Method of Determination of Parameters of Ferromagnetic Resonance from Experimental Data

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A calculation of the conditions for the extrema of the magnetic permeabilities in the region of ferromagnetic resonance is carried out. On the basis of the results obtained, formulas are derived for the determination of the gyromagnetic ratio and the relaxation time from experimental data. In the proposed method the calculation of the parameters of ferromagnetic resonance is carried out for fields of magnitudes corresponding to extrema of the magnetic permeabilities. Consequently, the necessity of calculating the maximum value of the magnetic permeability μ_k (or ρ'), which is the source of a large error (greater than 25%) in the determination of the relaxation time, disappears. It is shown that the method of calculation of g and T for fields of the extrema is applicable in cases where it is impossible to calculate the maximum values of the permeability μ_k (or ρ').

1. INTRODUCTION

THE phenomenon of ferromagnetic resonance was discovered and first studied in 1911-1913 by Arkad'ev[1,2]. While investigating the behavior of iron and nickel in high frequency electromagnetic fields (of wavelengths from 1.7 to 73 cm) he discovered, in the wavelength region from 2-6 cm, a band of dispersion of the magnetic permeability and maximum absorption of electromagnetic energy. He explained the results which he obtained as consequences of the resonance of elementary magnetic dipoles.

In these works Arkad'ev first introduced the complex magnetic permeability of the material

$$\mu' = \mu - i\rho'$$

and showed that as the frequency of the external magnetic field is changed under given conditions, there must exist a band of dispersion of μ and a maximum in ρ'.

Dorfman[3] was the first to explain the phenomenon of ferromagnetic resonance on the basis of quantum mechanics, while Landau and Lifshitz[4] carried out a theoretical calculation of the magnetic permeability of a single crystal for alternating fields without hysteresis. Starting from contemporary ideas of magnetic structure and processes of ferromagnetic magnetization, they obtained formulas determining the dependence of the real and imaginary parts of the complex magnetic permeability of the material on the frequency of the external field. It was shown in this work that for definite frequencies of the external field there must exist a resonance absorption of electromagnetic energy and a dispersion of the magnetic permeability. Frenkel'[5], in a quantum calculation of the resonance absorption in paramagnetics, also considered the question of the quantum treatment of ferromagnetic resonance.

Further theoretical works[6-8] developed the basic ideas of Arkad'ev, Landau and Lifshitz. In these works studies were made of the effect of the finite dimensions of the sample (demagnetizing field), the magnetic crystallographic anisotropy of a ferromagnetic, and eddy currents on the phenomenon of ferromagnetic resonance. In the works of Kittel[6] and Bloembergen[8], a calculation of the high-frequency magnetic permeability is carried out for the case of a homogeneously magnetized single crystal under the action of a weak alternating magnetic field. In this calculation no account is taken of hysteresis nor of possible inhomogeneous conditions for the various parts of the ferromagnetic.

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[7] J. H. Van Vleck, Phys. Rev. **78**, 266 (1950); Physica **17**, 234 (1951).

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However, the formulas obtained by these authors for the magnetic permeability seem at present to be quite correct and give satisfactory agreement with experimental results. According to reference 8, the dependence of the real and imaginary parts of the complex magnetic permeability of the substance on the constant magnetic field and on the frequency of the alternating magnetic field, with account being taken of the relaxation time, may be written (for a polycrystal) in the form:

$$\mu = \frac{\omega_1^2(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (4\omega^2 / T^2)} + 1, \quad (1)$$

$$\rho' = \frac{\omega_1^2(2\omega / T)}{(\omega_0^2 - \omega^2)^2 + (4\omega^2 / T^2)}, \quad (2)$$

where

$$\omega_0^2 = \gamma^2 [H + (N_y - N_z) I_s] \times [H + (N_x - N_z) I_s] + T^{-2}, \quad (3)$$

$$\omega_1^2 = 4\pi\gamma^2 I_s [H + (N_y - N_z) I_s]. \quad (4)$$

N_x, N_y, N_z are the demagnetizing factors along the corresponding axes, I_s is the saturation magnetization of the sample, T is the transverse relaxation time, ω is the angular frequency of the alternating magnetic field, H is the external constant magnetic field, $\gamma = eg/2mc$ is the gyromagnetic ratio for the electron, g is the Landé splitting factor. It is supposed here that the constant magnetic field is directed along the Z axis and the high-frequency magnetic field along the X axis. The effective magnetic permeabilities as given by the coefficient of absorption μ_k and the index of refraction μ_n, are connected with the magnetic permeabilities of the substance by the relations[9]:

$$\mu_k = \sqrt{\mu^2 + \rho'^2} + \rho', \quad (5)$$

$$\mu_n = \sqrt{\mu^2 + \rho'^2} - \rho'. \quad (6)$$

Ferromagnetic resonance under conditions of a simultaneous constant magnetic field and a high-frequency magnetic field perpendicular to it, both acting on the ferromagnetic, has been studied in many experimental works[10-16]. In these experiments the frequency of the alternating magnetic field was fixed and the constant magnetic field was changed. The results of the experiments with polycrystalline samples and single crystals, with metals and polyconductor (ferrite) ferromagnetics, basically confirmed the theoretical calculations and made it possible to establish the fact that the splitting factor g determined from experiments on ferromagnetic resonance is, in the overwhelming majority of cases, greater than two. The value of the magnetomechanical factor determined from gyromagnetic experiments, g', is always less than 2. Kittel[17], and subsequently Van Vleck[7], gave an explanation of this discrepancy depending on the spin-orbit interaction. However, their formula $g-2 = 2-g'$ is not confirmed by the experimental results: it turns out that in the majority of cases $g-2 > 2-g'$.

One of the most essential questions of the theory of ferromagnetic resonance is the question of the width of the absorption curve. The theoretical calculations give a result smaller than that obtained from the experimental data. In connection with this it should be noted that the above indicated discrepancies between the experimental and theoretical results should not be considered as due solely to the imperfections of the theory. It is also necessary to give attention both to the more exact development of the actual methods used in the measurements and to the development and substantiation of methods of determining the parameters of ferromagnetic resonance (g, T) from the experimental data.

In the determination of the splitting factor g it is customary under the condition of maximum absorption of electromagnetic energy to neglect the term containing the relaxation time[3]. This

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neglect could be considered legitimate if there were obtained from the experimental data at least an estimate of the relaxation time. However, the methods of determining the relaxation time are very often either insufficiently substantiated or involve large errors. The relaxation time is often determined by making use of the approximate relation $\mu_{k\,\max} = 2\rho'_{\max}$, from which it follows that:

$$1/T = \omega_1^2/\omega\mu_{k\,\max}.$$

A calculation of $\mu_{k\,\max}$ is essential for this method of determining the relaxation time. Such a calculation involves a large error, and under certain experimental conditions it is impossible to carry it out. Thus, for example, in the case where the measurement is carried out with the help of a cavity resonator, it is necessary to calculate its quality factor Q_0, which is determined only from the losses in the ferromagnetic walls[8]. A calculation of the quality factor can be carried out for resonators of simple form, but cannot be carried out for resonators of complicated form, since the structure of the high-frequency field in the cavity of the resonator is very inexactly known. Consequently, this method of determining the relaxation time cannot be applied with resonators of complicated form. In those cases where the calculation may be carried out, it gives too large a value Q_0, since, on account of irregularities and scratches commensurate with the depth of penetration of the field into the metal, the operating surface of the wall is greater than that obtained by calculation. Bloembergen[8] estimates that the error in the calculation of Q_0 does not exceed 10%, but he does not substantiate his estimate.

A comparison of the experimental and theoretical calues of the quality factor[18] shows that the indicated discrepancy is significantly larger than 10%, increases with increasing frequency and depends very strongly on the treatment of the wall surfaces. This latter fact makes the estimation of the error in such a calculation difficult. Even with an error of 10% in Q_0, the error in the calculation of $\mu_{k\,\max}$ reaches 25%. However, it is significantly larger, since Q_0 is calculated with an error of more than 10%, especially in the wavelength region $\lambda < 3$ cm.

In order to calculate $\mu_{k\,\max}$ one must know μ_k for some value of the external magnetic field. It is customary to take $\mu_k' = 1$ for magnetic fields in which suppression of ferromagnetic resonance occurs. Such a choice of μ_k imposes definite requirements on the manner of conduction of the experiment, since a suppression of the resonance must necessarily be obtained, that is, one must necessarily use a constant magnetic field significantly exceeding the resonance field. This requirement complicates the experiment and very often is not fulfilled even with the use of comparatively strong fields (curves for nickel and supermalloy[8] for a frequency of 9030 *mc* and for supermalloy[11,15,16] for a frequency of 24,000 *mc*). Bloembergen[8], in a calculation of the relaxation time for frequencies of the order of 24,000 *mc* (nickel and supermalloy), uses the value of μ_k for $H = 0$. The choice of this value ($\mu_k' = 0.8-0.9$ was taken in reference 8) was determined by the condition of the best coincidence of the experimental and theoretical curves in the region of minimum absorption. This manner of computing the relaxation time, based on the selection of a value of μ_k, is not completely convincing.

In connection with the fact that at the present time theoretical and experimental works are directed toward the detailed study of the phenomena of ferromagnetic resonance and the obtaining of qualitative results, there arises the necessity of developing such methods of measurement and of treatment of experimental data as will guarantee increased accuracy in the determination of the parameters of ferromagnetic resonance. Hence the aim of the present work is the analysis of the dependence of the magnetic permeabilities in the region of ferromagnetic resonance on the constant magnetic field and the determination of the conditions under which maxima and minima in the magnetic permeabilities of the substance (μ, ρ') and in the effective magnetic permeabilities (μ_n, μ_k) will occur. On the basis of the obtained conditions on the extrema, methods are found for determining the splitting factor and the relaxation time from the experimental data (absorption curve and dispersion curve). Theoretical curves giving the dependence of μ, ρ', μ_n and μ_k on the constant magnetic field H are given in the Figure.

The region of maximum absorption (maximum ρ' or μ_k) and the region of dispersion (minimum and maximum μ or maxima in μ_n) are investigated in the overwhelming majority of the experimental works. The structure of the expression and the course of the curves for μ and ρ' and for μ_n and μ_k

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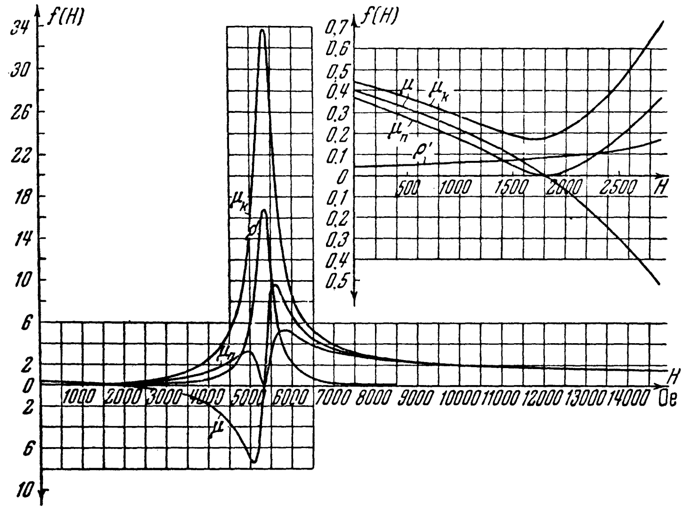

Theoretical dependence of the magnetic susceptibilities during ferromagnetic resonance. $\omega = 15.3 \times 10^{10} mc$, $4\pi I_0 = 6000$ gauss, $N_x = N_z = 0$, $g = 2.24$, $1/T = 5 \times 10^9$ sec^{-1}.

in the Figure are close to the formulas and the course of the curves for dispersion and absorption in optics. Consequently, we may surmise that the difference in the fields corresponding to the maximum and minimum $\mu(H)$ and the difference in the fields corresponding to the maxima in $\mu_n(H)$ are determined by the relaxation time.

In the investigation of ferromagnetic resonance the experimental data are obtained in the form of the dependences on the magnetic field of magnitudes proportional to the square root of μ_k or ρ' (for example, the attenuation of a cavity resonator with a ferromagnetic wall) and the square root of μ_n or μ (the resonance frequency of such a resonator). The positions of the extrema of these quantities will coincide with the extrema of the corresponding magnetic permeability. Hence, in order to determine the parameters of ferromagnetic resonance (γ, g, T) it is necessary to find the conditions for extrema in $\mu(H)$, $\rho'(H)$, $\mu_n(H)$ and $\mu_k(H)$.

Before beginning the determination of the conditions for the extrema, we note the following special feature of the calculations which are to be carried out: all the conditions for the extrema are obtained as approximations, an evaulation of the accuracy of which is carried out, proceeding from the magnitudes of the quantities characteristic of samples made from metallic ferromagnetics in the form of plates for the wavelength region above 1 cm. For the evaluation, we take the following values of the quantities: field corresponding to maximum absorption $H_0 = 10^3$ oersteds; $N_x = N_z = 0$ (small in comparison with N_y); $4\pi I_s = 10^4$ gauss; $\omega^2 = 10^{20}$ sec^{-2}; $\gamma^2 = 3 \times 10^4$ oersteds^{-2} sec^{-2}; $T^{-2} = 10^{20}$ sec^{-2}; $\omega_1^2 = 10^{22}$ sec^{-2}.

2. CONDITION FOR MAXIMUM $\rho'(H)$

The condition for an extremum in $\rho'(H)$ can be reduced to a second degree equation relating to the quantity $\alpha = \omega_0^2 - \omega^2$. Making use of an evaluation of the quantities entering into this equation, we obtain the following approximate magnitudes of the roots:

$$\alpha_1 = \frac{2\omega^2}{T^2\omega_1^2}, \quad \alpha_2 = 2\omega_1^2. \tag{7}$$

The first root gives the condition for maximum ρ',

$$\omega_0^2 = \omega^2\left(1 + \frac{2}{T^2\omega_1^2}\right),$$

or, with an exactitude of the order of 2%, $\omega_0 = \omega$. This equation determines the field $H_{0\rho'}$ corresponding to the maximum value of $\rho'(H)$.

The second root determines a field of order $8\pi I_s$, and for it this estimate of magnitude is incorrect.

3. CONDITIONS FOR EXTREMA IN $\mu(H)$

The condition for an extremum in $\mu(H)$ can be written in the form:

$$\alpha^3 - \beta\alpha^2 + \alpha + \beta = 0,$$

where

$$\omega_0^2 - \omega^2 = (2\omega/T)\,\alpha, \quad \beta = \omega_1^2 T/2\omega.$$

Using the ordinary method of solving cubic equations[19] we obtain approximate expressions for the roots, neglecting terms of order $1/\beta^2 \ll 1$,

$$\alpha_1 = -1 + \beta^{-1}, \quad \alpha_2 = 1 + \beta^{-1}, \quad \alpha_3 = \beta.$$

Then the conditions for the extrema of $\mu(H)$ can be written in the form

$$\omega_{01\mu}^2 - \omega^2 = -\frac{2\omega}{T}\left(1 - \frac{2\omega}{T\omega_{11\mu}^2}\right); \qquad (8)$$

$$\omega_{02\mu}^2 - \omega^2 = \frac{2\omega}{T}\left(1 + \frac{2\omega}{T\omega_{12\mu}^2}\right), \qquad (9)$$

where

$$\omega_{01\mu}^2 = \omega_0^2(H_{1\mu}), \qquad \omega_{02\mu}^2 = \omega_0^2(H_{2\mu}),$$

$$\omega_{11\mu}^2 = \omega_1^2(H_{1\mu}), \qquad \omega_{12\mu}^2 = \omega_1^2(H_{2\mu}),$$

$H_{1\mu}$ and $H_{2\mu}$ are the values of the constant magnetic field which correspond to the minimum and maximum values of $\mu(H)$. The root α_3 gives a condition analogous to (7).

4. CONDITIONS FOR MAXIMUM $\mu_k(H)$

The determination of the conditions for maximum $\mu_k(H)$ leads to the necessity of solving an algebraic equation of higher than the fourth degree. Consequently, in the solution of this equation it is necessary to use approximate methods, taking into account the character of the behavior of the functions $\mu(H)$ and $\rho'(H)$. In the region of maximum $\rho'(H)$ the values of $\mu(H)$ are near unity and are small in comparison with the values of $\rho'(H)$ in this region. Hence we may suppose that the conditions for maxima in $\mu_k(H)$ and $\rho'(H)$ are near each other. Starting from this, the condition for maximum $\mu_k(H)$ can be written in the form: $\omega_0^2 = \omega^2(1+\alpha)$, assuming that $\alpha \ll 1$. The equation $d\mu_k/dH = 0$ leads to a linear equation in α, and, on evaluating the quantities entering into it, we obtain:

$$\alpha = 4/3\omega^2 T^2.$$

Consequently, the condition for maximum $\mu_k(H)$ will be:

$$\omega_{0k}^2 = \omega^2\left(1 + \frac{4}{3\omega_{1k}^2 T^2}\right), \quad \omega_{1k}^2 = \omega_1(H_{k0}) \qquad (10)$$

This equation determines the constant magnetic field H_{0k} for which maximum $\mu_k(H)$ occurs, that is, maximum absorption of the energy of the high-frequency field in the case where the penetration depth is less than the thickness of the sample. In accordance with the accepted evaluation of the quantities involved, we may write, with an accuracy of 1-2%, $\omega_{0k} = \omega$. If we use the values of the quantities which were assumed for this evaluation, then $H_{0\rho'} - H_{0k} \approx 1$ oersted, that is, the difference in the fields corresponding to the maximum values of μ_k and ρ' lie within the limits of the error in the measurement of the field.

5. CONDITION FOR MAXIMA IN $\mu_n(H)$

An analysis of the equation $d\mu_n/dH = 0$ shows that it becomes considerably simpler and can be solved exactly if we assume

$$\frac{d\omega_1^2}{dH} = 0; \quad \mu = \frac{\omega_1^2(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + (4\omega^2/T^2)}. \qquad (11)$$

We can substantiate the first simplifying condition to a certain extent by the fact that the basic course of the dependence of the magnetic permeability on the constant magnetic field is determined mainly by the field entering into ω_0^2. The second condition receives some confirmation from the fact that the calculation is carried out for a region of field near the region of maximum absolute values of μ, which are determined by the first component in (1). The considerations introduced make it possible to suppose that the conditions for extrema in $\mu_n(H)$ found on the basis of the

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[19] L. Ia. Okunev, *Higher Algebra*, Gostekhizdat, 1949.

simplifying assumptions will be near the conditions for the extrema which are obtained if we take account of the exact expression for μ and the dependence of ω_1^2 on the field. Solving the equation $d\mu_n/dH = 0$ while taking account of (11), we obtain the following conditions for the extrema:

$$\omega_{01}^2 - \omega^2 = -2\sqrt{3}\,\omega/T, \tag{12}$$

$$\omega_{02}^2 - \omega^2 = 2\sqrt{3}\,\omega/T.$$

If we now take account of the dependence of ω_1^2 on H and the exact expression for μ in the form (1), then the solution of the equation $d\mu_n/dH = 0$ in the region of fields $H > H_{0k}$ can be sought in the form:

$$\omega_0^2 - \omega^2 = (2\sqrt{3}\,\omega/T)(1+\alpha),$$

assuming $\alpha \ll 1$. The equation $d\mu_n/dH = 0$ can be reduced to a linear equation in α (neglecting terms of order α^2 and higher in comparison with unity), and we then obtain

$$\alpha = \frac{8\sqrt{3}}{3}\frac{\omega}{T\omega_1^2}. \tag{13}$$

For the assumed estimate of magnitudes, $\alpha = 0.05$. The problem of the extrema in the region of fields $H < H_{0k}$ is solved analogously. As a result of these calculations, we obtain the conditions for the first and second maxima of $\mu_n(H)$ in the form:

$$\omega_{01n}^2 - \omega^2 = -\frac{2\sqrt{3}\,\omega}{T}\left(1 - \frac{8\sqrt{3}}{3}\frac{\omega}{T\omega_{11n}^2}\right), \tag{14}$$

$$\omega_{02n}^2 - \omega^2 = \frac{2\sqrt{3}\,\omega}{T}\left(1 + \frac{8\sqrt{3}}{3}\frac{\omega}{T\omega_{12n}^2}\right) \tag{15}$$

where

$$\omega_{01n}^2 = \omega_0^2(H_{1n}), \quad \omega_{02n}^2 = \omega_0^2(H_{2n}),$$

$$\omega_{11n}^2 = \omega_1^2(H_{1n}), \quad \omega_{12n}^2 = \omega_1^2(H_{2n}),$$

H_{1n} and H_{2n} are the magnitudes of the constant magnetic field corresponding to the first and second maxima of $\mu_n(H)$. From the evaluation of α there follows the possibility of using Eqs. (12) and (13) as approximate conditions for the extrema in $\mu_n(H)$. The error in using these approximate conditions may be estimated from Eqs. (14) and (15).

6. METHOD OF DETERMINING THE GYROMAGNETIC RATIO AND THE RELAXATION TIME FROM EXPERIMENTAL DATA

In the investigation of ferromagnetic resonance in metallic ferromagnetics, one most frequently meets with cases in which the penetration depth is less than the thickness of the sample, and the absorption is determined by μ_k and the phase relationships by μ_n. Hence, let us consider a means of determining the parameters of ferromagnetic resonance in this case, assuming that H_{0k}, H_{1n} and H_{2n} are known from experimental results. Using Eqs. (14) and (15), we obtain

$$\omega_{02n}^2 - \omega_{01n}^2 = \frac{4\sqrt{3}\,\omega}{T}\left(1 - \frac{4\sqrt{3}\,\omega}{T}\frac{\omega_{12n}^2 - \omega_{11n}^2}{\omega_{12n}^2\omega_{11n}^2}\right).$$

Taking into account the evaluation of the magnitudes entering into the second member in the parantheses, and assuming $H_{2n} - H_{1n} = 5\times 10^2$ oersteds, we obtain $\omega_{12n}^2 - \omega_{11n}^2 \approx 10^{21}$. Consequently, allowing an error of less than 0.3%, we can discard the second member in the parentheses. Then the expression for the determination of the relaxation time will be [using Eq. (3)]:

$$\frac{1}{T} \tag{16}$$

$$= \frac{\gamma^2 (H_{2n} - H_{1n})\,[H_{1n} + H_{2n} + (N_x + N_y - 2N_z) I_s]}{4\sqrt{3}\,\omega}.$$

In the determination of the relaxation time it is not always possible to use the formula obtained. Thus in the wavelength region $\lambda > 5$ cm the first maximum μ_n lies at comparatively weak fields, in which the sample is not magnetized to saturation. Hence the position of the first maximum μ_n does not correspond to the conditions considered in Sec. 5, and the magnitude of the field corresponding to this maximum cannot be used for a calculation of the relaxation time.

The experimental results in the indicated region allow the use of the field determining the maximum μ_k and the second maximum μ_n. Using Eqs. (10) and (15) and assuming $H_{2n} - H_{0k} \approx 10^2$ oersteds, we obtain the following expression for the determination of the relaxation time:

$$\frac{1}{T}=\frac{\omega_{02n}^2-\omega_{0k}^2}{2\sqrt{3}\,\omega}\left(1-\frac{22\sqrt{3}}{9}\frac{\omega}{T\omega_{12n}^2}\right). \quad (17)$$

The second member in the parentheses ~ 0.03; consequently, discarding it, we obtain an expression for the relaxation frequency ($1/T$) increased by approximately 3%:

$$\frac{1}{T} = \frac{\gamma^2(H_{2n}-H_{0k})\,[H_{0k}+H_{2n}+(N_x+N_y-2N_z)\,I_s]}{2\sqrt{3}\,\omega}. \quad (18)$$

In the study of ferromagnetic resonance in the region of comparatively short waves, a fairly strong field is necessary in order to obtain the second maximum of μ_n. If the second maximum of μ_n is not obtained experimentally, then we may use the fields corresponding to the first maximum of μ_n and to the maximum μ_k. As a result of an analogous calculation we obtain:

$$\frac{1}{T}=\frac{\omega_{0k}^2-\omega_{01n}^2}{2\sqrt{3}\,\omega}\left(1+\frac{22\sqrt{3}}{9}\frac{\omega}{T\omega_{11n}^2}\right), \quad (19)$$

or, approximately (the magnitude of $1/T$ will be decreased by 3%),

$$\frac{1}{T} = \frac{\gamma^2(H_{0k}-H_{1n})\,[H_{1n}+H_{0k}+(N_x+N_y-2N_z)\,I_s]}{2\sqrt{3}\,\omega}. \quad (20)$$

The formulas obtained for the calculation of the relaxation time are not of equally good accuracy. As follows from the very derivation of the formulas, the determination of the relaxation time by the two maxima in μ_n is considerably more exact. Consequently, if the conditions of the experiment allow the use of the two maxima, it is necessary to use Eq. (16). The results of the calculation of the relaxation time using Eqs. (18) and (20) can be made more exact if we introduce corrections in accordance with Eqs. (17) and (19) and calculate the first approximation for $1/T$. In the terms determining the correction, the value of $1/T$ should be taken from estimates according to Eqs. (18) and (20).

Any of the formulas (16), (18) and (20) can be written in the form

$$1/T=\gamma^2 p, \quad (21)$$

where p is determined, depending on the extrema with respect to which the calculation of the relaxation time has been carried out, by comparison of Eq. (21) with Eqs. (16), (18) and (20).

The condition for maximum μ_k may be written in the form

$$\omega^2=\gamma^2 q+T^{-2}, \quad (22)$$

$$q=[H_{0k}+(N_x-N_z)\,I_s]\,[H_{0k}+(N_y-N_z)\,I_s]. \quad (23)$$

From Eqs. (21) and (22) we obtain a formula for the determination of the gyromagnetic ratio:

$$\gamma^2=-\frac{q}{2p^2}+\frac{1}{p}\sqrt{\frac{q^2}{4p^2}+\omega^2}. \quad (24)$$

Formula (24) makes possible the determination of the gyromagnetic ratio from the values of the field corresponding to maxima of μ_k and μ_n, taking account of the $1/T^2$ term in expression (3) for ω_0^2. Naturally, the use of formula (24) for the calculation of the gyromagnetic ratio makes sense only in case the relaxation frequency $1/T$ is commensurate with the frequency of the electromagnetic field, ω. If $1/T\omega \ll 1$, then it is simpler to use the formula $\gamma=\omega/\sqrt{q}$.

If there exist conditions in the experiment such that the depth of penetration of the field into the ferromagnetic is considerably greater than the thickness of the sample, then the absorption is determined by ρ' and the phase relations by μ. In this case, using the conditions for extrema in ρ' and μ, and carrying out an analogous calculation, we obtain the following relationship:

$$\frac{1}{T}=\frac{\omega_{02\mu}^2-\omega_{01\mu}^2}{4\omega}\left(1+\frac{\omega}{T}\frac{\omega_{12\mu}^2-\omega_{11\mu}^2}{\omega_{12\mu}^2\,\omega_{11\mu}^2}\right).$$

If we assume that $H_{2\mu}-H_{1\mu}=3\times 10^2$ oersteds, then the second member in the parentheses is of the order of 10^{-3}; consequently, allowing an error of the order of 0.1%, we may write:

$$\frac{1}{T} = \frac{\gamma^2 (H_{2\mu} - H_{1\mu}) [H_{1\mu} + H_{2\mu} + (N_x + N_y - 2N_z) I_s]}{4\omega}. \quad (25)$$

On using the maxima of μ and ρ',

$$\frac{1}{T} = \frac{\omega^2_{02\,\mu} - \omega^2_{0\rho'}}{2\omega}\left(1 - \frac{\omega}{T\omega^2_{12\,\mu}}\right),$$

or, approximately (with an error of ~ 1%),

$$\frac{1}{T} = \frac{\gamma^2 (H_{2\mu} - H_{0\rho'}) [H_{0\rho'} + H_{2\mu} + (N_x + N_y - 2N_z) I_s]}{2\omega}. \quad (26)$$

For the case of the minimum of μ and the maximum of ρ',

$$\frac{1}{T} = \frac{\omega^2_{0\rho'} - \omega^2_{01\,\mu}}{2\omega}\left(1 + \frac{\omega}{T\omega^2_{11\,\mu}}\right),$$

or, with an error of ~ 1%,

$$\frac{1}{T} = \frac{\gamma^2 (H_{0\rho'} - H_{1\mu}) [H_{1\mu} + H_{0\rho'} + (N_x + N_y - 2N_z) I_s]}{2\omega}. \quad (27)$$

The remarks concerning the accuracy of Eqs. (16), (18) and (20) and concerning the introduction of the corrections for the calculation of the first approximation for the determination of the relaxation frequency also are completely applicable to formulas (25), (26) and (27).

CONCLUSIONS

The proposed method of treating the experimental data for the determination of the parameters of ferromagnetic resonance uses, besides the usual initial data (ω, N_x, N_y, N_z, I_s), the values of the constant magnetic field which correspond to the extrema in μ_k and μ_n (or ρ' and μ). The essential advantages of this method, in comparison with the method based on the calculation of the maximum value of μ_k (or ρ') are:

1. The necessity of calculating the resonator's quality factor Q_0, which depends on the losses in the ferromagnetic walls of the resonator, disappears. This considerably broadens the possibilities in the study of ferromagnetic resonance, since resonators of complicated form, for which such a calculation is impossible, can be used. We can dispense with the rather complicated technology of soldering the sample to the wall of the resonator (see, for example, reference 8) and instead can place the sample in a suitable place in the cavity of the resonator.

In the case of resonators of simple form, the error in the calculation of Q_0 has as basic source the error in the determination of the relaxation time, which exceeds 25%. The basic error in the determination of the relaxation time by the fields of the extrema is connected with the error in the determination of the difference of the fields corresponding to the extrema, which error depends to a considerable extent on the sharpness of the extrema. The difference of these fields ordinarily consists of a few hundred oersteds and can be determined with an accuracy of not less that 5%. This guarantees a more exact result for the relaxation time.

2. In the use of the proposed method of treating the experimental data in order to determine g and T, the suppression of the resonance is not required. Consequently, in carrying out the experiment we can use a considerably smaller region of constant magnetic fields. It is sufficient that the experimental data be obtained in a region which includes the extrema of μ_n (or μ), and it is even sufficient to obtain one of these extrema and the maximum of μ_k (or ρ').

3. In case the suppression of the resonance is obtained, the use of the values of g and T obtained by the proposed method makes possible the determination of the absolute value of the magnetic permeabilities μ_n, μ_k, μ, ρ' and the estimation of the quality factor Q_0 from experimental data.

It should be noted that it is necessary, in using the method of the fields of the extrema, to obtain the absorption and dispersion curves experimentally. This complicates the experiment somewhat, but with present day technique the measurements do not present great difficulties.

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274