value of the total spin in the corresponding state, while the subscript is the value of the spin projection on the axis of quantization. $\mathbf{p}_0, \mathbf{p}_I, \mathbf{p}_{II}$ and $\mathbf{p}$ are the relative momenta of neutron and proton in the initial, intermediate and final states, respectively, and $\vec{\varkappa}$ is the momentum of the photon.

If we calculate all the matrix elements and substitute in (6), we get

$$H' = \frac{2e}{Mc}\{p_{0\epsilon}{\to}q_{25+1}[(\alpha-3\beta)V_{p_0+p} \qquad (7)$$

$$+(2\beta-\gamma)V_{p_0+p}S(S+1)$$

$$+6\pi\gamma LS(S+1)-6\pi\gamma Nm_S^2]\,\delta_{m_1 o m_{1'}}\delta_{m_{1'} m_S}\delta_{1'S}\delta_{1^oS}$$

$$+(2\beta-\gamma)V_{p_0+p}S''(S''+1)$$

$$-p_{\epsilon}\,g_{25+1}[(\alpha-3\beta)V_{p_0+p}+6\pi\gamma LS''(S''+1)$$

$$-6\pi\gamma Nm_{S''}^2]\,\delta_{m_1 o m_{S^o}}\delta_{1^o S''}\delta_{m_{1''} m_S}\delta_{1''S}\}.$$

Here δ is the Kronecker delta and

$$L=\Big\{\frac{2}{\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2}$$

$$+\frac{\lambda}{\hbar^{-3}|\mathbf{p}_0+\mathbf{p}|^3}\arccos\frac{\lambda^2-\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2}{\lambda^2+\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2}\Big\},$$

$$N=\Big\{\frac{6\lambda^2+(4/\hbar^2)|\mathbf{p}_0+\mathbf{p}|^2}{\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2(\lambda^2+\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2)}$$

$$+\frac{3\lambda}{\hbar^{-3}|\mathbf{p}_0+\mathbf{p}|^3}\arccos\frac{\lambda^2-\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2}{\lambda^2+\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2}\Big\},$$

$$V_{p_0+p}=2\pi/[\lambda^2+\hbar^{-2}|\mathbf{p}_0+\mathbf{p}|^2].$$

Summing H' over the values of total spin and spin projection in the intermediate states, squaring, summing over values of spin and spin projection in the final state, and then averaging over all initial spin states, we obtain, after some transformations, the differential effective cross section in the form:

$$d\sigma=\frac{e^2g_3^2}{64\pi^4\hbar^5}\frac{p}{p_0}\frac{1}{\omega_\varkappa^3}(\mathbf{p}_{0\epsilon}-\mathbf{p}_{\epsilon})^2\Big\{3[(1-2\gamma)V_{p_0+p} \qquad (8)$$

$$+12\pi\gamma L]^2+72\pi^2\gamma^2N^2-24\pi\gamma N[(1-2\gamma)V_{p_0+p}$$

$$+12\pi\gamma L]+(\alpha-3\beta)^2\frac{g_1^2}{g_3^2}V_{p_0+p}^2\Big\}\varkappa^2\,d\varkappa\,d\Omega_\varkappa\,d\Omega_p.$$

Summing this expression over the two photon polarizations, and integrating over the angle of emergence of the photon and the azimuthal angle of the scattered particle, substituting the values of V_{p_0+p}, L and N, we find

$$d\sigma=2\frac{g_3^2}{p_0^2c^2}\frac{e^2}{\hbar c}\frac{p^2}{1-p^2}(\mathbf{n}_0-\mathbf{p})^2\Big\{\frac{1}{\eta^2}[(1-2\gamma)^2$$

$$+(g_1^2/3g_3^2)(\alpha-3\beta)^2+48\gamma^2+8\gamma(1-2\gamma)]$$

$$+\frac{96\gamma^2\lambda^2}{\xi^2\eta^2}+\frac{72\gamma^2\lambda^4}{\xi^4\eta^2}+\frac{24\gamma^2\lambda(3\lambda^2+2\xi^2)}{\xi^5\eta}\arccos\frac{\zeta}{\eta}$$

$$+\frac{18\gamma^2\lambda^2}{\xi^6}\arccos^2\frac{\zeta}{\eta}\Big\}dp\sin\theta\,d\theta. \qquad (9)$$

We have introduced the symbols

$$\xi=(1/\hbar)|\mathbf{n}_0+\mathbf{p}|,\quad \eta=\lambda^2+\xi^2,\quad \zeta=\lambda^2-\xi^2;$$

$\mathbf{n}_0$ is a unit vector along $\mathbf{p}_0$, p is measured in units of p_0, λ in units of $p_0/\hbar$, and θ is the angle of scattering.

We note that if we set $\alpha=1$, $\beta=\gamma=0$ (the central forces), we get a formula for the differential cross section similar to that obtained in earlier papers.

Investigation of the differential cross section (9) shows that the inclusion of noncentral forces gives a sharper maximum of the radiation in the region of scattering angles close to π even for low photon energies.

In conclusion, it is a pleasant duty to express my thanks to L. N. Rozentsveig for suggesting the problem and for many valuable suggestions, and also to Prof. A. I. Akhiezer for his interest in the work.

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