

stood the number of levels with energy of excitation not exceeding  $E$ . In the immediate neighborhood of the fission energy  $E_f$  we have  $N^*(E - E_f) \approx 1$  and, therefore,

$$\Gamma_f \sim D / 2\pi. \quad (2)$$

As is known<sup>3</sup>, the classical statistics are applicable for the condition  $T \gg \hbar\omega$ , where  $T$  is the temperature and  $\omega$  is the vibration frequency of the nuclear configuration, associated with fission. In the region  $T \ll \hbar\omega$  it is impossible to use the estimates (1) and (2). In this region, in order to make an estimate of the fission width, valid in all temperature intervals, we represent it in the form  $\Gamma_f \approx (\hbar\omega/2\pi)\bar{w}$  where  $w$  is the probability density on the dividing degree of freedom of the energy, exceeding the fission threshold  $E_f$ . We estimate this probability, having applied a microcanonical distribution<sup>3</sup> to the nucleus. This distribution takes into account not only the law of conservation of energy, but also the law of conservation of momentum—all states, compatible with the conservation laws, being equally probable. Finally, we obtain

$$\Gamma_f \approx \frac{\hbar\omega}{2\pi} \frac{N^*(E - E_f)}{N^*(E)} \quad (3)$$

As it was shown by Landau<sup>4</sup>, the density of levels with fixed moment varies, in principle, according to the law  $e^{S(E)}$  where  $S$  is the entropy. Since  $dS/dE = 1/T$ , we have  $N^*(E) \approx T/D^*$  where  $D^*$  is the separation of the levels, which are bound. In view of the fact that, near the threshold,  $N^*(E - E_f) \approx 1$ , we obtain

$$\Gamma_f \sim (\hbar\omega/T)(D^*/2\pi). \quad (4)$$

In the classical region,  $T \gg \hbar\omega$ , the estimates (3) and (4) pass to that obtained by Bohr and Wheeler (1) and (2), since in this case, as it is not difficult to see,  $D^* \approx (T/\hbar\omega)D$ . On the other hand, in the opposite limiting case  $T \ll \hbar\omega$  we have  $D^* = D$  and therefore,

$$\Gamma_f \sim (\hbar\omega/T)(D/2\pi). \quad (5)$$

The actual values of the quantities  $\hbar\omega$  and  $T$  for heavy nuclei are not known with any accuracy. We assume for computation that both these values are

of the order of 1 mev; however, they can vary several fold. From this point of view the estimate (2) must be considered as the lower estimate of the fission width.

If the full energy of the composite nucleus  $E$  significantly surpasses the fission energy, then, as it is not difficult to see from (3)

$$\Gamma_f \sim \frac{\hbar\omega}{2\pi} \exp\left\{-\frac{E_f}{T}\right\}.$$

This rate was also obtained by Kramers<sup>2</sup>; evidently, it is not applicable in the immediate vicinity of the fission threshold.

<sup>1</sup> N. Bohr and J. Wheeler, Phys. Rev. 56, 426 (1939)

<sup>2</sup> H. Kramers, Physica 7, 284 (1940)

<sup>3</sup> L. Landau and E. Lifshitz, *Statistical Physics*, State Technical Publication, 1951

<sup>4</sup> L. Landau, J. Exper. Theoret. Phys. USSR 7, 819 (1937)

Translated by A. A. Silvidi  
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## Radiation of Molecules in the Presence of a Strong High-Frequency Field

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(Submitted to JETP editor July 11, 1955)

J. Exper. Theoret. Phys. USSR 29, 878-880

(December, 1955)

**T**HE absorption of high-frequency radiation in a molecular gas was examined in references 1,2. It will be shown in the present paper that, besides the absorption (and saturation of levels) at a frequency  $\omega \approx \omega_{12} = (E_1 - E_2)/\hbar \equiv \omega_0$  ( $E_1$  and  $E_2$  are energy levels of the molecules), there appears a radiation at a frequency  $\Omega_0$  [see Eq. (6)], which depends on the matrix element of the dipole moment, corresponding to the transition  $E_1 \rightarrow E_2$ , and on the field potential of the frequency  $\omega$ .

We assume that the following conditions are satisfied<sup>1-3</sup>: a)  $T \ll 1/\omega \ll \tau$ , where  $T$  is the duration of the molecular collision, and  $\tau$  is the mean time between collisions; b)  $|\omega - \omega_0| \lesssim 1/\tau$  and  $|\omega - \omega_{mn}| \gg 1/\tau$ , where  $m, n = 1, 2$ ; c) molecular collisions re-establish the Boltzman energy

distribution; d)  $\omega_0 \gg \Omega_0$ .

For any ensemble of quantum mechanical systems the density matrix satisfies the following equation:

$$i\hbar(\partial\rho/\partial t) = H\rho - H\rho. \quad (1)$$

In our case the Hamiltonian  $H$  of the molecule in the external field  $F(t) = F \sin \omega t$  equals

$$H = H_0 - \vec{\mu}F(t) = H_0 + V \sin \omega t, \quad (2)$$

where  $H_0$  is the Hamiltonian of the free molecule, and  $\vec{\mu}$  is the operator of the dipole moment. Selecting the representation in which  $H_0$  is diagonal, we get

$$i\hbar \frac{\partial \rho_{mn}}{\partial t} = \sin \omega t \sum_l (V_{nl} \rho_{lm} - \rho_{nl} V_{lm}), \quad (3)$$

where  $n, m = 1, 2$  [according to condition (b)].

According to condition (c), at the moment of collision  $t = t_0$ :

$$\rho_{12} = \rho_{21} = 0, \quad D \equiv \rho_{22} - \rho_{11} = D_0, \quad (4)$$

where  $D_0 = \rho_{22}^0 - \rho_{11}^0$  is the difference in the populations of levels  $E_2$  and  $E_1$  for a free molecule.

In satisfying the conditions (a) and (d) the solution of equation (3), which satisfies the initial condition (4), assumes the form<sup>2</sup>:

$$D = D_0 (h\Omega_0)^{-2} \{ |V_{12}|^2 \cos \Omega_0 (t - t_0) + (h\delta)^2 \}, \quad (5)$$

where

$$\Omega_0^2 = h^{-2} |V_{12}|^2 + \delta^2, \quad \delta = \omega_0 - \omega. \quad (6)$$

The mean dipole moment of a molecule equals

$$\langle \mu \rangle = \text{Sp}(\rho \vec{\mu}) = \rho_{11} \vec{\mu}_{11} + \rho_{22} \vec{\mu}_{22} + \rho_{12} \vec{\mu}_{21} + \rho_{21} \vec{\mu}_{12}. \quad (7)$$

On the basis of the correspondence principle we are now in the position to find the radiation from and absorption by the molecular gas. The last two terms in (7) show the contribution to the absorption of electromagnetic radiation of the frequency  $\omega^{1,2}$ . We are going to examine, therefore, the first two terms of (7):

$$\langle \vec{\mu}_1 \rangle = \rho_{11} \vec{\mu}_{11} + \rho_{22} \vec{\mu}_{22} = 1/2 [\vec{\mu}_{11} \quad (8)$$

$$+ \vec{\mu}_{22} + (\vec{\mu}_{22} - \vec{\mu}_{11}) D_0 \Omega_0^{-2} \delta^2] \\ + \frac{1}{2} \frac{(\vec{\mu}_{22} - \vec{\mu}_{11}) D_0 |V_{12}|^2}{h^2 \Omega_0^2} \cos \Omega_0 (t - t_0).$$

Here we have used Eq. (5) and the fact that

$\rho_{11} + \rho_{22} = 1$ . The contribution to radiation is given by the variable part of  $\langle \vec{\mu}_1 \rangle$

$$\langle \vec{\mu} \rangle = \vec{\mu}_0 \cos \Omega_0 (t - t_0), \quad (9)$$

$$\vec{\mu}_0 = 1/2 (\vec{\mu}_{22} - \vec{\mu}_{11}) D_0 (h\Omega_0)^{-2} |V_{12}|^2.$$

Taking into account that the times between the collisions  $\tau$  are distributed according to the law

$$w(\tau) d\tau = (1/\bar{\tau}) e^{-\tau/\bar{\tau}} d\tau,$$

it is possible to find the spectral intensity of the radiation

$$I(\Omega) \sim [(\Omega_0 - \Omega)^2 + \bar{\tau}^{-2}]^{-1}. \quad (10)$$

The total radiation intensity equals

$$I_{\text{total}} = (\Omega_0^4 / 3c^3) \mu_0^2. \quad (11)$$

It is easy to understand the origin of the radiation at the frequency  $\Omega_0$  of Eq. (6) on the basis of simple quantum electrodynamic considerations. The system consisting of molecules and an electromagnetic field is actually described by the

Hamiltonian  $\mathcal{H} = H_0 + H_\gamma^0 + 1/2 V$ , where  $H_\gamma^0$  is the Hamiltonian of the free field, and  $V/2$  is the energy of interaction [the coefficient 1/2 is necessary to maintain consistency with Eq. (2)]. We shall try to find the solution of the equation

$$\mathcal{H}\psi = \mathcal{E}\psi \quad (12)$$

in the form of superposition of the solutions of the equation

$$(H_0 + H_\gamma^0) \psi^{(0)} = \mathcal{E}^{(0)} \psi^{(0)}. \quad (13)$$

We write

$$\psi = c_1 \psi_1^{(0)} + c_2 \psi_2^{(0)}, \quad (14)$$

$$\psi_1^{(0)} = \psi_{1m}^{(0)} \Phi_{N_0}(N); \quad \psi_2^{(0)} = \psi_{2m}^{(0)} \Phi_{N_0+1}(N);$$

$$\mathcal{E}_1^{(0)} = E_1 + N_0 \hbar \omega; \quad \mathcal{E}_2^{(0)} = E_2 + (N_0 + 1) \hbar \omega;$$

$$\mathcal{E}_1^{(0)} \approx \mathcal{E}_2^{(0)},$$

where  $\psi_{1m}^{(0)}$ ,  $\psi_{2m}^{(0)}$  are wave functions of the free molecule  $\Phi_{N_0}(N)$ ,  $\Phi_{N_0+1}(N)$  are wave functions of the free field, and  $N$  is the number of photons. Substituting (14) in (12), multiplying first by  $\psi_{1m}^{(0)*}$ , then by  $\psi_{2m}^{(0)*}$ , and integrating, we get two equations. The condition of their consistency gives:

$$\mathcal{E}_{1,2} = \frac{1}{2}(\mathcal{E}_1^{(0)} + \mathcal{E}_2^{(0)}) \pm \frac{1}{2} \sqrt{(\mathcal{E}_1^{(0)} - \mathcal{E}_2^{(0)})^2 + |V_{12}|^2}$$

Transitions of the system molecule + field with radiation of quanta of energy  $\mathcal{E}_1 - \mathcal{E}_2 = \hbar\Omega_0$  are thus possible. It is also not difficult to demonstrate that the matrix element of the dipole moment corresponding to the transition at the frequency  $\Omega_0$  is proportional to  $\vec{\mu}_{22} - \vec{\mu}_{11}$ .

Let us note that the measurements of the frequency  $\Omega_0$  offer the possibility of the experimental determination of the matrix element  $|V_{12}|$ , which is proportional to the product of the dipole moment and the magnitude of the field intensity  $F$ . Such measurements offer a method for the precise determination of the field intensity of frequency  $\omega$ , if the matrix element  $|\vec{\mu}_{12}|$  is known. If conversely, the field intensity of frequency  $\omega$  is known with sufficient precision, it is possible to determine  $\vec{\mu}_{12}$  precisely.

Let us evaluate the order of magnitude of possible frequencies  $\Omega_0 \sim |\vec{\mu}_{12}|F/\hbar$ . Let  $|\vec{\mu}_{12}| \sim 10^{10} \text{ sec}^{-1}$  and  $F = 1 \text{ cgs unit} = 300 \text{ v/cm}$ ; then  $\Omega_0 \sim 10^9 \text{ sec}^{-1}$ . If, however,  $F = 10 \text{ cgs units}$ , then  $\Omega_0 = 10^{10} \text{ sec}^{-1}$ . By changing the field intensity at the frequency  $\omega$ , we can change the frequency  $\Omega_0$ , which presents some convenience in the experimental handling of the problem.

In conclusion, let us note that radiation at the frequency  $\Omega_0$  will be observed only for molecules whose dipole moments  $\vec{\mu}_{11}$  and  $\vec{\mu}_{22}$  differ from zero.

## Anomalous Skin Effect with Arbitrary Collision Integral

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(Submitted to JETP editor July 14, 1955)

*J. Exper. Theoret. Phys. USSR* **29**, 876-878  
(December, 1955)

IN previous publications<sup>1,2</sup> an expression was obtained for the surface impedance of metal in the case of anomalous skin effect\*. It was then assumed that the integral of collisions can be written with the help of the relaxation time  $\tau$  in the form:

$$(\partial f / \partial t)_{st} = (f - f_0) / \tau, \quad (1)$$

where  $f$  is the electron distribution function,  $f_0(\epsilon)$  is the Fermi equilibrium distribution function.

Introduction of the relaxation time can be rigorously established only at high temperatures ( $T \gg \Theta$ ,  $\Theta$  the Debye temperature). At lower temperatures, the collision integral, in general, cannot be written in form (1), and one must consider an arbitrary collision integral. In the present contribution it is proved that the formula for impedance, obtained in reference 1, is valid for an arbitrary collision integral. Let us note that the left-hand member of (1) is not assumed to be small in comparison with  $\omega f$  ( $\omega$  - frequency of external field).

The complete system of equations has the form:

$$\frac{d^2 E_x(z)}{dz^2} \quad (2)$$

$$= \frac{4\pi i \omega}{c^2} \frac{2e^2}{(2\pi\hbar)^3} \int n_x [\psi(z; \mathbf{n}) - \psi(z; -\mathbf{n})] dS_p^{**}$$

$$n_z \frac{\partial \psi(z; \mathbf{n})}{\partial z} + \check{L}^{-1} \psi(z; \mathbf{n}) \quad (3)$$

$$= E_\beta(z) n_\beta \equiv E_x n_x + E_y n_y;$$

$$\psi(z; \mathbf{n})|_{z=+\infty} = 0,$$

$$\psi(0; n_x, n_y, n_z) = q \psi(0; n_x, n_y, -n_z); \quad n_z > 0$$

$$\mathbf{n} = \mathbf{v} / v; \quad \mathbf{v} = \nabla_p \epsilon(\mathbf{p}). \quad (4)$$

<sup>1</sup> R. Karplus and J. Schwinger, *Phys. Rev.* **73**, 1020 (1948)

<sup>2</sup> H. Snyder and P. Richards, *Phys. Rev.* **73**, 1178 (1948).

<sup>3</sup> J. Van Vleck and V. Weisskopf *Rev. Mod. Phys.* **17**, 227 (1945).