ment with experimental data. It is of interest to look over the scheme of the α-decay ThC → ThC″ (Fig. 2) in accordance with the newest experimental data compiled in Selinov's monograph[9]. These data are in much better agreement than the previous ones with the scheme of nuclear rotators. For the interpretation of some even newer empirical data[10] it turns out to be necessary to introduce yet a fourth rotator with a rotation constant $B = 33$ kev. This constant, just as the first three, agrees with the simple natural proportion ($B_1 : B_2 : B_3 : B_4 = 3 : 4 : 5 : 6$) pointed out in our earlier paper**[1].

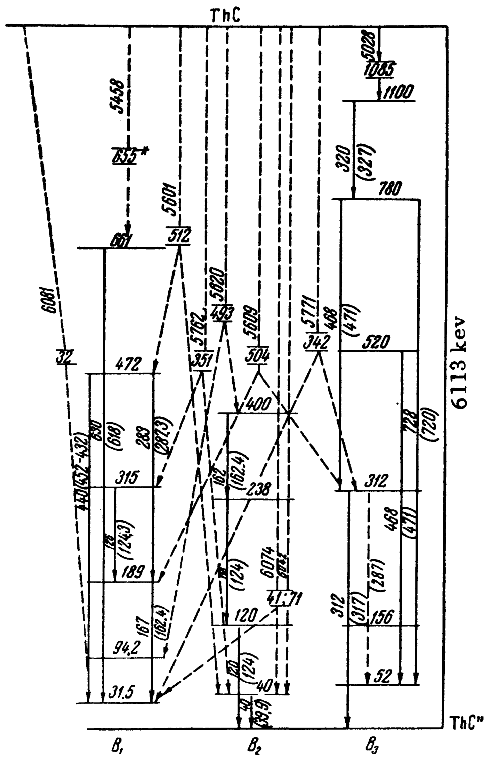

FIG. 2. The α-particle decay ThC $\overset{\alpha}{\to}$ ThC″. The assignment of excitation energies of ThC″ by rotators and the resulting emission of γ-quanta. Full lines: γ-rays; dashed lines: α-particles. Numbers between horisontal lines: excitation energy of ThC″ in kev; numbers above lines: energy of α-particles in kev*.

* $\Delta E = 655$ kev can also be explained by the scheme $520 + 94 + 40$ (B_3, B_1, B_2).

The fact that much experimental information on the α- and γ-spectra of naturally radioactive elements can be interpreted theoretically on the basis of the scheme of nuclear rotators indicates that this scheme is of some significance.

* In connection with Wigner's formula, it is convenient to remove from the statistical weight $(2j + 1)$ that statistical weight which corresponds to forbidden m.

** The quadrupolarity of the γ-rays can be explained, in connection with Fig. 1 (of the previous paper[1]), by the unity spin of the γ-photon and the circularly polarized radiation.

1 S. G. Ryzhanov, J. Exper. Theoret. Phys. USSR 23, 417 (1952)

2 F. Rasetti, *Elements of Nuclear Physics*, p. 92 (read in U. S. edition)

3 H. Bethe, *Physics of the Nucleus*, Part 2, p. 194 (published in U. S.)

4 See reference 3, p. 178-198

5 S. G. Ryzhanov, J. Exper. Theoret. Phys. USSR 24, 361 (1953)

6 W. Heisenberg, *Theory of the Atomic Nucleus*, p. 38

7 L. D. Landau and E. M. Lifshitz, *Quantum Mechanics*, I, pp. 123-126 (OGIZ, 1948)

8 See reference 7, pp. 404-409

9 I. P. Selinov, *Atomic Nuclei and Nuclear Transmutations*, I, p. 198 (GITTL, 1951)

10 F. Rasetti and E. C. Booth, Phys. Rev. 91, 315 (1953)

Translated by F. Ajzenberg-Selove
182

Green's Function in Scalar Electrodynamics in the Region of Small Momenta

A. A. LOGUNOV
Moscow State University

(Submitted to JETP editor June 24, 1955)
J. Exper. Theoret. Phys. USSR 29, 871-874 (December, 1955)

IN the usual perturbation theory, in addition to the fundamental divergences which occur at high energies of the virtual quanta, there arise divergences in the integration over the virtual quanta with q^2, close to zero. This divergence bears the name of the infrared "catastrophe", and is connected with the improper application of perturbation theory to the given processes. The study of any particular process presents no difficulty if the Green's function and the operator of the peak part are computed. Inasmuch as we do not have to take into account the virtual electron-positron pairs at $k^2 \sim m^2$, the photon Green's function will be the Green's functions of the free photon field.

In this note we investigate the behavior of the Green's function of the particle when $k^2 \sim m^2$ in the case of scalar electrodynamics:

$$\Big\{M^2-\Big[i\frac{\partial}{dx^k}+\sqrt{4\pi}\,eA_k(x)+i\sqrt{4\pi}\,e\int \mathfrak{G}_{\beta,k}(\xi,x\,|\,A)\frac{\delta}{\delta A_\beta(\xi)}\Big]^2\Big\}G(x,y\,|\,A)=\delta(x-y), \quad (1)$$

$\mathfrak{G}_{\beta,k}(\xi,x\,|\,A)$ is the Green's function of the photon. Representing $G(x,y\,|\,A)$ in the form

$$G(x,y\,|\,A)=\sum_n\frac{1}{n!}\int G_n(x,\,y\,|\,x_1\ldots x_n)\,A(x_1)\ldots A(x_n)\,dx_1\ldots dx_n,$$

where G_n, by virtue of the homogeneity of the space, is translationally invariant:

$$G_n(x,y\,|\,x_1,\ldots x_n)=G_n(x+a,\,y+a;\,x_1+a,\ldots,x_n+a),$$

we have

$$G(x,y\,|\,A)=G(y-x\,|\,T_xA)=T_xG(y-x\,|\,A),$$

where

$$T_xA(\xi)=A(\xi+x).$$

Since

$$G(y-x\,|\,A)=\frac{1}{(2\pi)^4}\int e^{ik(y-x)}G(k\,|\,A)\,dk,$$

we get

$$i\frac{\partial}{\partial x^\nu}G(x,y\,|\,A)=T_x\frac{1}{(2\pi)^4}\int e^{ik(y-x)}\Big[k^\nu g^{\nu\nu}+\int p^\nu g^{\nu\nu}A_\cdot(p)\frac{\delta}{\delta A(p)}\,dp\Big]G(k\,|\,A)\,dk. \quad (2)$$

After substituting Eq. (2) in Eq. (1) and making several elementary computations, we get

$$\Big\{M^2-\Big[k+\Pi+g\int A(p)\,dp+ig\int \mathfrak{G}(p\,|\,A)\frac{\delta}{\delta A(p)}\,dp\Big]^2\Big\}G(k\,|\,A)=1, \quad (3)$$

where

$$g^{00}=1,\ g^{ii}=-1,$$

$$\Pi_\nu=\int p^\nu g^{\nu\nu}A_\alpha(p)\frac{\delta}{\delta A_\alpha(p)}\,dp;\ g=\frac{\sqrt{4\pi}\,e}{(2\pi)^2}.$$

Carrying out the renormalization of Eq. (3), we get

$$\Big\{M^2-\Big[k+\Pi+g\int A(p)\,dp+ig\int \mathfrak{G}(p\,|\,A)\frac{\delta}{\delta A(p)}\,dp\Big]^2\Big\}G(k\,|\,A)=Z_2^{-1} \quad (4)$$

In our approximation we can take for $\mathfrak{G}(p\,|\,A)$ [2]

$$\mathfrak{G}_{\alpha\beta}(p)=\frac{1}{p^2}\Big[d_l(p^2)\Big(-g^{\alpha\beta}+g^{\alpha\alpha}g^{\beta\beta}\frac{p^\alpha p^\beta}{p^2}\Big)-d_l(p^2)g^{\alpha\alpha}g^{\beta\beta}\frac{p^\alpha p^\beta}{p^2}\Big]. \quad (5)$$

Making use of the method of proper time [1] we can rewrite Eq. (4) in the form

$$i\frac{\partial U(\nu,k|A)}{\partial\nu}=(M^2-\hat{\mathscr{H}})\,U(\nu,k|A), \quad (6)$$

$$U(0,k|A)=-Z_2^{-1};\quad G(k|A)=-i\lim_{\varepsilon\to 0}\int_0^\infty e^{-\varepsilon\nu}U(\nu,k|A)\,d\nu.$$

Limiting ourselves to terms no higher than second order in g^2 and linear in $A_\alpha(p)$, we obtain

$$\hat{\mathscr{H}}=k^2+\int dp\,[2(kp)+p^2]\,A_\alpha(p)\frac{\delta}{\delta A_\alpha(p)}+g\int dp\,(2k+p)\,A(p)+ig\int dp\,[2k^\beta+p^\beta]\,\mathfrak{G}_{\alpha\beta}(p)\frac{\delta}{\delta A_\alpha(p)}+ig^2\int dp\,g^{\alpha\alpha}\mathfrak{G}_{\alpha\alpha}(p). \quad (7)$$

The constants M, Z_2^{-1} in our approximation are represented with an accuracy which does not exceed g^2. We have

$$U(\nu, k|A) = -Z_2^{-1} \exp\{iS(\nu, k|A)\}.$$

Then:

$$\frac{\partial S(\nu,k|A)}{\partial \nu} = (k^2 - M^2 + g\int dp\,(2k+p)A(p) \tag{8}$$

$$+ ig^2 \int dp g^{\alpha\alpha} \mathfrak{G}_{\alpha\alpha}(p) + i\int dp\,[2(kp) + p^2] A_\alpha(p)$$

$$\times \frac{\delta}{\delta A_\alpha(p)} S(\nu,k|A) + ig^2 \int dp\, \mathfrak{G}_{\alpha\beta}(2k^\beta + p^\beta) \frac{\delta S}{\delta A_\alpha(p)}$$

We set

$$\frac{\delta S(\nu,k|A)}{\delta A_\alpha(p)} = \tau^\alpha(\nu,k,p).$$

Then, by varying Eq. (8) we obtain

$$\frac{\delta \tau^\alpha(\nu,k,p)}{\delta \nu} = g(2k^\alpha + p^\alpha) \tag{9}$$

$$+ i[2(kp) + p^2]\,\tau^\alpha(\nu,k,p), \quad \tau^\alpha(0,k,p) = 0,$$

whence

$$\tau^\alpha(\nu,k,p) = ig\,\frac{(2k^\alpha + p^\alpha)}{2(kp) + p^2} \tag{10}$$

$$\times \quad (1 - \exp\{i\nu\,[2(kp) + p^2]\}),$$

and, consequently,

$$S(\nu, k|A) = \int_0^\nu H(\nu, k|A)\, d\nu, \tag{11}$$

$$H(\nu, k|A) = k^2 - M^2 + ig^2 \int dp\, g^{\alpha\alpha}\, \mathfrak{G}_{\alpha\alpha}(p)$$

$$+ g\int dp\,(2k+p)\,A(p)$$

$$+ i\int dp\,[2(kp) + p^2]\,A_\alpha(p)\,\tau^\alpha(\nu, k, p)$$

$$- g\int dp\, \mathfrak{G}_{\alpha\beta}(p)\,(2k^\beta + p^\beta)\,\tau^\alpha(\nu, k, p).$$

Substituting Eq. (10) in (11) and assuming

$$d_l(p^2) = \int_0^\infty e^{i\gamma p^2} v_l(\gamma)\, d\gamma, \tag{12}$$

$$d_t(p^2) = \int_0^\infty e^{i\gamma p^2} v_t(\gamma)\, d\gamma,$$

we obtain, after some computation,

$$H(\nu, k|0) = k^2 - M^2 + ig^2 \int dp g^{\alpha\alpha}\, \mathfrak{G}_{\alpha\alpha}(p) \tag{13}$$

$$+ ig^2 A(0) - ig^2 A(\nu) - 4g^2 \int_0^\nu F(\nu)\, d\nu,$$

$$A(\nu) = -\frac{\pi^2}{\nu^2} \int_0^1 d\omega\, e^{-\nu k^2 \omega}\, \omega\, [2i\nu k^2 \tag{14}$$

$$+ 2 - i\nu k^2 \omega] \int_0^{(\nu/\omega)-\nu} d\beta \int_0^\beta v_l(\gamma)\, d\gamma,$$

$$F(\nu) = \frac{3}{2}\,\frac{\pi^2}{\nu^2}\, k^2 \int_0^1 e^{-i\nu k^2 \omega}\omega\, d\omega \int_0^{(\nu/\omega)-\nu} d\beta \int_0^\beta v_t(\gamma)\, d\gamma, \tag{15}$$

whence

$$U(\nu, k|0) = -Z_2^{-1} \exp\Big[i\Big(k^2 - M^2 \tag{16}$$

$$+ ig^2 \int dp g^{\alpha\alpha} \mathfrak{G}_{\alpha\alpha}(p) + ig^2 A(0)\Big)\nu$$

$$+ g^2 \int_0^\nu A(\nu)\, d\nu - 4ig^2 \int_0^\nu d\beta \int_0^\beta F(\gamma)\, d\gamma\Big].$$

Carrying out renormalization of the mass we obtain

$$U(\nu, k|0) = -Z_2^{-1} \exp\Big[i\,(k^2 - m^2 - ig^2 \int_0^\nu A(\nu)\, d\nu \tag{17}$$

$$+ 4g^2 \int_0^\nu d\beta \int_\beta^\infty F(\gamma)\, d\gamma\Big],$$

and hence

$$G(k|0) = iZ_2^{-1}\lim_{\varepsilon\to 0}\int_0^\infty e^{i\Phi(\nu)}d\nu; \quad (18)$$

$$\Phi(\nu) = (k^2 - m^2)\nu + i\varepsilon\nu - ig^2\int_0^\nu A(\nu)\,d\nu + 4g^2\int_0^\nu d\beta\int_\beta^\infty F(\gamma)\,d\gamma.$$

We now write the integral in Eq. (18) in the form

$$\lim_{\varepsilon\to 0}\int_0^\infty e^{i\Phi(\nu)}\,d\nu$$

$$= i\xi_0 e^{i\Phi(i\xi_0)}\int_\theta^\infty \exp\left\{-i\xi_0^2\int_1^\lambda \Phi''(i\xi_0\omega)(\lambda-\omega)\,d\omega\right\}d\lambda,$$

where $\Phi'(i\xi_0) = 0$. But, since

$$A'(\nu)\underset{|\nu|\to\infty}{\sim} -\frac{2\pi^2}{\nu}d_e(0), \quad F(\nu)\underset{|\nu|\to\infty}{\sim} -i\frac{3}{2}\frac{\pi^2}{\nu^2}d_t(0), \quad (19)$$

then for $k^2 \sim m^2$,

$$\xi_0 = 2\pi^2 g^2\frac{[3d_t(0) - d_l(0)]}{k^2 - m^2},$$

and, consequently, for the corresponding choice of constant Z_2^{-1}, we obtain[3]

$$G(k|0) = \sim [m^2 - k^2]^\gamma,$$

$$\gamma = -1 - \frac{e^2}{2\pi}[3d_t(0) - d_l(0)].$$

The increase of the singularity of the Green's function for $k^2 \sim m^2$ in comparison with the Green's functions of the free scalar field lead to the conclusion that the probability of radiation of one or a finite number of photons with frequency $\omega \to 0$ is equal to zero[4].

As is known, in the usual perturbation theory, this probability is infinite. We hope later to apply this method to the calculation of the operator of the peak part.

In conclusion, I express my deepest gratitude to Academician N. N. Bogoliubov under whose direction the work was completed.

[1] V. A. Fock, Phys. Z. Sowjet Union 12, 404 (1937)

[2] L. D. Landau, A. A. Abrikosov and I. M. Khalatnikov, Dokl. Akad. Nauk SSSR 95, 497 (1954)

[3] A. A. Abrikosov, Dissertation, Institute for Physical Problems, Academy of Sciences, USSR, 1955

[4] F. Bloch and A. Nordsieck, Phys. Rev. 52, 54 (1937)

Translated by R. T. Beyer
287

The Problem of the Asymptote of the Green Function in the Theory of Mesons with Pseudoscalar Coupling

E. S. Fradkin
P. N. Lebedev Institute of Physics, Academy of Sciences, USSR
(Submitted to JETP editor April 18, 1955)
J. Exper. Theoret. Phys. USSR 29, 377-379 (September, 1955)

In investigations[1] using the non-renormalized equations the asymptote of the Green function was found for the case of weak pseudoscalar interaction. In this note the asymptote of the non-renormalized equations for the same problem was found. Moreover, the asymptote found agrees with the renormalized expression obtained in investigation[1].

In contrast to the work of reference 1, the equations for the Green function in our form do not contain any infinities and, when finding the asymptote, it is not necessary to find the small corrections to the Green function (see reference 1), which simplifies the calculation considerably.

It can be shown that, in the first approximation with respect to g^2, the following approximate system of completely renormalized equations results from the system of interlinked renormalized equations (it should be noted that, when $\lim_{p^2\to\infty} g^2_{\text{prim}} \ln\frac{p^2}{m^2} = \text{const}$, the equations obtained in the same case fully express the asymptote of the exact equations):

$$\Gamma_\sigma(p, p-l, l) = \tau_\sigma\gamma_5 \quad (1)$$

$$+ \frac{g^2}{\pi i}\int [\Gamma_\mu(p, p-k, k)\,G(p-k)\,\Gamma_\sigma(p-k, p-k-l, l)$$

$$\times G(p-k-l)\,\Gamma_\nu(p-k-l, p-l, -k)$$

$$-\Gamma_\mu(p^0, p^0-k, k)\,G(p^0-k)\,\Gamma_\sigma(p^0-k, p^0-k, 0)$$

$$\times G(p^0-k)\,\Gamma_\nu(p^0-k, p^0, -k)]\,D_{\mu\nu}(k)\,d^4(k);$$