

where $V_0 \cos \varphi$ is the accelerating potential; $\bar{\tau}$ — the mean value of the quanta emitted by the electron at the given energy E ; $a_1 - \bar{a}_1$ — the deviation (fluctuation) of the number of quanta emitted per unit time, from its mean value; the coefficient Q is determined by the accommodation parameters

$$Q = 2 + \bar{\psi}_\Phi - \bar{\psi}_D + \bar{\psi}_H / |n|, \quad (6)$$

where $\bar{\psi}_F$ and $\bar{\psi}_D$ are the averages of $\psi(\theta)$ over focussing ($n < 0$) and defocussing ($n > 0$) sectors. The calculations show that $Q \sim 4$. Note that the coefficient E of $d\eta/dt$ in (5) can be neglected in that range of energy where radiation (and its fluctuation) is important. Assuming a linear time dependence, we can solve Eq. (5). For the mean square of η , we find:

$$\bar{\eta}^2 = \frac{55 V^3}{64} \frac{\hbar c}{e^2} \frac{q}{\sigma} \frac{\bar{\psi}_H}{Q |n|} \text{ctg } \varphi_0 \frac{mc^2}{E}, \quad (7)$$

where the factor $\bar{\psi}_H / Q |n|$ is approximately equal to $1/|n|$. Equation (7) can be used to find the azimuthal dimensions of the electron concentration, which are of some interest for evaluation of loss by coherent radiation.

The largest radial deviation ρ_{max} of the instantaneous orbit is determined by (1), when $\psi(\theta)$ is given its maximum value $\psi_{\text{max}}(\theta)$. Using (1), (2), (5), and (7) we obtain the mean square value:

$$\bar{\rho}_{\text{max}}^2 = (55 V^3 / 96) (\hbar R / mc) (\psi_{\text{max}}^2 / Q |n|^2) (E / mc^2)^2. \quad (8)$$

Note that exactly the same correction characterizes the instantaneous orbit in a strong focussing betatron, in which the radiation losses compensate on the average. The evaluations show that, near the center of steadiness ($\sqrt{|n|} v \approx \pi/2$) the factor $\psi_{\text{max}}^2 / Q |n|^2 \approx 10 / |n|^2$, while in the case of weak focussing^{2,3} it is replaced by the expression $1 / (1-n)(3-4n)$, which, for $n \sim 0.6-0.7$ is ≈ 10 . The small dependence of θ_{max}^2 on E or t is explained by the influence of powerful extinction linked to the large magnitude of the mean radiation losses. This extinction has a simple physical meaning. It can be shown that it corresponds to the fact that when the orbit is displaced along the radius, the particle radiates in such a way that the change of its energy tends to restore the instantaneous orbit in its equilibrium position.

If, for instance, we let $H_{\text{max}} \approx 10^4$ oersteds, then, according to Eq. (8), we get the evaluation $(\bar{\rho}_{\text{max}}^2 / \text{cm}^2)^{1/2} \approx E_{\text{BeV}}^{2/3} / |n|$, which shows that even for $E \approx 5-10$ BeV is only of the order

of a centimeter. The considered effect has thus, by itself, no appreciable effect on acceleration.

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Translated by E. S. Troubetzkoy
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Auger Effect in Heavy Atoms

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(Submitted to JETP editor September 15, 1955)

J. Exper. Theoret. Phys. USSR **30**, 209-210

(January, 1956)

EXCITED atoms, in which one of the interior electrons (say a K-electron) is missing, undergo transitions to a lower energy state by means of radiation of a quantum or by a nonradiative transition with the emission of an electron (Auger Effect). The total number of transitions per unit time $(1/c)_\epsilon$ has been obtained for the nonrelativistic case for arbitrary Z with the aid of the Coulomb function.¹

Only in the case of the interaction of L-electrons (for $Z = 47$) has the screening of the atomic nucleus been taken into account.²

In first order perturbation theory,

$$\frac{1}{\tau} = \frac{2\pi}{k} |V(n_1 l_1 m_1, n_2 l_2 m_2 | 100, k l' m')|$$

$$- V(n_2 l_2 m_2, n_1 l_1 m_1 | 100, k l' m')|^2$$

and

$$\frac{1}{\tau} = \frac{2\pi}{k} |V(n_1 l_1 m_1, n_2 l_2 m_2 | 100, k l' m')|^2$$

for the interaction of electrons with parallel and antiparallel spins, respectively. Here we have the matrix elements of the operator $V = 1/|r_1 - r_2|$, which corresponds to a transition from a state with quantum numbers $n_1, l_1, M_1; m_2, l_2, m_2$ to a state

with quantum numbers $l, 0, 0; l', m'$ and with momentum k of the emerging electron (atomic units are used here). In the radial integral which enters into the matrix element, the important region is evidently $r_1, r_2 \ll 1/Z$. In this region the wave function of the initial state coincides with the Coulomb function with accuracy to a normalization factor A_{nl} . To find A_{nl} , we make use of the fact that the initial and Coulomb functions are quasi-classical for $r \gg 1/Z$. If we write out the quantization rule and the normalization condition for them, we get, after some simple transformations,

$$A_{nl} = B_{nl} \sqrt{(n^3/Z^2) (\partial E_{nl} / \partial n)},$$

where B_{nl} is the normalization coefficient of the Coulomb function, E_{nl} is the energy of the corresponding level. In this case, it is assumed in the calculations that for $n = 1, 2$, $A_{nl} = B_{nl}$.

For large n_2 and for $n_1 = 2$,

$$1/\tau = A_{l_1 l_2} \partial E_{n_2 l_2} / \partial n_2,$$

where $A_{l_1 l_2}$ is almost independent of n_2 . Therefore, replacing the sum over n_2 , beginning with $n_2 = 3$, by an integral, we can write the total number of Auger-transitions per unit time in the form:

$$\begin{aligned} (1/\tau)_\Sigma &= (1/\tau)_{L-L} + \sum_{l_1=0}^1 \sum_{l_2=0}^2 A_{l_1 l_2} \int_{n_2=3}^{\infty} (\partial E_{n_2 l_2} / \partial n_2) dn_2 \\ &+ \sum_{l_1=0}^1 A_{l_1 3} \int_{n_2=4}^{\infty} (\partial E_{n_2 3} / \partial n_2) dn_2 + \dots \\ &= (1/\tau)_{L-L} - \sum_{l_1=0}^1 \sum_{l_2=0}^2 A_{l_1 l_2} E_{3 l_2} - \sum_{l_1=0}^1 A_{l_1 3} E_{4 3} - \dots, \end{aligned}$$

where small terms of the type

$$A_{l_1 l_2} (\partial E_{n_2 l_2} / \partial n_2) (\partial E_{n_1 l_1} / \partial n_1) \quad (n_1 > 2).$$

are discarded.

In the first approximation,

$$(1/\tau)_\Sigma = (1/\tau)_{L-L} - \sum_{l_1=0}^1 \sum_{l_2=0}^2 A_{l_1 l_2} E_{3 l_2}.$$

We can put $E_{3 l_2}$ in the form:³

$E_{3 l_2} = -(Z - s_l)^2 / 18$, where s_l is the screening constant

For $(1/\tau)_{L-L}$, making use of the well-known results of reference 2, we obtain for $Z = 47$, after some computation,

$$(1/\tau)_\Sigma = 45.9 \text{ atomic units} \quad (1)$$

A quantity defined from experiment is the coefficient

$$\alpha_K = Z^4 (1/\tau)_\Sigma / (1/\tau)_{\text{rad}},$$

where $(1/\tau)_{\text{rad}}$ is the number of radiative transitions per unit time. For $Z = 47$: $(1/\tau)_{\text{rad}} = 0.197$ atomic units.⁴ Making use of Eq. (1), we obtain $\alpha_K = 1.14 \times 10^6$.⁶ The experimental value is⁴ $\alpha_K = 1.14 \times 10^6$. Calculation with the help of the Coulomb functions gives $\alpha_K = 1.65 \cdot 10^6$.

The authors express their deep gratitude to Prof. A. B. Migdal who directed the present research.

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Translated by R. T. Be yer
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Scattering of Fast Neutrons by Semi-transparent Nonspherical Nuclei

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(Submitted to JETP editor September 27, 1955)

J. Exper. Theoret. Phys. USSR **30**, 210-212
(January, 1956)

THE scattering of fast neutrons by an opaque nonspherical nucleus with spin zero has been studied by Drozdov^{1,2}. The scattering of fast neutrons by an even-even semitransparent nonspherical nucleus is considered in the present work.

According to Bohr and Mottelson^{3,4}, the even-even nuclei in their rotational states have the form of an ellipsoid of revolution and the wave function of such a state is a spherical harmonic $Y_{lm}(\omega)^*$, where l, m are the spin of the nucleus and the projection of that spin, ω represents the angles ϑ, φ which characterize the direction of the axes of symmetry of the ellipsoid. The rotational levels are determined by the formula $E_l = (\hbar^2/2I)l(l+1)$, $l = 0, 2, 4, \dots$, where I is the effective moment of