Concerning Rayski's Bilocal Field Theory

V. A. ZHIRNOV
P. N. Lebedev Physical Institute
Academy of Sciences, USSR
(Submitted to JETP editor, December 2, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 425-426
(February, 1956)

Wave equations for a Ψ-function depending on x_i and u_i are investigated in some of Rayski's papers[1,2] in a way similar to that used by Ginzburg and Tamm [3] and by Yukawa.[4] Ginzburg and Silin[5] have already made some critical notes to Ref. 2, showing in particular the incompatibility of Eqs. (22′) and (22″) of that paper, as well as some other shortcomings.

The system of equations

$$L_1\Psi \equiv \left(\frac{\partial^2}{\partial x_i \partial x_i} - \varkappa^2 - 4c\left[-\frac{\partial^2}{\partial u_i \partial u_i}\right.\right. \tag{1}$$

$$\left.\left. + \frac{1}{u_i u_i}\left\{\lambda^4\left(\frac{\partial^2}{\partial x_i \partial u_i}\right)^2 + \left(u_i\frac{\partial}{\partial u_i}\right)^2 + 2u_i\frac{\partial}{\partial u_i}\right\}\right]\right)\Psi(x_i, u_i) = 0,$$

$$L_2\Psi \equiv u_i\frac{\partial}{\partial x_i}\Psi(x_i, u_i) = 0,$$

$$L_3\Psi \equiv \left(\lambda^2\frac{\partial^2}{\partial x_i \partial x_i} - \lambda^{-2}u_i u_i\right)\Psi(x_i, u_i) = 0,$$

proposed in some new papers[6] is a compatible one, as L_1, L_2, and L_3 commute with each other. However even this new system has some shortcomings. The mass spectrum derived from (1)

$$m_0^2 = \frac{\varkappa^2}{2} \pm \sqrt{\frac{\varkappa^4}{4} + \frac{4c}{\lambda^4}l(l+1)}, \qquad l = 0, 1, 2, \ldots \tag{2}$$

not only contains imaginary masses $m_0^2 < 0$, which must be thrown out because of the condition of boundedness of the solution $\Psi \sim e^{\pm|m_0|t}$ in 4-space (see Ref. 5), but also gives the value $m_0 = 0$, in contradiction to the recipe for finding the mass spectrum (going to the center-of-mass system). Furthermore, interaction with a field cannot be introduced into the system (1) in the usual simple way, and presents an additional problem. In addition to this, we shall show that the system (1), as well as the analogous set in Ref. 5, contains solutions with spacelike momenta, corresponding to signal velocities faster than light. To do this, let us go to the system of reference where $E = 0$, $p_1 = p_2 = 0$, $p_3 \neq 0$, and let us introduce generalized spherical coordinates in u_i- space:

$$\begin{aligned} &u_1 = u\cosh\chi\sin\theta\cos\varphi, \quad u_3 = u\cosh\chi\cos\theta, \quad 0 < u < \infty, \quad 0 \leqslant \theta \leqslant \pi, \\ &u_2 = u\cosh\chi\sin\theta\sin\varphi, \quad u_4 = iu\sinh\chi, \quad -\infty < \chi < \infty, \quad 0 \leqslant \varphi \leqslant 2\pi. \end{aligned} \tag{3}$$

Then from Eqs. (1) we get

$$\left[p_3^4 + \varkappa^2 p_3^2 - \frac{4c}{\lambda^4}\left(\frac{1}{\cosh\chi}\frac{\partial}{\partial\chi}\left(\cosh\chi\frac{\partial}{\partial\chi}\right) - \frac{1}{\cosh^2\chi}\frac{\partial^2}{\partial\varphi^2}\right)\right]\Psi(x_i, u_i) = 0. \tag{4}$$

Separating the variables, $\Psi(x, u) = \Psi(x)R(\chi)Y(\varphi)$, we find

$$\left(p_3^4 + \varkappa^2 p_3^2 - \frac{4c}{\lambda^4}\mu\right)\Psi(x) = 0, \tag{5}$$

$$\frac{\partial^2}{\partial\varphi^2}Y(\varphi) = -\lambda^2 Y(\varphi),$$

$$\left(\frac{\partial^2}{\partial\chi^2} + \frac{\sinh\chi}{\cosh\chi}\frac{\partial}{\partial\chi} + \frac{\lambda^2}{\cosh^2\chi} - \mu\right)R(\chi) = 0.$$

By putting $v = R\cosh^{\lambda}\chi$, $y = \sinh\chi$, the last equation of (5) becomes

$$(1 + y^2)\frac{d^2v}{dy^2} - 2(\lambda - 1)y\frac{dv}{dy} + (\lambda^2 - \lambda - \mu)v = 0. \tag{6}$$

If we try to solve Eq. (6) by a finite sum $v = \sum_{n=0}^{n_{\max}} a_n y^n$, we get the eigenvalues $\mu = l(l+1)$. $l = 0, 1, 2, \ldots$ Then from the first of equations (5) we can find the spectrum of eigenvalues of the momentum:

$$p_3^2 = -\frac{\varkappa^2}{2} \pm \sqrt{\frac{\varkappa^4}{4} + \frac{4c}{\lambda^4} l(l+1)}, \qquad (7)$$

$$l = 0, 1, 2, \ldots$$

It is clear from (7) that for $E=0$ we can have $p_3^2 > 0$, i. e., there is a system of reference where $E^2 = p^2 - a^2$ for real E, p, and a, and $v = \partial E/\partial p = p(p^2 - a^2)^{-1/2} \geq 1$. Insofar as solutions with velocity greater than that of light can not be eliminated by the condition of finiteness, it is clear that equations (1), as they have physically unallowable solutions, cannot be considered satisfactory.

In conclusion, I express my thanks to Professor V. L. Ginzburg for valuable advice.

[1]J. Rayski, Proc. Phys. Soc. (London) 64, 957 (1951); Acta. Phys. Pol. 11, 109 (1951).

[2]J. Rayski, Nuovo Cimento 10, 1729 (1953).

[3]V. L. Ginzburg and I. E. Tamm, J. Exptl. Theoret. Phys. (U.S.S.R.) 17, 227 (1947).

[4]H. Yukawa, Phys. Rev. 77, 219 (1950); 91, 415, 416 (1953).

[5]V. L. Ginzburg and V. P. Silin, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 116 (1954).

[6]J. Rayski, Fortsh. Physik 2, 165 (1954); Acta Phys. Pol. 14, 107 (1955); Nuovo Cimento 2, 255 (1955).

Translated by E. J. Saletan
81

The Leading Role of the Vibration Properties of Plasma and the Effect of the Gas Atoms on these Properties

I. P. Bazarov
Moscow State University

(Submitted to JETP editor, February 21, 1955)
J. Exptl. Theoret. Phys. (U.S.S.R.) 30, 426
(February, 1956)

In Eq. (5) of our earlier work[1], the second term of the right-hand member should have a plus sign, and not a minus sign. The necessity for this can be easily recognized if one compares the particular case of Eq. (5) in the form of Eq. (10) with Eq. (2) which is identical to (10) and in which the sign of the second term is indicated properly; the identity of (2) and (10) are indicated in Ref. 1. The incorrectness of the sign is noted in Ref. 2, although it does not change all the Eqs. (7) – (10) which follow, or the conclusions drawn from them. The basic result of the work – the prediction of the existence of anodic strata under several conditions of charge ($l_1 < 0$) – does not depend on the sign of the second term of Eq. (5) (only the numerical value of the period d can depend on this sign), and this result was actually confirmed by experiment, as was indicated in Ref. 1. This results in the fact that, although in the developed theory of strata[1,3] the effect of the atoms of gas on the vibration of the plasma is not taken into consideration (for which reason, in part, we obtain only qualitative agreement of theory with experiment in certain cases), this theory correctly indicates, however, the leading role of the vibration properties of the plasma under certain conditions of gaseous discharge – the collective interaction between charged particles.

Calculation of the effect of atoms of the gas on the behavior of the plasma is important and necessary; however the means, borrowed from a number of other research papers, by which this calculation is carried out in Ref. 2 is unsatisfactory, since the approximation employed in Ref. 2 in the kinetic equation of the Boltzmann term for collision expressed by $-(f - f_0)/\tau = -f_1/\tau$ demands, within the frame of reference of linear approximation, the disappearance of the integral $\int f_1 d\mathbf{v}$, while in the other equation of the solved system this integral, with the same approximation, must not be equal to zero. For this reason, Ref. 2 bears a solely computational character, with no knowledge as to what sort of approximation it has with relation to a real plasma, since the solved system of equations suffers from internal contradictions.

I take advantage of this opportunity to point out that in Ref. 1, Eq. (4), the term $4\pi\rho$ must have a minus sign (and not a plus sign, as printed) and paragraph 4 should be paragraph 3. These errors bear no relationship to the correctness of the work within the frame of reference of the initial approximations.

[1]P. Bazarov, J. Exptl. Theoret. Phys. (U.S.S.R.) 21, 711 (1951).

[2]A. A. Luchina, J. Exptl. Theoret. Phys. (U.S.S.R.) 28, 18 (1955); Soviet Phys. JETP 1, 12 (1955).

[3]A. A. Vlasov, *The theory of many particles*, Gostekhizdat, 1950.

Translated by A. Certner
82