where

$$\epsilon^{\mathrm{eff}}_{\alpha\beta} = \epsilon_{\alpha\beta} + \frac{1}{2} i S_\alpha \frac{\omega^{0\,2}_{z\beta}}{\omega^2} \frac{1-(\omega^{0\,2}_{zz}/\omega^{0\,2}_{z\beta})(\epsilon^0_{z\beta}/\epsilon^0_{zz})}{\omega\tau(\epsilon_{zz}/\epsilon^0_{zz})+4i}; \tag{10}$$

$$S_\alpha = \frac{8}{\bar v}\,\frac{1}{\int \frac{dS}{v}} \int_0^{2\pi} N_\alpha\, d\varphi \int_0^{\pi/2} tg\,\theta \left\{\frac{\bar v}{v(\varphi,\theta)} - \frac{\bar v}{v\left(\varphi,\frac{\pi}{2}\right)}\right\} d\theta;\quad \bar v = \frac{1}{S}\int v\, dS;$$

$$\mathbf{v} = (v\sin\theta\cos\varphi,\ v\sin\theta\sin\varphi,\ v\cos\theta);$$

$$N_\alpha = \begin{cases}\cos\varphi\ (\alpha = x) \\ \sin\varphi\ (\alpha = y).\end{cases} \tag{11}$$

The integration in Eq. (11) is carried out over the Fermi surface $\epsilon(\mathbf{p}) = \epsilon_0$.

In the simplest case of perfect reflection ($q = 1$) of the electrons from the metal surface the tensor of surface impedance is

$$Z_{\alpha\beta} = -\frac{4\pi i\omega}{c^2}\,\frac{\partial E_\alpha(0)}{\partial E'_\beta(0)}\quad \frac{8 i\omega}{c^2\,(l\delta^2)^{4/3}} \tag{12}$$

$$\times \int_0^\infty \frac{t\,dt}{t^3\delta_{\alpha\beta} + 3ik_{\alpha\beta} - (\omega^2\epsilon^{\mathrm{eff}}_{\beta\alpha}/c^2)(l\delta^2)^{2/3}};$$

$$k_{\alpha\beta} = \frac{8\pi}{S}\int_0^\pi \frac{M_\alpha N_\beta\, d\varphi}{K\left(\varphi,\frac{\pi}{2}\right)}$$

[$K(\phi, \theta)$ is the Gaussian curve of the Fermi surface].

It is seen from Eq. (10) that the effective dielectric constant is a complex quantity. Hence, it is not sufficient to know X and R for the measurement of ϵ^0 and the determination of the sign of ϵ. We note that the direction of the principal axes of the surface impedance tensor $z_{\alpha\beta}$ depends on the frequency.

These results [Eqs. (10)-(12)] apply not only to single crystals but also to polycrystals with sufficiently large crystal dimensions.

In conclusion, I consider it my pleasant duty to express my thanks to I. M. Lifshitz for his discussions of the results of the research.

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The Role of Spin in the Study of the Radiating Electron

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IN his recently published paper, Nelipa[1] claims that the ratio of the magnitude of the integrated radiation of the electron to the magnitude of the integrated radiation of a spinless particle is equal to $1 + (mc^2/E)^2$. Our calculations[2], which take into account quantum corrections of all orders, show that this is not so.

We obtained the following formulas, which may be applied to the radiation of all the spectrum, for arbitrary energies of the electron or of a spinless particle.

$$dW^{(1/2)} = \frac{ce^2}{\pi\sqrt{3}}\left(\frac{mc}{h}\right)^2 \tag{1a}$$

$$\times\ \xi d\xi\left[\int_{\frac{\xi}{1-\xi}\frac{1}{\zeta}}^{\infty} K_{5/3}(x)\,dx + \frac{\xi^2}{1-\xi}K_{2/3}\left(\frac{\xi}{1-\xi}\frac{1}{\zeta}\right)\right];$$

$$dW^{(0)} = \frac{ce^2}{\pi\sqrt{3}}\left(\frac{mc}{h}\right)^2 \xi d\xi \int_{\frac{\xi}{1-\xi}\frac{1}{\zeta}}^{\infty} K_{5/3}(x)\,dx;$$

$$\xi = \frac{h\omega}{E},\quad \zeta = \frac{3}{2}\frac{h}{Rmc}\left(\frac{E}{mc^2}\right)^2. \tag{1b}$$

For $h = 0$ these formulas give the classical formula for the differential spectrum:

$$dW = \frac{3\sqrt{3}}{4\pi}\frac{ce^2}{R^2}\left(\frac{E}{mc^2}\right)^4 y\,dy\int_y^\infty K_{5/3}(x)\,dx, \tag{2}$$

$$y = \frac{\omega}{\omega_c},\quad \omega_c = \frac{3}{2}\frac{c}{R}\left(\frac{E}{mc^2}\right)^3,$$

obtained by Ivanenko and Sokolov[3], and later by Schwinger[4]. If one considers only quantities of first order in h (first quantum correction), one obtains the quantum-theoretical formulas for the differential spectrum obtained by Sokolov and Ternov[5] and Schwinger[6] which are exact to the first order in h.

Formulas for total radiation energy, which are exact for arbitrary energies of the radiating particles, have the following form (see also reference 7):

$$W^{(1/2,\,0)} = W_{\rm cl}\,\varphi^{(1/2,\,0)}(\zeta); \qquad (3)$$

$$\varphi^{(1/2)}(\zeta) = \frac{3\sqrt{3}}{8}\pi\left\{\frac{\partial}{\partial\zeta}\frac{\Phi_{5/3}\left(\frac{i}{\zeta}\right)}{\zeta^2}\right. \qquad (3a)$$

$$\left.-\frac{\zeta^2}{3}\frac{\partial^3}{\partial\zeta^3}\frac{\Phi_{2/3}\left(\frac{i}{\zeta}\right)}{\zeta} - \frac{\sqrt{3}}{2\pi}\frac{1}{\zeta^2}\right\},$$

$$\varphi^{(0)}(\zeta) = \frac{3\sqrt{3}}{8}\pi\left\{\frac{\partial}{\partial\zeta}\frac{\Phi_{5/3}\left(\frac{i}{\zeta}\right)}{\zeta^2} - \frac{\sqrt{3}}{2\pi}\frac{1}{\zeta^4}\right\}; \qquad (3b)$$

$$\Phi_\nu(z) = i^{-\nu}[J_\nu(z) - \mathbf{J}_\nu(z)] + i^\nu[J_{-\nu}(z) - \mathbf{J}_{-\nu}(z)],$$

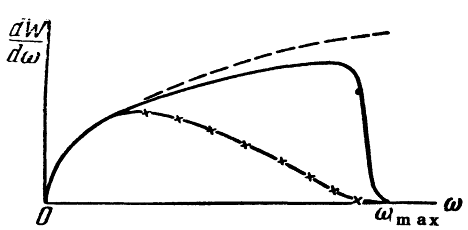
where J_ν is the Bessel function and $\mathbf{J}_\nu$ the Anger's function.

For relatively small energies ($\zeta \ll 1$) of the radiating particles, we obtain from the above formulas:

$$\frac{W^{(1/2)}}{W^{(0)}} \approx 1 + \frac{8}{3}\zeta^2 - \frac{275\sqrt{3}}{18}\zeta^3 + \ldots,$$

which shows that the spin is involved only in quantities ζ^2, i.e., in quantities of second order in h. This is why Schwinger's spinless calculation[4], correct to the first order in h, gives the same result as the first quantum correction for particles with spin made by Sokolov, Klepikov and Ternov[7]. Nelipa's evaluation cannot be correct because it does not depend on h.

For extreme relativistic energies ($\zeta \gg 1$) the radiation of a spinless particle differs essentially from that of an electron. This may be seen on the graph, where the spectra of the electron (solid line) and of a spinless particle (dashes-crosses) have been plotted. For comparison under the same circumstances, the spectrum given by classical theory has also been plotted (dotted line). The limit of the spectrum in the high frequency region is given by the energy-momentum conservation law ($\omega_{\rm max} \approx E/h$).



The particularities of the spectra are obvious and do not need further explanations. As it was noted by Sokolov, the main difference between radiation spectra of an electron and a spinless particle is related to the appearance of the electronic magnetic moment which takes place for high energies ($\zeta \gg 1$). This question will be considered in more detail elsewhere.

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Investigation of the Structure of Extensive Air Showers at Sea Level

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IN reference 1 a qualitative indication was found that the spatial distribution of the electron component of extensive air showers at sea level is essentially different from that expected from the point of view of the electron-photon picture of the development of showers in the atmosphere. In the summer of 1953 we carried out in Moscow a detailed investigation of the spatial distribution of different components of extensive air showers at small distances from the axis of the shower by the method of correlated hodoscopes*. We report below preliminary results.

To study the spatial distribution of electrons of the shower, density indicators were used, consisting of groups of 24 counters of identical areas, each of which was contained in a hodoscopic cell of a neon hodoscope[2]. In the apparatus were used counters of three different areas: 24, 100 and 330 cm[2], which made it possible to study the