

where  $J_0$  is the zero order Bessel function. From this it follows that if  $\xi$  is distributed normally, then  $A$  follows the Rayleigh distribution.

Hence, knowing  $f_\xi(u)$ , one can use the Fourier transform to obtain the distribution of  $\xi$ :

$$w_\xi(\xi) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f_\xi(u) e^{-i u \xi} du, \quad (2)$$

while the Hankel transform gives the distribution of the amplitude  $A$ . This elegant result is based on the following conditions: 1)  $A$  and  $\theta$  are independent and 2)  $\theta$  is distributed uniformly in the interval  $(0, 2\pi)$ . It is not difficult to show that both these are necessary conditions for  $\xi(t)$  to be stationary, taking the two-dimensional distribution  $w(A, \theta)$  to be stationary. The latter must have the form  $w(A, \theta) dA d\theta = w_A(A) dA d\theta / 2\pi$  or else the process  $\xi(t)$  will not be stationary.

In the present note I should like to show that (1) and (2) can be used to derive a formula which gives  $w_\xi(\xi)$  directly in terms of  $w_A(A)$ . Indeed, according to (1) we have

$$f_\xi(u) = \int_0^\infty w_A(A) J_0(Au) dA.$$

In conjunction with (2) this gives

$$w_\xi(\xi) = \int_0^\infty w_A(A) dA \frac{1}{2\pi} \int_{-\infty}^{+\infty} J_0(Au) e^{-i \xi u} du.$$

The inner integral is  $1/\pi \sqrt{A^2 - \xi^2}$  when  $A \geq |\xi|$  and zero for  $A < |\xi|^2$ , so that

$$w_\xi(\xi) = \frac{1}{\pi} \int_{|\xi|}^\infty \frac{w_A(A) dA}{\sqrt{A^2 - \xi^2}}, \quad (3)$$

or, introducing the new variable of integration  $x$ ,  $A = |\xi| \operatorname{ch} x$

$$w_\xi(\xi) = \frac{1}{\pi} \int_0^\infty w_A(|\xi| \operatorname{cosh} x) dx \quad (4)$$

It is easy to verify that the Rayleigh distribution for  $A$ :

$$w_A(A) = \frac{A}{\sigma^2} e^{-A^2/\sigma^2}$$

implies by formula (4), the normal distribution for  $\xi$  with mean square  $\overline{\xi^2} = \sigma^2$ . If the amplitude is fixed, i.e.,  $w_A(A) = \delta(A - A_0)$  then from (3) it follows that

$$w_\xi(\xi) = \begin{cases} 1/\pi \sqrt{A_0^2 - \xi^2} & (|\xi| \leq A_0), \\ 0 & (|\xi| > A_0). \end{cases}$$

The uniform distribution for  $A$ , i.e.,  $w_A(A) = 1/A_0$  for  $A \leq A_0$  and  $w_A(A) = 0$  for  $A > A_0$  gives, according to (4)

$$w_\xi(\xi) = \begin{cases} \frac{1}{2\pi A_0} \ln \left( \frac{A_0 + \sqrt{A_0^2 - \xi^2}}{A_0 - \sqrt{A_0^2 - \xi^2}} \right) & (|\xi| \leq A_0) \\ 0 & (|\xi| > A_0). \end{cases}$$

The exponential distribution  $w_A(A) = \alpha e^{-\alpha A}$  for the amplitude  $A$  leads to a Macdonald function of zero order for the distribution of  $\xi$ .

$$w_\xi(\xi) = \frac{\alpha}{\pi} \int_0^\infty e^{-\alpha |\xi| \cosh x} dx = \frac{\alpha}{\pi} K_0(\alpha \xi).$$

<sup>1</sup> A. Blanc-Lapierre, M. Savelli and A. Tortrat, *Ann. Télécomm.* **9**, 237 (1954).

<sup>2</sup> I. M. Ryzhik and I. S. Gradshteyn, *Tables of Integrals, sums, series and products*, Moscow, Leningrad, 1951, p. 268.

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## The Fermi-Yang Hypothesis

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ACCORDING to the Fermi-Yang hypothesis a  $\pi$ -meson is considered as a composite particle consisting of a proton and an anti-neutron in a bound state. In the work of Fermi and Yang<sup>1</sup> the interaction between a nucleon and an anti-nucleon is approximated by a potential well whose width is equal to the nuclear Compton wavelength,  $r_0 = \hbar/Mc$ , and whose depth is determined by the requirement that the lowest energy eigenvalue of the system equals the meson rest energy  $E = \mu c^2$ . With this condition they obtain the value  $V_0 = 26.5 Mc^2$  for the depth of the well in the

formation of the  ${}^1S_0$  proton-anti-neutron system.

In recent years, a series of particles have been discovered with mass intermediate between that of nucleons and  $\pi$ -mesons. Such a family of particles may also be considered from the point of view of the Fermi-Yang hypothesis, i.e., as nucleon-anti-nucleon systems in bound states. It is interesting to compare the value of the interaction potential obtained in this case with the enormously strong potential obtained by Fermi and Yang by considering the  $\pi$ -meson as a composite particle.

Fermi and Yang have considered the problem in the two particle approximation. The Schrödinger equation for the proton-anti-neutron system is as follows:

$$[-\hbar^2 \nabla^2 (\vec{\alpha}_P - \vec{\alpha}_A) \vec{\nabla} + Mc^2(\beta_P + \beta_A) - V(r)(1 - \vec{\alpha}_A \vec{\alpha}_P)] \psi = E\psi, \quad (1)$$

where  $\psi$  is a 16-components wave function,  $r$  is the relative coordinate and the indices  $P$  and  $A$  refer to the proton and the anti-neutron. Given an interaction potential  $V(r)$  in the form of a well

of width  $r_0$  and depth  $V_0$ , the continuity condition on the logarithmic derivative of the wave function at  $r = r_0$  gives<sup>2</sup> for the state  ${}^1S_0$ :

$$k \operatorname{ctg} k r_0 = -k_0, \quad (2)$$

where

$$k^2 = \frac{E(8V_0^2 - 2V_0E + 4M^2c^4 - E^2)}{4(\hbar c)^2(2V_0 - E)}, \quad (3)$$

$$k_0^2 = \frac{4M^2c^4 - E^2}{4(\hbar c)^2} \quad (4)$$

The value of the potential depth  $V_0$  can be found from Eq. (2) for various stipulated values of  $E = \mu c^2$  and  $r_0 = \hbar c / \kappa$  ( $\kappa$  is the rest mass of the composite particle; if it is assumed that the Fermi-Yang force arises from quanta exchange, then the parameter  $\kappa$  corresponds to their rest energy). The value of  $V_0$  is given in the Table below for various values of  $E$  and  $\kappa$ . All quantities are given in units of  $Mc^2$ . For  $\pi$ -mesons  $\mu \approx 0.15 M$  (i.e.,  $E \approx 0.15$ ) and with  $r_0 = \hbar / Mc$  ( $\kappa = 1$ ) we have the value obtained by Fermi and Yang  $V_0 \approx 26.5$ ;

$\kappa = 1.2$		$\kappa = 1.0$		$\kappa = 0.8$		$\kappa = 0.6$		$\kappa = 0.4$		$\kappa = 0.2$	
$E$	$V_0$										
0.15	37.7	0.13	30.5	0.09	31.5	0.12	14.4	0.11	7.8	0.12	2.0
0.72	7.3	0.46	8.7	0.37	7.4	0.83	1.1	0.54	1.2		
1.09	4.5	0.81	4.6	0.96	2.3						
1.25	3.7	1.26	2.4	1.14	1.3						
1.54	2.5	1.43	1.5								
1.66	1.9										

for  $\tau$ ,  $\kappa$ -,  $\chi$ -particles for  $r_0 = 2\hbar / Mc$  ( $\kappa = 0.5$ ),  $V_0 \approx 2.2$ , i.e., about 2 bev. The maximum possible value of the masses  $\mu = E/c^2$  of the constructed particles diminishes with increasing value of  $r_0$ .

The present calculation was suggested by Prof. Ia. P. Terletskii.

<sup>1</sup> E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).

<sup>2</sup> L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, vol. 1, p. 140, GFTI, 1948.

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## On the Development by Means of Leaders of the Process of Breakdown of Liquids

(Reply to the Remarks of G. A. Vorob'ev)

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**I**N one of the 1954<sup>1</sup> issues of JETP there were published some remarks by G. A. Vorob'ev on my article<sup>2</sup> which dealt with the question of pre-breakdown currents in liquids. These remarks referred to