and hence

$$G(k|0) = iZ_2^{-1} \lim_{\varepsilon \to 0} \int_0^\infty e^{i\Phi(\nu)} d\nu; \qquad (18)$$

$$\Phi(\nu) = (k^2 - m^2)\nu + i\varepsilon\nu - ig^2 \int_0^\nu A(\nu)\, d\nu + 4g^2 \int_0^\nu d\beta \int_\beta^\infty F(\gamma)\, d\gamma.$$

We now write the integral in Eq. (18) in the form

$$\lim_{\varepsilon \to 0} \int_0^\infty e^{i\Phi(\nu)} d\nu = i\xi_0 e^{i\Phi(i\xi_0)} \int_\theta^\infty \exp\left\{-i\xi_0^2 \int_1^\lambda \Phi''(i\xi_0\omega)(\lambda - \omega)\, d\omega\right\} d\lambda,$$

where $\Phi'(i\xi_0) = 0$. But, since

$$A'(\nu) \underset{|\nu| \to \infty}{\sim} -\frac{2\pi^2}{\nu} d_e(0), \quad F(\nu) \underset{|\nu| \to \infty}{\sim} -i\frac{3}{2}\frac{\pi^2}{\nu^2} d_t(0), \qquad (19)$$

then for $k^2 \sim m^2$,

$$\xi_0 = 2\pi^2 g^2 \frac{[3d_t(0) - d_l(0)]}{k^2 - m^2},$$

and, consequently, for the corresponding choice of constant Z_2^{-1}, we obtain[3]

$$G(k|0) = \sim [m^2 - k^2]^\gamma,$$

$$\gamma = -1 - \frac{e^2}{2\pi}[3d_t(0) - d_l(0)].$$

The increase of the singularity of the Green's function for $k^2 \sim m^2$ in comparison with the Green's functions of the free scalar field lead to the conclusion that the probability of radiation of one or a finite number of photons with frequency $\omega \to 0$ is equal to zero[4].

As is known, in the usual perturbation theory, this probability is infinite. We hope later to apply this method to the calculation of the operator of the peak part.

In conclusion, I express my deepest gratitude to Academician N. N. Bogoliubov under whose direction the work was completed.

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The Problem of the Asymptote of the Green Function in the Theory of Mesons with Pseudoscalar Coupling

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In investigations[1] using the non-renormalized equations the asymptote of the Green function was found for the case of weak pseudoscalar interaction. In this note the asymptote of the non-renormalized equations for the same problem was found. Moreover, the asymptote found agrees with the renormalized expression obtained in investigation[1].

In contrast to the work of reference 1, the equations for the Green function in our form do not contain any infinities and, when finding the asymptote, it is not necessary to find the small corrections to the Green function (see reference 1), which simplifies the calculation considerably.

It can be shown that, in the first approximation with respect to g^2, the following approximate system of completely renormalized equations results from the system of interlinked renormalized equations (it should be noted that, when $\lim_{p^2 \to \infty} g^2_{\text{prim}} \ln \frac{p^2}{m^2} = \text{const}$, the equations obtained in the same case fully express the asymptote of the exact equations):

$$\Gamma_\sigma(p, p-l, l) = \tau_\sigma\gamma_5 \qquad (1)$$

$$+ \frac{g^2}{\pi i} \int [\Gamma_\mu(p, p-k, k)\, G(p-k)\, \Gamma_\sigma(p-k, p-k-l, l) \times G(p-k-l)\, \Gamma_\nu(p-k-l, p-l, -k)$$

$$- \Gamma_\mu(p^0, p^0-k, k)\, G(p^0-k)\, \Gamma_\sigma(p^0-k, p^0-k, 0) \times G(p^0-k)\, \Gamma_\nu(p^0-k, p^0, -k)]\, D_{\mu\nu}(k)\, d^4(k);$$

$$\frac{\partial G^{-1}(p)}{\partial p_\sigma} = \gamma_\sigma - \frac{g^2}{\pi i}\int\Big[\Gamma_\mu(p, p - k, k)\,\frac{\partial G(p-k)}{\partial p_\sigma}\,\Gamma_\nu(p-k, p, -k) - \Gamma_\mu(p^0, p^0-k, k)\,\frac{\partial G(p^0-k)}{\partial p_\sigma} \times \Gamma_\nu(p^0-k, p^0, -k)\Big] D_{\mu\nu}(k)\, d^4(k); \quad (2)$$

$$\frac{\partial D^{-1}_{\mu\nu}(k)}{\partial k^2} = \delta_{\mu\nu} + \frac{g^2}{\pi i} T_\rho \frac{1}{2}\int\Big\{\frac{\partial}{\partial k_\sigma}\Big[G(p)\Gamma_\mu(p, p-k, k) \times \frac{\partial G(p-k)}{\partial k_\sigma}\Gamma_\nu(p-k, p, -k)\Big] - \frac{\partial}{\partial k^0_\sigma}\Big[G(p)\Gamma_\mu(p, p-k^0, k^0)\frac{\partial G(p-k^0)}{\partial k^0_\sigma} \times \Gamma_\nu(p-k^0, p, -k^0)\Big]\Big\} d^4p \quad (3)$$

Here g is the renormalized charge, ${p^0}^2 = m^2$, ${k^0}^2 = \mu^2$ are the experimental values of the masses.

The system of equations (1) - (3) should satisfy the following boundary conditions:

$$(p^0 - m)\, G(p^0) = 1, \qquad (k^0 - \mu^2)\, D_{\mu\nu}(k^0) = \delta_{\mu\nu}, \quad (4)$$

τ_μ is the operator of the isotopic spin, T_ρ in Eq. (3) is taken in the spinor, as well as in the isotopic matrices.

We will go over to the new variables $D_{\mu\nu}(k) = \delta_{\mu\nu}D(k)$, $\Gamma_\mu = \tau_\mu\Gamma$; then Eqs. (1) - (3) will go over into the following system of equations:

$$\Gamma(p, p-l, l) = \gamma_5 - \frac{g^2}{\pi i}\int\{\Gamma(p, p-k, k)\, G(p-k)\,\Gamma(p-k, p-k-l, l) \times G(p-k-l)\,\Gamma(p-k-l, p-l, -k) - \Gamma(p^0 p^0 - k, k)\, G(p^0-k)\,\Gamma(p^0-k, p^0-k, 0) \times G(p^0-k)\,\Gamma(p^0-k, p^0, -k)\}\, D_{\nu\nu}(k)\, d^4k; \quad (5)$$

$$\frac{\partial G^{-1}(p)}{\partial p_\sigma} = \gamma_\sigma - \frac{3g^2}{\pi i}\int\Big[\Gamma(p, p-k, k)\,\frac{\partial G(p-k)}{\partial p_\sigma}\,\Gamma(p-k, p, -k) - \Gamma(p^0, p^0-k, k)\,\frac{\partial G(p^0-k)}{\partial p^0_\sigma} \times \Gamma(p^0-k, p^0, -k)\Big] D(k)\, d^4k; \quad (6)$$

$$\frac{\partial G^{-1}(k)}{\partial k^2} = 1 + \frac{g^2}{\pi i} \times \frac{1}{2}\,\mathrm{Sp}\int\Big\{\frac{\partial}{\partial k_\sigma}\Big[G(p)\,\Gamma(p, p-k, k)\,\frac{\partial G(p-k)}{\partial k_\sigma} \times \Gamma(p-k, p, -k)\Big] - \frac{\partial}{\partial k^0_\sigma}\Big[G(p)\,\Gamma_\nu(p, p-k^0, k^0)\,\frac{\partial G(p-k^0)}{\partial k^0_\sigma} \times \Gamma(p-k^0, p, -k^0)\Big]\Big\} d^4p. \quad (7)$$

Following reference 1, we will look for an asymptote for $G(p)$, $D(p)$ and Γ at high momenta in the form

$$G(p) = \beta(p^2)/\hat{p}, \quad D(p) = d(p^2)/p^2, \quad \Gamma = \gamma_5\alpha(f^2), \quad (8)$$

where f is the largest vector on which Γ is dependent. As a result of calculations we obtain logarithmically from Eqs. (5) - (7) the following equations:

$$\alpha(\xi) = 1 + \frac{g^2}{4\pi}\int_\xi^0 d(z)\,\alpha^3(z)\,\beta^2(z)\, dz,$$
$$\frac{1}{\beta(\xi)} = 1 - \frac{3g^2}{8\pi}\int_\xi^0 \alpha^2(z)\,\beta(z)\, d(z)\, dz,$$
$$\frac{1}{d(\xi)} = 1 + \frac{g^2}{\pi}\int_\xi^0 \alpha^2(z)\,\beta^2(z)\, dz,$$
$$\xi = \ln(-p^2/m^2). \quad (9)$$

The system of equations (9) is equivalent to the following system:

$$\frac{1}{\beta}\frac{d\beta}{d\xi} = \frac{3g^2}{8\pi}\alpha^2(\xi)\beta^2(\xi)d(\xi), \qquad (10)$$
$$\frac{1}{\alpha}\frac{d\alpha}{d\xi} = \frac{g^2}{4\pi}\alpha^2(\xi)\beta^2(\xi)d(\xi),$$
$$\frac{1}{d}\frac{dd(\xi)}{d\xi} = -\frac{g^2}{\pi}\alpha^2(\xi)\beta^2(\xi)d(\xi).$$

The boundary conditions for β, α, d are equal to logarithmic accuracy, namely $\alpha(0) = \beta(0) = d(0)=1$ and it follows from Eq. (10) that:

$$\beta(\xi) = \alpha^{-3/2}(\xi), \qquad (11)$$
$$d(\xi) = \alpha^{-4}(\xi),$$
$$\alpha(\xi) = \left(1 - \frac{5g^2}{4\pi}\xi\right)^{1/5}.$$

With the aid of Eq. (11) it is easy to find the relationship in this approximation between the primed charge g_{prim} and the renormalized charge. Actually, it is known that

$$g^2_{prim} = \lim_{L\to\infty}(g^2\beta^2(L)\alpha^2(L)d(L)) \qquad (12)$$

$$= \lim_{L\to\infty}\left[g^2 \Big/ \left\{1 - \frac{5g^2}{4\pi}\ln\left(-\frac{L^2}{m^2}\right)\right\}\right]$$

or

$$g^2 = \lim_{L\to\infty}\left[g^2_{prim} \Big/ \left\{1 + \frac{5g^2_{prim}}{4\pi}\ln\left(-\frac{L^2}{m^2}\right)\right\}\right]. \qquad (13)$$

It is evident from Eqs. (12) and (13) that, at least in our approximation, no matter what kind the primed charge g_{prim} is, the experimental charge is equal to zero. This explains the fact that the solution of Eq. (11) at a finite g^2 changes sign at high momenta; moreover, a fictitious pole appears in $d(\xi)$, although, according to the formal general properties of the theory, $d(\xi)$ cannot become negative at large ξ.

One can easily become convinced that, if we substitute for g^2 its value which follows from theory in this approximation, then $d(\xi)$, as one would expect, does not change sign; however, the primed charge at $L\to\infty$ is completely shielded, and g^2 becomes equal to zero. The resultant difficulty, inherent in contemporary theories with point interaction, will be discussed in more detail in a separate paper.

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The Interaction of Extraordinary and Ordinary Waves in the Ionosphere and the Effect of Multiplication of Reflected Signals

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It is known that the electromagnetic field of a wave traveling in an inhomogeneous magnetoactive medium (the ionosphere), generally speaking, cannot be represented by the superposition of independent extraordinary and ordinary waves. A consideration of the inhomogeneity of the medium leads to the conclusion that during the propagation of waves of one type in the medium, waves of another type appear. Strictly speaking, this interaction exists over the entire extent of the inhomogeneous medium; however, under ionospheric conditions (a slowly changing medium) the observable interaction appears only in limited regions, outside of which it is exceedingly slight. It is essential to speak of a division of the field into ordinary and extraordinary waves only under conditions of slight interaction. As a result of this it is possible to describe these waves in terms of geometrical optics; the interaction itself defines the very special nature of the field in regions of slight interaction separated by a region of considerable interaction.

With normal incidence of an electromagnetic wave upon a plane, laminated, ionized medium located in an external magnetic field, the strongest interaction between extraordinary and ordinary waves is observed during quasi-longitudinal propagation, when the angle between the direction of propagation and the direction of the external field is small. This interaction, which in the ionosphere produces the so-called multiplication of signals effect, can be explained in the following manner. The ordinary wave falling upon an inhomogeneous layer reaches the region where the index of refraction of the ordinary wave $n_1(z)$ and that of the extraordinary wave $n_2(z)$ are very close in value. In this region intense interaction of the two types of waves takes place; as a result of this interaction, an ordinary wave partly penetrates the region of imaginary values of $n_1(z)$ as an extraordinary wave; here the index of refraction of the latter $n_2(z)$ takes on real values and it is partly reflected as an ordinary wave. A wave of the second type passing through the region of interaction is reflected from a superincumbent region of