The Strong Coupling Theory of Meson Fields. II

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In an earlier paper the problem of nucleons interacting with a meson field has been treated by perturbation theory, using as the expansion parameter the reciprocal coupling constant. On that basis in the present paper the interaction of two nucleons, the scattering of mesons from nucleons, and the bound states of mesons are calculated for the case of the infinitely heavy nucleons.

1. IN a previous paper[1] the theory of a meson field interacting with infinitely heavy nucleons has been treated by perturbation theory using the inverse coupling constant as the expansion parameter. We shall here be concerned with the solution of the equations of the zeroth and first approximations.

We first shall obtain the solutions of the zero order approximation, Eq. (14) in I (notation the same as in I):

$$\left\{\frac{g^2}{2}\int\sum_{\alpha}^{3}[|O_\alpha(\mathbf{r})|_{\lambda\lambda}-2O_\alpha(\mathbf{r})] \times |O_\alpha(\mathbf{r}')|_{\lambda\lambda}G(\mathbf{r},\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}'\right\}\psi_\mu^0=E_\mu^0\psi_\mu^0; \tag{1}$$

which describes a neutral pseudoscalar, a charged scalar or a charged pseudoscalar field interacting with one nucleon.

For the first case (1) has the form

$$[A+(\mathbf{B},\vec{\sigma})]\psi_\mu^0=E_\mu^0\psi_\mu^0;$$

$$A=\frac{I_p}{2}(\vec{\eta})^2;\quad \mathbf{B}=-I_p\vec{\eta};$$

$$I_p=\frac{g^2}{\varkappa^2}\int\frac{\partial^2G(\mathbf{r},\mathbf{r}')}{\partial x\,\partial x'}U(\mathbf{r})\,U(\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}'\approx\frac{g^2}{\varkappa^2a^3};$$

$$\vec{\eta}=\{\vec{\sigma}\}_{\lambda\lambda}.$$

We shall now introduce a coordinate system $\bar{x}$, $\bar{y}$, $\bar{z}$ such that $\bar{z}$ is parallel to $\vec{\eta}$ (the angle between $\bar{z}$ and $\vec{\eta}$ is zero or π). Then $\eta_{\bar{x}}=\eta_{\bar{y}}=0$ and (1) becomes

$$(A+B_{\bar{z}}\,\sigma_{\bar{z}})\,\psi_\mu^0=E_\mu^0\psi_\mu^0,\quad |B_{\bar{z}}|=B. \tag{2}$$

The Hamiltonian H^0 can be diagonalized also by the known unitary transformation[2]

$$\Lambda=\begin{vmatrix}\cos\frac{\vartheta}{2}e^{-i[(\alpha/2)-(\pi/4)]} & i\sin\frac{\vartheta}{2}e^{-i[(\alpha/2)-(\pi/4)]}\\ i\sin\frac{\vartheta}{2}e^{i[(\alpha/2)-(\pi/4)]} & \cos\frac{\vartheta}{2}e^{i[(\alpha/2)-(\pi/4)]}\end{vmatrix};\quad \vartheta=\arccos\frac{\eta_z}{\eta},\quad \alpha=\operatorname{arctg}\frac{\eta_y}{\eta_x}.$$

The eigenvalues of (2) are: $E_1^0=A+B_{\bar{z}}$; $E_2^0=A-B_{\bar{z}}$. In the first case we obtain from

$$\eta_\alpha=(\psi_{\lambda\lambda}^{0*}(s,\eta_\alpha),\ \tau_\alpha\psi_{\lambda\lambda}^0(s,\eta_\alpha)) \tag{3}$$

[see Eq. (15) in I] for $\lambda=1$, $\eta_{\bar{z}}^{(1)}=1$; for $\lambda=2$, $\eta_{\bar{z}}^{(2)}=-1$. The Hamiltonians H_1^0 and H_2^0 and their respective eigenvalues are

$$H_1^0=I_p\left(\frac{1}{2}-\sigma_{\bar{z}}\right);\quad \psi_{11}^0=\begin{vmatrix}1\\0\end{vmatrix};\quad \psi_{12}^0=\begin{vmatrix}0\\1\end{vmatrix};$$

$$E_{11}^0\equiv E_1^0=-\frac{I_p}{2};\quad E_{12}^0=\frac{3}{2}I_p.$$

$$H_2^0=I_p\left(\frac{1}{2}+\sigma_{\bar{z}}\right);\quad \psi_{22}^0=\begin{vmatrix}0\\1\end{vmatrix};\quad \psi_{21}^0=\begin{vmatrix}1\\0\end{vmatrix};$$

$$E_{22}^0=-\frac{I_p}{2};\quad E_{21}^0=\frac{3}{2}I_p.$$

[1] B. T. Geilikman, J. Exper. Theoret. Phys. USSR **29**, 417 (1955); hereinafter quoted as I.

[2] L. de Broglie, *L'électron magnétique*, Paris, 1934, p. 127.

In the case $H_2^0 = A - I_p(\vec{\eta}, \vec{\sigma})$, clearly all directions in $\vec{\sigma}$ space are equivalent, and therefore the direction of $\vec{\eta}$, and therewith $\bar{z}$, is arbitrary. However, for any given direction of $\bar{z}$ the projections of $\vec{\eta}$ are well defined: for $\lambda = 1$, $\eta_{\bar{z}} = 1$; for $\lambda = 2$, $\eta_{\bar{z}} = -1$. This is similar to the case of the angular momentum vector **L** of a particle moving in a central field, where any frame of reference can be used, and therefore the direction of $|\mathbf{L}|_{nn}$ is also arbitrary. In fact, projecting **L** on the z-axis, we obtain $|\mathbf{L}_z|_{nn} = m\hbar$ and $|\mathbf{L}_x + i\mathbf{L}_y|_{nn} = h\,|\,e^{i\varphi}\left(\frac{\partial}{\partial\vartheta} + i\operatorname{ctg}\vartheta\frac{\partial}{\partial\varphi}\right)|_{nn} = 0$; $|\mathbf{L}_x|_{nn} = |\mathbf{L}_y|_{nn} = 0$, or, the vector $|\mathbf{L}|_{nn}$ is parallel to the z-axis. Despite the arbitrary direction of the z-axis the projections $|\mathbf{L}|_{nn}$ onto it are fully determined and correspond to a certain set of ψ-functions. The direction used for the projection does not enter into our expressions of the energy in any order of approximation in analogy to the case of a particle in a central field.

Similarly, in the case of a symmetric scalar field,

$$H^0 = A + (\mathbf{B}, \vec{\tau}); \quad A = \frac{I_s}{2}(\vec{\eta})^2;$$

$$\mathbf{B} = -I_s\vec{\eta}; \quad \vec{\eta} = \{\vec{\tau}\}_{\lambda\lambda};$$

$$I_s = g^2\int G(\mathbf{r}, \mathbf{r}')\, U(\mathbf{r})\, U(\mathbf{r}')\, d\mathbf{r}\, d\mathbf{r}'; \quad I_s \approx g^2/a.$$

For the axis $\bar{z}$, parallel to $\vec{\eta}$, we find

$$H^0 = A + B_{\bar{z}}\tau_{\bar{z}}; \quad \eta_{\bar{x}} = \eta_{\bar{y}} = 0.$$

For $\lambda = 1$:

$$\eta_{\bar{z}}^{(1)} = 1; \; H_1^0 = I_s\left(\frac{1}{2} - \tau_{\bar{z}}\right); \quad \psi_{11}^0 = \begin{vmatrix}1\\0\end{vmatrix};$$

$$\psi_{12}^0 = \begin{vmatrix}0\\1\end{vmatrix}; \; E_{11}^0 = -\frac{I_s}{2}; \quad E_{12}^0 = \frac{3}{2}I_s.$$

For $\lambda = 2$:

$$\eta_{\bar{z}}^{(2)} = -1; \; H_2^0 = I_s\left(\frac{1}{2} + \tau_{\bar{z}}\right); \quad \psi_{22}^0 = \begin{vmatrix}0\\1\end{vmatrix};$$

$$\psi_{21}^0 = \begin{vmatrix}1\\0\end{vmatrix}; \quad E_{22}^0 = -\frac{I_s}{2}; \; E_{21}^0 = \frac{3}{2}I_s.$$

Actually, the direction of the $\bar{z}$-axis is determined by the experimental conditions, i.e., by a not-charge-invariant perturbation. Under usual circumstances the charge of a nucleon, i.e., τ_z, is well defined. Therefore, here $\vec{\eta}$ is parallel to the z-axis and the $\bar{z}$-axis coincides with the z-axis.

In the case of a nonsymmetric scalar field ($\tau_3 = \gamma_1 + \gamma_2\tau_z$), there exists a preferred direction in τ-space, namely the z-axis, and therefore the direction of the vector $\{\vec{\tau}\}_{\lambda\lambda}$ is not arbitrary, and $E_{11}^0 \neq E_{22}^0$ (the self energy of the proton and the neutron differ).

We shall now investigate the symmetric pseudoscalar field:

$$H^0 = A + \sum_{\alpha,k}^{3,3} B_{\alpha k}\tau_\alpha\sigma_k; \tag{4}$$

$$A = \frac{I_p}{2}\sum_{\alpha,k}^{3.3}\xi_{\alpha k}^2; \quad B_{\alpha k} = -I_p\xi_{\alpha k};$$

$$\xi_{\alpha k} = |\tau_\alpha\sigma_k|_{\lambda\lambda}.$$

We shall choose a system of coordinates in the $\vec{\tau}$ and $\vec{\sigma}$ space such that the matrix $\xi_{\alpha k}$ is diagonal[3,4]: $\xi_{\alpha k} = \xi_k\delta_{\alpha k}$. Then

$$H^0 = A + \sum_k^3 B_{kk}\tau_k\sigma_k.$$

We now apply a unitary transformation to H^0:[3]

$$S = \frac{1}{\sqrt{2}}(\sigma_{\bar{x}} + i\tau_{\bar{y}}).$$

$$S^{-1}H^0S = A - B_1\tau_{\bar{z}} - B_2\sigma_{\bar{z}} - B_3\tau_{\bar{z}}\sigma_{\bar{z}} \qquad (B_{kk} \equiv B_k).$$

The eigenvalues are

$$E_1^0 = A - B_1 - B_2 - B_3;$$

$$E_2^0 = A + B_1 + B_2 - B_3;$$

$$E_3^0 = A - B_1 + B_2 + B_3;$$

$$E_4^0 = A + B_1 - B_2 + B_3.$$

According to Eqs. (3) and (15) of I we have, for $\lambda = 1$,

[3] G. Wentzel, Helv. Phys. Acta **16**, 551 (1943).

[4] W. Pauli and S. Dancoff, Phys. Rev. **62**, 85 (1942).

$$\xi_1^{(1)} = \xi_2^{(1)} = \xi_3^{(1)} = -1;$$

$$H_1^0 = I_p\left(\frac{3}{2} - \tau_z - \sigma_z - \tau_z\sigma_z\right);$$

$$\psi_{11}^0 = u_1;\ \psi_{12}^0 = u_2;$$

$$\psi_{13}^0 = u_3;\ \psi_{14}^0 = u_4,$$

with

$$u_1 = \begin{vmatrix}1\\0\\0\\0\end{vmatrix};\ u_2 = \begin{vmatrix}0\\1\\0\\0\end{vmatrix};\ u_3 = \begin{vmatrix}0\\0\\1\\0\end{vmatrix};\ u_4 = \begin{vmatrix}0\\0\\0\\1\end{vmatrix};$$

$$E_{11}^0 = -\frac{3}{2}I_p;\ E_{12}^0 = E_{13}^0 = E_{14}^0 = \frac{5}{2}I_p.$$

For $\lambda = 2$:

$$\xi_1^{(2)} = \xi_2^{(2)} = 1;\ \xi_3^{(2)} = -1;$$

$$H^0 = I_p\left(\frac{3}{2} + \tau_z + \sigma_z - \tau_z\sigma_z\right);$$

$$\psi_{22}^0 = u_4;\ \psi_{21}^0 = u_3;\ \psi_{23}^0 = u_2;$$

$$\psi_{24}^0 = u_1;\ E_{22}^0 = -\frac{3}{2}I_p;$$

$$E_{21}^0 = E_{23}^0 = E_{24}^0 = \frac{5}{2}I_p.$$

For $\lambda = 3$:

$$\xi_1^{(3)} = -1;\ \xi_2^{(3)} = \xi_3^{(3)} = 1;$$

$$H_3^0 = I_p\left(\frac{3}{2} - \tau_z + \sigma_z + \tau_z\sigma_z\right);$$

$$\psi_{33}^0 = u_2;\ \psi_{31}^0 = u_1;\ \psi_{32}^0 = u_3;$$

$$\psi_{34}^0 = u_4;\ E_{33}^0 = -\frac{3}{2}I_p;$$

$$E_{31}^0 = E_{32}^0 = E_{34}^0 = \frac{5}{2}I_p.$$

For $\lambda = 4$:

$$\xi_1^{(4)} = \xi_3^{(4)} = 1;\ \xi_2^{(4)} = -1;$$

$$H_4^0 = I_p\left(\frac{3}{2} + \tau_z - \sigma_z + \tau_z\sigma_z\right);$$

$$\psi_{44}^0 = u_3;\ \psi_{41}^0 = u_4;\ \psi_{42}^0 = u_2;\ \psi_{43}^0 = u_1;$$

$$E_{44}^0 = -\frac{3}{2}I_p;\ E_{41}^0 = E_{42}^0 = E_{43}^0 = \frac{5}{2}I_p.$$

Knowing $\psi_{\lambda\lambda}^0$ it is possible to find the diagonal matrix elements of the spin operators $\{\tau'_\alpha \vec{\sigma}'\}_{\lambda\lambda} = \{S^{-1}\tau_\alpha\vec{\sigma}S\}_{\lambda\lambda}$ which enter the expressions for the magnetic moments [Eq. (26) in I]. We then obtain for M_z in the zeroth approximation

$$M_z = -\left(\frac{g^2\alpha}{\hbar c}\frac{e}{\varkappa^2 a} + \frac{e\hbar}{4mc}\right)|\tau_z\sigma_z|_{\lambda\lambda}$$

$$= \mp\left(\frac{g^2\alpha}{\hbar c}\frac{e}{\varkappa^2 a} + \frac{e\hbar}{4mc}\right).$$

The magnetic moments of the proton and the neutron differ only in sign.

It may be noted that the field φ_α^0 is fully determined by the parameters η_α, η_k, $\xi_{\alpha k}$ which are found when solving Eq. (1), and that the η_α, $\xi_{\alpha k}$ differ for different λ. For example, in the case of a scalar symmetric field $\varphi_x^0 = \varphi_y^0 = 0$ for $\lambda = 1, 2$; $(\varphi_z^0)_{\lambda=1} = \frac{g}{c\sqrt{4\pi}} \int G(\mathbf{r}, \mathbf{r}')\, U(\mathbf{r}')\, d\mathbf{r}'$; $(\varphi_z^0)_{\lambda=2} = -(\varphi_z^0)_{\lambda=1}$. As a result of the fact that φ_α^0 is fully determined by solving the equations of the zeroth order, the character of the fields φ_α is purely oscillatory in all its degrees of freedom (the Hamiltonian $H_{\lambda 2}$ splits into a sum of harmonic oscillator Hamiltonians). This is not so in the theory of molecules (see I). Since the electronic energy $E_m^0(\mathbf{R}_i)$ depends only on the distance between the nucleon positions, $|\mathbf{R}_i - \mathbf{R}_j|$, the minimizing of the energy yields only $3n - 6$ ($3n - 5$ for $n = 2$) relative distances. In these $3n - 6$ degrees of freedom the motion is oscillatory in character; the remainder are translatory (motion of the center of mass) and rotational (rotation of the molecule as a whole) in character.

In all three cases of symmetric coupling, the eigenvalues $E_{\lambda\lambda}^0$ are the same for the different λ as to be expected. This degeneracy disappears only in the higher approximations when real mesons appear. However, for a particular Hamiltonian H_λ^0 the eigenvalues $E_{\lambda\lambda}^0$ are not degenerate:

$E^0_{\lambda\mu} \neq E^0_{\lambda\lambda}$ for $\mu \neq \lambda$. A mathematical degeneracy is absent.

2. We shall now consider two nucleons interacting with a pseudoscalar neutral field. According to Eq. (30) of I the zero order Hamiltonian for this case is

$$H^0 = A + (\mathbf{B}_\mathrm{I}, \vec{\sigma}_\mathrm{I}) + (\mathbf{B}_\mathrm{II}, \vec{\sigma}_\mathrm{II}); \tag{5}$$

$$A = \frac{I_p}{2}[(\vec{\eta}_\mathrm{I})^2 + (\vec{\eta}_\mathrm{II})^2] + \frac{1}{2}\sum_{i,k}^{3,3} K_{ik}\,\eta_{\mathrm{I}i}\eta_{\mathrm{II}k};$$

$$B_{\mathrm{I}k} = -\eta_{\mathrm{I}k} I_p - \sum_l^3 \eta_{\mathrm{II}l} K_{kl};$$

$$B_{\mathrm{II}k} = -\eta_{\mathrm{II}k} I_p - \sum_l^3 \eta_{\mathrm{I}l} K_{lk};$$

$$\vec{\eta}_{\mathrm{I,II}} = \{\vec{\sigma}_{\mathrm{I,II}}\}_{\lambda\lambda};$$

$$K_{ik} = \frac{g^2}{\varkappa^2}\int U_\mathrm{I}(\mathbf{r} - \mathbf{r}_\mathrm{I})\, U_\mathrm{II}(\mathbf{r}' - \mathbf{r}_\mathrm{II})$$

$$\times \frac{\partial^2 G(\mathbf{r}, \mathbf{r}')}{\partial x_i \partial x'_k}\, d\mathbf{r} d\mathbf{r}' = \delta_{ik} K_1 + n_i n_k K_2;$$

$$\mathbf{n} = \frac{\mathbf{r}_\mathrm{II} - \mathbf{r}_\mathrm{I}}{r_\mathrm{I,II}} \equiv \frac{\mathbf{r}_\mathrm{I,II}}{r_\mathrm{I,II}};$$

For

$$\mathbf{r}_\mathrm{I,II} \gg a \quad K_1 \approx -\frac{g^2}{\varkappa^2}\frac{1}{r_\mathrm{I,II}}\frac{d}{dr_\mathrm{I,II}} G(\mathbf{r}_\mathrm{I}, \mathbf{r}_\mathrm{II});$$

$$K_2 \approx r_\mathrm{I,II}\frac{dK_1}{dr_\mathrm{I,II}}.$$

Therefore,

$$H^0 = \frac{I_p}{2}[(\vec{\eta}_\mathrm{I})^2 + (\vec{\eta}_\mathrm{II})^2] + K_1(\vec{\eta}_\mathrm{I}, \vec{\eta}_\mathrm{II}) + K_2(\mathbf{n}, \vec{\eta}_\mathrm{I})(\mathbf{n}, \vec{\eta}_\mathrm{II}) - I_p[(\vec{\eta}_\mathrm{I}, \vec{\sigma}_\mathrm{I}) + (\vec{\eta}_\mathrm{II}, \vec{\sigma}_\mathrm{II})] - K_1[(\vec{\sigma}_\mathrm{I}, \vec{\eta}_\mathrm{II}) + (\vec{\sigma}_\mathrm{II}, \vec{\eta}_\mathrm{I})] - K_2[(\vec{\sigma}_\mathrm{I}, \mathbf{n})(\vec{\eta}_\mathrm{II}, \mathbf{n}) + (\vec{\sigma}_\mathrm{II}, \mathbf{n})(\vec{\eta}_\mathrm{I}, \mathbf{n})].$$

We now choose in the $\vec{\sigma}_\mathrm{I}$, $\vec{\sigma}_\mathrm{II}$ space a frame of reference such that the axes $\bar{z}_\mathrm{I}$ and $\bar{z}_\mathrm{II}$ are parallel to $\mathbf{B}_\mathrm{I}$ and $\mathbf{B}_\mathrm{II}$, respectively. Then $\eta_{i\bar{x}} = \eta_{i\bar{y}} = 0$ (i = I, II) or, $\vec{\eta}_\mathrm{I} \,||\, \mathbf{B}_\mathrm{I}$ and $\vec{\eta}_\mathrm{II} \,||\, \mathbf{B}_\mathrm{II}$. Now, $\mathbf{B}_i = a_i\vec{\eta}_\mathrm{I} + b_i\vec{\eta}_\mathrm{II} + c_i\mathbf{n}$; so $\vec{\eta}_\mathrm{I} \,||\, \mathbf{n}$ and $\vec{\eta}_\mathrm{II} \,||\, \mathbf{n}$, and finally

$$H^0 = A + B_{\mathrm{I}\bar{z}}\sigma_{\mathrm{I}\bar{z}} + B_{\mathrm{II}\bar{z}}\sigma_{\mathrm{II}\bar{z}};$$

$$B_{\mathrm{I}\bar{z}} = -I_p\eta_{\mathrm{I}\bar{z}} - K_p\eta_{\mathrm{II}\bar{z}};$$

$$B_{\mathrm{II}\bar{z}} = -I_p\eta_{\mathrm{II}\bar{z}} - K_p\eta_{\mathrm{I}\bar{z}}; \quad K_p = K_1 + K_2.$$

The eigenvalues are

$$E_1^0 = A + B_{\mathrm{I}\bar{z}} + B_{\mathrm{II}\bar{z}}; \quad E_2^0 = A - B_{\mathrm{I}\bar{z}} - B_{\mathrm{II}\bar{z}};$$

$$E_3^0 = A - B_{\mathrm{I}\bar{z}} + B_{\mathrm{II}\bar{z}};$$

$$E_4^0 = A + B_{\mathrm{I}\bar{z}} - B_{\mathrm{II}\bar{z}}.$$

For $\lambda = 1$:

$$\eta^{(1)}_{\mathrm{I}\bar{z}} = \eta^{(1)}_{\mathrm{II}\bar{z}} = 1; \quad E^0_{11} = -I_p - K_p.$$

For $\lambda = 2$:

$$\eta^{(2)}_{\mathrm{I}\bar{z}} = \eta^{(2)}_{\mathrm{II}\bar{z}} = -1; \quad E^0_{22} = -I_p - K_p.$$

For $\lambda = 3$:

$$\eta^{(3)}_{\mathrm{I}\bar{z}} = -1; \quad \eta^{(3)}_{\mathrm{II}\bar{z}} = 1; \quad E^0_{33} = -I_p + K_p.$$

For $\lambda = 4$:

$$\eta^{(4)}_{\mathrm{I}\bar{z}} = 1; \quad \eta^{(4)}_{\mathrm{II}\bar{z}} = -1; \quad E^0_{44} = -I_p + K_p.$$

We see that $\vec{\eta}_\mathrm{I}$, $\vec{\eta}_\mathrm{II}$ are parallel to $\mathbf{n}$, which is a preferred direction in this case of two nucleons. Similarly, we obtain for a scalar symmetric field [Eq. (30) in I]

$$H^0 = A + (\mathbf{B}_\mathrm{I}, \vec{\tau}_\mathrm{I}) + (\mathbf{B}_\mathrm{II}, \vec{\tau}_\mathrm{II}); \tag{6}$$

$$A = \frac{I_s}{2}[(\vec{\eta}_\mathrm{I})^2 + (\vec{\eta}_\mathrm{II})^2] + K_s(\vec{\eta}_\mathrm{I}, \vec{\eta}_\mathrm{II});$$

$$\mathbf{B}_\mathrm{I} = -\vec{\eta}_\mathrm{I} I_s - \vec{\eta}_\mathrm{II} K_s;$$

$$\mathbf{B}_\mathrm{II} = -\vec{\eta}_\mathrm{II} I_s - \vec{\eta}_\mathrm{I} K_s;$$

$$K_s = g^2\int U_\mathrm{I}(\mathbf{r})\, U_\mathrm{II}(\mathbf{r}')\, G(\mathbf{r}, \mathbf{r}')\, d\mathbf{r} d\mathbf{r}'.$$

For the choice of axes z_I and z_{II} parallel to B_I and B_{II}, respectively, we find

$$H^0 = A + B_{I\bar{z}}\tau_{I\bar{z}} + B_{II\bar{z}}\tau_{II\bar{z}}.$$

For $\lambda = 1$:

$$\eta^{(1)}_{I\bar{z}} = \eta^{(1)}_{II\bar{z}} = 1;\ E^0_{11} = -I_s - K_s;$$

For $\lambda = 2$:

$$\eta^{(2)}_{I\bar{z}} = \eta^{(2)}_{II\bar{z}} = -1;\ E^0_{22} = -I_s - K_s;$$

For $\lambda = 3$:

$$\eta^{(3)}_{I\bar{z}} = -1;\ \eta^{(3)}_{II\bar{z}} = 1;\ E^0_{33} = -I_s + K_s;$$

For $\lambda = 4$:

$$\eta^{(4)}_{I\bar{z}} = 1;\ \eta^{(4)}_{II\bar{z}} = -1;\ E^0_{44} = -I_s + K_s.$$

In all cases we obtain an interaction of the Yukawa type. But the $E^0_{\lambda\lambda}$ for two nucleons are already not the same: $E^0_{11} = E^0_{22}$ and $E^0_{33} = E^0_{44}$, but $E^0_{11} \neq E^0_{33}$ [these cases differ by having parallel or antiparallel (ordinary and isotopic) spins].

3. We shall now investigate the first approximation for the simple case of the symmetric scalar field. Equation (20) of I, describing the motion of a meson interacting with a nucleon, for $\lambda = 1$ has the form

$$(\varkappa^2 - \Delta)\varphi_{\alpha k} - d_s U(\mathbf{r}) \sum_{\beta=1}^{3} (|\tau_\beta|_{12}|\tau_\alpha|_{21} + |\tau_\alpha|_{12}|\tau_\beta|_{21}) \int U(\mathbf{r}')\varphi_{\beta k}(\mathbf{r}')\,d\mathbf{r}' = \frac{\omega_k^2}{c^2}\varphi_{\alpha k}; \quad (7)$$

$$d_s = 2\pi g^2 / I_s.$$

In the frame of reference $\bar{x}$, $\bar{y}$, $\bar{z}$ we have $|\tau_{\bar{x}}|_{12} = |\tau_{\bar{x}}|_{21} = 1$; $|\tau_{\bar{y}}|_{12} = -|\tau_{\bar{y}}|_{21} = -i$; $|\tau_{\bar{z}}|_{12} = |\tau_{\bar{z}}|_{21} = 0$ and we obtain from (7)

$$(\varkappa^2 - \Delta)\varphi_{\bar{x}} - 2 d_s U(\mathbf{r}) \int \varphi_{\bar{x}}(\mathbf{r}')U(\mathbf{r}')\,d\mathbf{r}' = \frac{\omega^2}{c^2}\varphi_{\bar{x}}; \quad (8)$$

$$(\varkappa^2 - \Delta)\varphi_{\bar{y}} - 2 d_s U(\mathbf{r}) \int \varphi_{\bar{y}}(\mathbf{r}')U(\mathbf{r}')\,d\mathbf{r}' = \frac{\omega^2}{c^2}\varphi_{\bar{y}};$$

$$(\varkappa^2 - \Delta)\varphi_{\bar{z}} = \frac{\omega^2}{c^2}\varphi_{\bar{z}}.$$

The equations for $\lambda = 2$ are identical.

It should be mentioned that only the full field $\Phi_\alpha = \varphi^0_\alpha + \varphi_\alpha$ obeys charge invariance but not the fields φ^0_α and φ_α separately. Because $\varphi^0_{\bar{x}} = \varphi^0_{\bar{y}} = 0$ and only $\varphi^0_{\bar{z}} \neq 0$, the component $\varphi_{\bar{z}}$ is also singled out.

We see that in the case of $\varphi_{\bar{x}}$ and $\varphi_{\bar{y}}$ the meson is being attracted by the nucleon. In the limiting case $a \to 0$ and for $\epsilon' = \epsilon - \mu c^2 \ll \mu c^2$ we obtain (taking, for example, $U = 3/4\pi a^3$ for $r \leq a$; $U = 0$ for $r > a$, and $I_s \approx 3g^2/2a$)

$$-\frac{\hbar^2}{2\mu}\Delta\varphi_{\bar{x}} - \frac{4\pi\hbar^2}{3\mu}\lim_{a\to 0} aU(\mathbf{r})\varphi_{\bar{x}} = \varepsilon'\varphi_{\bar{x}}.$$

The depth of the potential well $U_0 \approx \hbar^2/\mu a^2$, and so the condition $U_0 \ll \hbar^2/\mu a^2$ for the absence of bound states (see I) is not fulfilled. Therefore, with the above choice of $U(\mathbf{r})$ there are isobars possible even for $a \to 0$; $\epsilon' \approx g^0$.

Under usual experimental conditions the charge of a nucleon, i.e., τ_z, is fully determined, and therefore $\vec{\eta}$ is parallel to the z-axis. Therefore, here the $\bar{z}$-axis coincides with the z-axis, and $\varphi_{\bar{z}} = \varphi_z$. From Eq. (8) one can see that in the first approximation neutral mesons (φ_z) do not interact with fixed nucleons. (This does not apply to a pseudoscalar field.)

If $\epsilon \geq \mu c^2$ we obtain for φ_x ($a_x = 1$) the following equation [see (23) in I)

$$\varphi_x = e^{i(\mathbf{k},\mathbf{r})} + \frac{d_s}{2\pi}\int \frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}U(\mathbf{r}')\,d\mathbf{r}' \int U(\mathbf{r}'')\varphi_x(\mathbf{r}'')\,d\mathbf{r}''$$

and a similar expression for φ_y. Here one has to take $d_s = 2\pi g^2/[I_s - (\epsilon/2)]$ [see (25) in I). The solution has the form

$$\varphi_x = e^{i(\mathbf{k},\mathbf{r})}$$
$$+\frac{d_s}{2\pi}\left(1-\frac{d_s}{2\pi}\int\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}U(\mathbf{r})\,U(\mathbf{r}')\,d\mathbf{r}d\mathbf{r}'\right)^{-1}$$
$$\int e^{i(\mathbf{k},\mathbf{r}')}U(\mathbf{r}')\,d\mathbf{r}'\int\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}U(\mathbf{r}')\,d\mathbf{r}'.$$

Expanding in a series in powers of ka and κa we obtain for $r \to \infty$

$$\varphi_x \approx e^{i(\mathbf{k},\mathbf{r})}$$
$$-\frac{1-k^2a^2/5}{(\varepsilon/2g^2)+\varkappa+ik-3a(\varkappa^2+k^2)/8}\,\frac{e^{ikr}}{r}\,(r_1=0).$$

For the scattering cross section for charged mesons, not considering absorption processes, we then obtain $\left(\varphi_- = \frac{1}{\sqrt{2}}(\varphi_x - i\varphi_y);\ \varphi_+ = \frac{1}{\sqrt{2}}(\varphi_x + i\varphi_y)\right)$:

$$d\sigma = \frac{(1-k^2a^2/5)^2 d\Omega}{[(\varepsilon/2g^2)^2+\varkappa-3a(\varkappa^2+k^2)/8]^2+k^2}; \quad (9)$$

$$d\Omega = 2\pi\sin\vartheta d\vartheta.$$

We now shall go to the limit $a = 0$. Here, naturally, the self energy $E^0_{\lambda\lambda} = \infty$, but the cross section $d\sigma$ as well as the bound state energy ϵ' of the meson (see above) approach a finite limit.

$$d\sigma = \frac{d\Omega}{[(\varepsilon/2g^2)^2+\varkappa]^2+k^2}.$$

For a neutral pseudoscalar field Eq. (20) of I has the form

$$(\varkappa^2-\Delta)\varphi - 2d_p\left[\frac{\partial U}{\partial x}\int\frac{\partial U'}{\partial x'}\varphi(\mathbf{r}')\,d\mathbf{r}\right.$$
$$\left.+\frac{\partial U}{\partial y}\int\frac{\partial U'}{\partial y'}\varphi(\mathbf{r}')\,d\mathbf{r}'\right] = \frac{\omega^2}{c^2}\varphi;$$
$$d_p = 2\pi\frac{g^2}{\varkappa^2}\left(I_p - \frac{\varepsilon}{2}\right)^{-1}.$$

The solution for $\epsilon \geq \mu c^2$ is

$$\varphi = e^{i(\mathbf{k},\mathbf{r})}$$
$$+\left(\frac{2\pi}{d_p}-\int\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial U}{\partial x}\frac{\partial U'}{\partial x'}\,d\mathbf{r}\,d\mathbf{r}'\right)^{-1}$$
$$\times\left[\int\frac{\partial U(\mathbf{r}')}{\partial x'}e^{i(\mathbf{k},\mathbf{r}')}\,d\mathbf{r}'\int\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial U(\mathbf{r}')}{\partial x'}\,d\mathbf{r}'\right.$$
$$\left.+\int\frac{\partial U(\mathbf{r}')}{\partial y'}e^{i(\mathbf{k},\mathbf{r}')}\,d\mathbf{r}'\int\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\frac{\partial U(\mathbf{r}')}{\partial y'}\,d\mathbf{r}'\right];$$

$$\{\vec{\sigma}\}_{\lambda\lambda}\parallel[\mathbf{k}\times\mathbf{k}'].$$

Up to terms $(ka)^2$ and $(\kappa a)^2$ the meson scattering cross section is given by

$$d\sigma = \frac{k^4\cos^2\vartheta\,(1-k^2a^2/5)^2\,d\Omega}{[(\varepsilon\varkappa^2/2g^2)+I_0(\varkappa^2+k^2)/3-\varkappa^3/3]^2+k^6/9}, \quad (10)$$

$$I_0 = \int U(\mathbf{r})\,U(\mathbf{r}')\frac{d\mathbf{r}\,d\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}\approx\frac{3}{2a};$$

for $a \to 0 \quad d\sigma \to 0$.

In the case of two nucleons, their being in a bound state can lead to an additional interaction between them in the order g^0. Actually, the energy ϵ depends, generally speaking, on the distance r_{12} and can even become zero at a certain distance $r_{12} = r_k$. The energy of the system has [in the absence of real mesons ($n_{k\rho} = 0$)] up to terms of order g^0 the form

$$E^0_\lambda + E_{\lambda 2} = E^0_\lambda(r_{12}) + \sum_{\varepsilon_i<\mu c^2}\frac{\varepsilon_i(r_{12})}{2} + \sum_{\hbar\omega_k>\mu c^2}\frac{\hbar\omega_k}{2}.$$

This way the term $\sum_{\epsilon_i<\mu c^2}\epsilon_i/2$ in the zero order energy of the meson field, as well as E^0_λ depends on r_{12}.

4. The results here obtained differ significantly from earlier work (see, for example, references 3-8). However, it can be shown that the quoted

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[5] G. Wentzel, Helv. Phys. Acta **13**, 269 (1940); **14**, 633 (1941).

[6] R. Serber and S. Dancoff, Phys. Rev. **63**, 143 (1943).

[7] W. Pauli and S. Kusaka, Phys. Rev. **63**, 400 (1943).

[8] W. Pauli, *Meson Theory of Nuclear Forces.*

papers contain essential errors. It should be pointed out, first, that the replacement of the operators $\vec{\sigma}$ and $\vec{\tau}$ by classical unit vectors in reference 8 seems to be inadmissible. The transition to the classical case corresponds to a situation with large quantum numbers. However, the spin operators have just two quantum states, and a transition to classical mechanics seems to be meaningless.

In references 3-7 the operators $\vec{\sigma}$ and $\vec{\tau}$ have been retained and the quantum mechanical Hamiltonian has been used. For the zeroth order Hamiltonian the correct potential enervy V_f, diagonalized in the spin variables, has been obtained. However, it has been incorrectly assumed that the level $E^0=(5/2\,I)$ for the pseudoscalar symmetric field and $E^0=(3/2\,I)$ for the scalar symmetric or the pseudoscalar neutral field obtains for the real state of the system. As has been shown above, the energy is actually the same for all cases of symmetric coupling ($E^0_{\lambda\lambda}=E^0_{\lambda'\lambda'}$). The relations ($E^0=(5/2I)$ and $E^0=(3/2I)$ hold for the eigenvalues of the auxiliary orthonormal functions $\psi^0_{\lambda\mu}$ for $\mu\neq\lambda$ (see I).

A more serious shortcoming in references 3-7 seems to be the assumption that because of the large difference $E^0=(3/2I)(5/2I)$ between the unperturbed and the perturbed level, one can neglect transitions from the unperturbed to the perturbed level. Therefore, in the higher approximations the spin operator $O_\alpha\Phi_\alpha$ is replaced by its diagonal elements (see, for example, Sec. 5 of reference 4 and III A of reference 6). Now the perturbation $H^{(1)}$ is of first order g, and therefore, its nondiagonal elements yield in the second approximation terms of zero order in g [see (19) in I; these nondiagonal elements actually correspond to transitions to the real level $E^0_{\lambda\lambda'}$, but in a state $\psi^0_{\lambda\mu}$ with $\lambda\neq\mu$]. In this way even the terms of order g^0 have not been correctly obtained in references 3-7, and the energy of the isobars, which has geen given in these references, actually is of the order g^{-2}. Furthermore, only one spin state of the system with $\lambda=\mu$ is considered, i.e., one projection of $\langle\vec{\sigma}\rangle$, $\langle\vec{\tau}\rangle$, $\langle\tau_\alpha\vec{\sigma}\rangle$; as has been seen above, the direction of the vectors is arbitrary. But it was inferred in these papers that, by solving the equation of zero order, one can only obtain the absolute value of the vectors $\langle\vec{\sigma}\rangle$, $\langle\vec{\tau}\rangle$, $\langle\tau_\alpha\vec{\sigma}\rangle$; therefore, angular variables were introduced in isospace to describe the rotational degrees of freedom of the vectors $\langle\vec{\sigma}\rangle$, $\langle\vec{\tau}\rangle$, $\langle\tau_\alpha\vec{\sigma}\rangle$ (a, b, c or $e_{\alpha k}$ for the pseudoscalar and ϑ for the scalar field, see references 4-6). However, as shown above, the vectors $\langle\vec{\sigma}\rangle$, $\langle\vec{\tau}\rangle$, $\langle\tau_\alpha\vec{\sigma}\rangle$ are fully determined solving (1), although for symmetric coupling the direction of these vectors turns out to be arbitrary. By the introduction of the angular variables the number of the degrees of freedom is arbitrarily increased while no conditions are imposed on the old variables, the spin variables of the nucleons and the field oscillators *. Finally, corrections to the energy are not calculated in references 3-7 in the straightforward and unique way of perturbation theory, but by applying a series of unitary transformations which are different for the different cases. Here the calculations are rendered very involved by the fact that the zero order field φ^0_α is an operator and not a classical quantity; therefore, the field of the real mesons and the corresponding momentum turn out not to be canonical variables. The involved character of the calculations explains the occurrence of a number of actual mistakes in these references which have been pointed out by the author[9].

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* *Note added in proof:* These critical remarks apply to a great extent to recently published papers by J. Pekar, J. Exper. Theoret. Phys. USSR 27,398, 411,579 (1954).

[9] B. Geilikman, Dokl. Akad. Nauk SSSR **90**, 991 (1953).

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