# Concerning the Cross Section of Overcharging of Slow Ions in their own Gas

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THE value of the effective cross section of overcharging for collisions between ions and atoms of the same gas can be determined from experiments on ion beams as well as from the measurements of the drift velocity of ions in an electric field in a gas. Recent methods, however, require a knowledge of the character of the interaction between ions and atoms. As is well known, with the collision of an ion and atom there occurs a resonance effect along with the possibility of charge exchange. As shown in theoretical calculations[1-4] the effective cross section of overcharging in this case is so large that exchange of momentum is not essential for the majority of collisions. Therefore, a convenient model for describing the interaction of an ion with an atom of the same gas appears to be the proposed Sena model for pure overcharging. This is confirmed also by the measurements of Ziegler[6]. On the basis of this model, wherein it is proposed that the cross section of overcharging does not depend on the relative velocity, Sena obtains, in an elementary way, a formula for the drift velocity of ions in a large field. An exact expression for this case, obtained by means of a solution of the kinetic equation[7] has the form

$$\bar{u} = (2eE/\pi mqN)^{1/2}, \tag{1}$$

where $\bar{u}$ is the drift velocity, $e$ and $m$ the charge and mass of the ions, $q$ the cross section of overcharging, $E$ the field intensity, and $N$ is the atomic concentration. Expression (1) differs from the formula of Sena in the value of the multiplier. In the case of small fields it is possible to make use of the well-known formula[8]

$$\bar{u} = \frac{3\pi^{1/2}eE}{8(mkT)^{1/2}N}\left[\int_0^\infty Qe^{-\lambda^2}\lambda^5 d\lambda\right]^{-1}, \tag{2}$$

where $T$ is the gas temperature and $k$ is the Boltzmann constant,

$$Q = 2\pi\int_0^\pi (1 - \cos\theta)\,\sigma(\theta)\sin\theta\, d\theta, \tag{3}$$

$\sigma(\theta)\sin\theta\, d\theta\, d\phi$ is the effective differential cross section for scattering, $\theta$ is the angle of scattering in the center of mass system. In the case under consideration $\sigma(\theta)$ has the form of a sharp maximum near $\theta = 0$ and near $\theta = \pi^2$. The first corresponds to elastic scattering and the second to overcharging. For angles perceptibly different from 0 and $\pi$, $\sigma(\theta)$ is small. Therefore, it is possible to write the approximation

$$\sigma(\theta) = \frac{1}{2\pi}\left[q\delta(1 + \cos\theta) + q_1\delta(1 - \cos\theta)\right], \tag{4}$$

where $q$ is the cross sectoin for pure overcharging and $q_1$ the cross section for scattering; $\delta(x)$ is a delta function. With the hypothesis that $q$ is not dependent on velocity, we obtain from (2) and (3)

$$\bar{u} = 3\pi^{1/2}eE/16\,(mkT)^{1/2}qN. \tag{5}$$

Equation (5) differs in its multiplicative factor from the approximate formula obtained in reference (7) by 10%.

By making use of Eq. (1) it is possible to calculate the cross section of overcharging from the experimental data of Hornbeck and Varney[9,10] for the motion of the ions of an inert gas in a strong field. In the case of a weak field it is possible to make use of Eq. (5) and calculate the cross section of overcharging from the experimental data of Biondi and Chanin[11] for the motion of ions. The values calculated by such a method are shown in the third and fourth columns of the Table. In the first and second columns, there are given for comparison the total cross section $q_n$ and the cross section of overcharging $q$ obtained by Ziegler from experiments with ion beams at ion of ~ 1 ev. The magnitude of the cross section in the Table is given in $10^{-15}$ cm$^2$. The cross section obtained from the value of the drift velocity in large fields corresponds to an energy of ion drift of the order of 1-2 ev. They are not in bad agreement with the data of Ziegler (with the exception of helium). The cross sections obtained from the data for the drift velocity in small fields also corresponds to the thermal energy of the ions, but does not exceed the cross sections near large energies. Thus the data in the Table show that the model for pure overcharge with velocity independence of cross section yields fairly good results in a sufficiently wide interval of ion energies. In the work of reference 7 it is shown that this model give good results for the drift velocity, according to the data of references 9 and 10, for all values of the ratio $E/p$ (where $p$ is the reduced pressure) reached in the experiment if they use the cross sections calculated from their data for large fields and the true thermal motion of the atoms.

In the work of Hornbeck and Wannier[12,13] it is attempted to explain the results of measurements of the drift velocity of ions in inert gases by means of a model which assumes isotropic scattering of the ions and atoms. They use this model to calculate the cross section from the drift velocity near large fields and arrive at a value which is in poor agreement with the results of Ziegler (Ziegler attempted to improve the agreement by subtracting (without presenting a reason) the gas kinetic cross section from the actual cross section). Furthermore, this model is not entirely in agreement with the relation obtained by Ziegler between the total cross section and the cross section of overcharging. Recently Wannier[14] rejected this model, and recognized that the model of pure overcharging is actually better. However, the cross section of overcharging calculated by him from the drift velocity in large fields [ with the help of Eq. (1)] is not correct, apparently due to an error in conversion. The cross sections of overcharging calculated by the formula of Demkov[3] for He, Ne, A, Kr, Xe are equal respectively to 2.6, 3.0, 4.1, 4.6, and $5.3 \times 10^{-15} \mathrm{cm}^2$. As is evident from the Table, agreement is best of all for He and Ne, and worse for A, Kr, and Xe. This also explains certain divergences between the theoretical and experimental curves for the cross section of the drift velocity of ions in these gases given in reference 7. This divergence is, in our opinion, entirely attributable to the inaccuracy of Demkov's calculation. We note that the determination of the cross section of overcharging made by Sena[15] leads to still worse agreement with the experimental values of the cross section.

| Gas | $q_n$[6] | $q$[6] | $q$[9, 10] | $q$[11] |
|---|---|---|---|---|
| He . . . . . . . | — | 4.1 | 2.6 | 3.6 |
| Ne . . . . . . . | 5 | 4.4 | 3.15 | 4.2 |
| A . . . . . . . | 9 | 5.5 | 6.5 | 7.5 |
| Kr . . . . . . . | — | 9 | 7.6 | 9.2 |
| Xe . . . . . . . | 13 | 10.3 | 9.3 | 11.3 |

The calculated mobilities of Massey and Mohr[1], and of Holstein[4] appear to be more accurate, but up to now calculations have only been made for small fields. They agreed well with results of experimental work[11].

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# The Effect of Pressure on the Superconductivity of Cadmium

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THE effect of pressure on the displacement of the critical temperature of a superconductor has been studied by many authors[1-7]. However, they studied only superconductors whose transition temperatures are above 1°K. It seemed interesting to study the effect of pressure on the properties of superconductors with low transition temperatures. Such a superconductor is cadmium, which goes into the superconducting state at 0.54°K. We measured the critical magnetic field vs. the temperature of samples of polycrystalline cadmium without pressure as well as with pressure.

To obtain temperatures in the interval between 0.06 - 0.6°K we used the method of adiabatic demagnetization of a paramagnetic salt. The pressure was obtained by freezing water in a bomb of fixed volume[8]. Heat contact between the bomb