at the moving contacts. The measurement can be obtained with the aid of a ballistic galvanometer and amplifier. With the estimated magnitudes above, this apparatus ought to give the value of e/m for conduction electrons with an error not exceeding 1%.

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The Radiation of a Rapidly Moving Electric Image of a Uniformly Moving Charge

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OF the wide class of problems on radiation effects accompanying the rapid passage of charges near conducting or dielectric surfaces of given arbitrary form, we consider the simplest concrete example: the calculation of the radiation caused by the change of the image of a charge with non-relativistic but sufficiently large velocity falling upon a conducting sphere of radius R. In the non-relativistic case we can employ certain formulas of electrostatics connected with the magnitude and coordinates of the inducing charge and the image charges: $x_1 x_0 = R^2$; $e_1 = -e_0 R/x_0 = -e_2$; $x_2 = 0$ (origin of coordinates measured from the center of the sphere, and the zero subscript denoting quantities relating to the inducing charge).

The dipole moment of the image charge is equal to the induced dipole moment of the sphere $p = e_1 x_1 = e_0 R^3 / x_0^2$, and has a second derivative with respect to time, different from zero even for $\dot{x}_0 = -\beta_0 c = \text{const}$. The total energy radiated for the change of dipole moment, due to the motion of the inducing change from infinity to the surface of the sphere, is

$$\Delta\mathcal{E} = \frac{2}{3c^3}\int \ddot{p}^2 dt = \frac{24}{7}\frac{e_0^2}{R}\beta_0^3.$$

Such energy of the first burst of radiation precedes the radiation of the transient decelerating source (concerning transient radiation for a plane boundary, see references 1 and 2). We compare the received radiation of the image with the radiation of a charge in complete braking in the electric field of a parallel plate condenser. For the path of charge parallel to the field

$$\delta\mathcal{E} = \frac{2}{3}\frac{e_0^3 E\beta_0}{m_0 c^2},$$

$$\frac{\Delta\mathcal{E}}{\delta\mathcal{E}} = \frac{36}{7}\frac{m_0 c^2\beta_0^2}{e_0 ER} = \frac{72}{7}\frac{\mathcal{E}_{\text{kin}}}{e_0 ER};$$

for $eER = \mathcal{E}_{\text{kin}}$; $\Delta\mathcal{E} \simeq 10\,\delta\mathcal{E}$.

It is evident that by suitable choice of the form (concave or convex) of the conducting surface, an accelerated or "super light" collapse of the field can be realized, redistributing the charge, even for a constant velocity of motion of the inducing charge (not exceeding that of light).

The employment of a bunch of charged particles as an inducing charge can increase the radiation effect by many orders of magnitude.[3] This justifies the interest in the study of the potentialities of transformation of the velocities and accelerations of image charges, and in the investigation of annihilation radiation associated with the uniting of the bunch with the induced charge.

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On the Problem of Rotational Levels and the Spectra of Heavy Nuclei. II

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IN the present communication, calculations concerning the relative intensities of α-particle groups from RdAc → AcX, based on the model of nuclear rotators[1], are presented, and are compared with experimental data[2,3]. The quantum-mechanical theory of α-decay, presented in

Bethe's monograph[4], is placed in the framework of the calculation. According to this theory, the probability of emission of an α-particle with reduced kinetic energy (reduced because of the excitation ΔE of the daughter nucleus) is equal to the probability of emission to the ground state multiplied by a factor approximately equal to

$$\exp(-170\Delta E\, E_{\alpha m}^{3/2}), \qquad (1)$$

where $E_{\alpha m}$ is the maximum disintegration energy, expressed (as is ΔE) in mev. If the α-particle carries an orbital angular momentum $l\hbar$, then the probability for its emission from the nucleus is decreased by

$$\exp\left[\frac{2l(l+1)}{g}\sqrt{1-x}\right]. \qquad (2)$$

For a heavy nucleus, $g \approx 12$, $x = E_\alpha / E_0$, where E_α is the disintegration energy for the particular group and $E_0 = 2Ze^2/R_c \approx 25$ mev. The calculation of the relative intensity of a given α-group (characterized by a particular excitation energy ΔE) leads to the multiplication of the factor (2) by the corresponding normalization of the statistical weight of the chosen l, and summed over all l possible for the group being investigated, with the consequent multiplication by the factor (1). In order to compare these intensities with the intensity of the ground state group (the normal state of the daughter nucleus), it is necessary to divide them by factor (2) for the ground state, which is assigned a statistical weight of one. In order to find the various l's possible for each group considered, and to calculate the corresponding "statistical weights", the following nuclear model is used: the nucleons, forming closed (or rather, almost closed) shells, distribute themselves into groups, forming "rotators"[5]. In addition to these groups, there are a small number of non-shell nucleons (formed, so to speak, by the nuclear atmosphere), for which the corresponding individual energy levels can be represented by Heisenberg's scheme[6]. The nuclear rotators and the non-shell nucleons are treated below as almost independent subsystems. The calculation of the non-shell nucleons is necessary in order to obtain the correct value for the ground state spins of heavy nuclei. In α-decay, the formation of α-particles in the nucleus leads to the regrouping of non-shell nucleons, whose energy enters as a very small part in the maximum disintegration energy.

The simplest regrouping consists in the jump of one nucleon between neighboring levels, near the direct transitions in the middle part of the scheme[6]. The resulting momentum of the α-particle can be obtained by the addition of the rotator momenta and the changed momenta of the non-shell nucleons participating in the energy and momentum balance of the nucleus. Such an addition of momenta is carried out according to the known quantum mechanical law[7]. Each possible value j of the sum of the momenta j_1 and j_2 being added has a definite "statistical weight" expressed by Wigner's well-known formula[8]. Using the notation of the monograph cited earlier (Bethe's), one can obtain the desired statistical weight of a given j by adding the $|C^j_{m_1 m_2}|^2$ (over all possible $m_1 m_2$ and $m = m_1 + m_2$ corresponding to the j_1, j_2 and j_3 permitted by selection laws). Selection laws consist of the law of conservation of parity and the selection law for the numbers m of rotators. The latter derives from the type of rotation symmetry of the electric quadrupole moments of the rotators (when some rotator is excited, its m can change either by 0 or ±2 or by ±1, depending upon the type of symmetry)*.

It is not without interest to bring in some concrete data in the disintegration RdAc → AcX being investigated, according to the scheme presented as Fig. 3 in reference 1. With the best agreement with the empirical data, it is assumed that the change Δj for the non-shell nucleons in a wide range ΔE (from 32 to 280 kev) is equal to three momentum units:

a) The ground state group, $E_{\alpha m} = 6.16$ mev, $\Delta E = 0$. The rotator is not excited.

The non-shell nucleon, participating in the momentum balance, changes by $\Delta j = 3$; this momentum value leads to an α-particle with $l = 3$.

b) The group with $\Delta E = 32$ kev. One rotator, B_1, is excited and gets a momentum $J = 1$. In this case $\Delta j = 3$. The possible values for the momentum of the α-particle are $l = 2$ and 4. Their statistical weights are in the ratio 3 to 6.

c) The group with $\Delta E = 62$ kev. Two rotators (of the B_1 type) are excited, each with a momentum $J = 1$. The resulting momentum can have the values 0 and 2. Adding these momentum values with $\Delta j = 3$, we obtain $l = 1$, 3 and 5 with a multiplicity 1 for $l = 1$ and 5 and a multiplicity 2 for $l = 3$, and statistical weights in the ratios 3:8:10.

d) The group with $\Delta E = 84$ kev. Two rotators are also excited (B_1 and B_3, each with $j = 1$) and the calculation is analogous to the previous one, with but one change, however: the rotator B_3 has a different rotation symmetry than B_1. For the case under consideration m can take values 1 and − 1 and then, as in the previous case, this quantity can have values +2, 0 and − 2, which carry almost twice as much statistical weight.

e) The group with $\Delta E = 129$ kev. Three rotators (B_1, B_2, B_3), each with $J = 1$, are excited. Their resulting momentum has values of 1 or 3. Essentially, the maximum statistical weight has to be one. The addition with Δj virtually leads to case c).

The analysis of the remaining groups is carried out in similar fashion.

Figure 1 shows two experimental curves of the dependence of the intensity of the α-groups on the excitation energy (the shape of the α-spectrum), drawn according to the data of Rasetti[2] and Bethe[3] (dotted lines). The solid line gives the calculated dependence. All three curves are normalized to the same total number of α-particles Taking into account the rather large scattering of the experimental data, one must admit that the agreement with the theory presented is satisfactory. Better agreement can be attained only on the basis of a more complete theory of α-decay, founded on the quantum many-body problem, instead of the present theory which is "one-bodied" in essence.

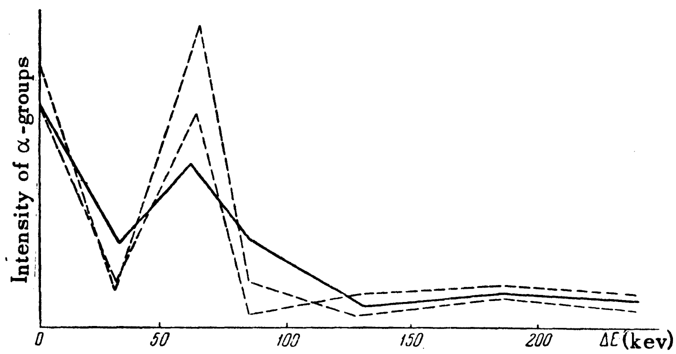

FIG. 1

The experimental data[2,3] show that, starting with $\Delta E = 290$ kev and higher (up to 380 kev) the shape of the α-spectrum almost perfectly reproduces its shape in the interval 0-290 kev. According to Bethe[4] the influence of factor (1), leading to a sharply decreasing intensity, can be compensated by a slight increase in the radius of the potential well for the α-particles, due to the fact that the α-particle already interacts with a relatively highly excited nucleus. The shape of the curve further on can be obtained excellently on the basis of the model of nuclear rotators developed here, if one assumes that the change in momentum of the non-shell nucleons $\Delta j = 1$. Indeed, the group $\Delta E = 290$ kev is connected with the excitation of two rotators: A_2 with $J = 3$ and B_3 with $J = 1$. The addition of the momenta of these two rotators leads to case (b) and gives a resulting momentum of four which, when added to $\Delta j = 1$, leads to a momentum of 3 (which has an overwhelmingly large " statistical weight "). Therefore, this group finally leads to case (a). The following group with $\Delta E = 312$ kev is connected with the excitation of one rotator B_3 with $J = 3$, the addition of which to $\Delta j = 1$ gives, in accordance with (b), a resulting momentum of four. This group is a repetition of the group with $\Delta E = 32$ kev. The group $\Delta E = 337$ kev (rotators B_1 with $J = 2$ and B_3 with $J = 3$), as can be easily shown, repeats the group with $\Delta E = 62$ kev. More interesting is the last group with $\Delta E = 383$ kev. This value of ΔE can be split into 312 kev (rotator B_3 with $J = 3$), 40 kev (rotator B_2 with $J = 1$) and 31 kev (rotator B_4 with $J = 1$). The addition of the first two momenta gives, according to (b), a resulting momentum of four, which, when added to the momentum of the third rotator ($J = 1$), gives with overwhelming probability the resulting momentum of three. Its addition to $\Delta j = 1$ leads to case (b) and this group repeats the groups with $\Delta E = 32$ kev and $\Delta E = 120$ kev.

Analogous calculations have been performed for two α-groups ThC → ThC″ with satisfactory agreement with experimental data. It is of interest to look over the scheme of the α-decay ThC → ThC″ (Fig. 2) in accordance with the newest experimental data compiled in Selinov's monograph[9]. These data are in much better agreement than the previous ones with the scheme of nuclear rotators. For the interpretation of some even newer empirical data[10] it turns out to be necessary to introduce yet a fourth rotator with a rotation constant $B = 33$ kev. This constant, just as the first three, agrees with the simple natural proportion ($B_1: B_2: B_3: B_4 = 3: 4: 5: 6$) pointed out in our earlier paper[1]***.

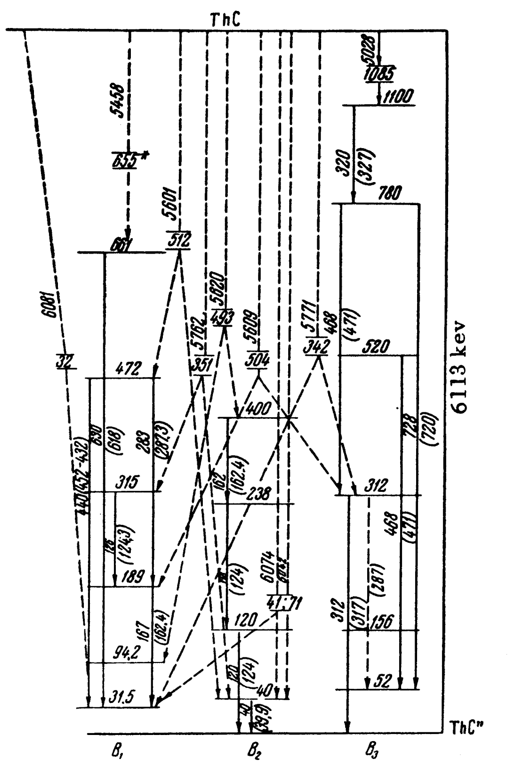

FIG. 2. The α-particle decay ThC → ThC″. The assignment of excitation energies of ThC″ by rotators and the resulting emission of γ-quanta. Full lines: γ-rays; dashed lines: α-particles. Numbers between horisontal lines: excitation energy of ThC″ in kev; numbers above lines: energy of α-particles in kev*.

* $\Delta E = 655$ kev can also be explained by the scheme 520 + 94 + 40 (B_3, B_1, B_2).

The fact that much experimental information on the α- and γ-spectra of naturally radioactive elements can be interpreted theoretically on the basis of the scheme of nuclear rotators indicates that this scheme is of some significance.

* In connection with Wigner's formula, it is convenientto remove from the statistical weight $(2j + 1)$ that statistical weight which corresponds to forbidden m.

** The quadrupolarity of the γ-rays can be explained, in connection with Fig. 1 (of the previous paper [1]), by the unity spin of the γ-photon and the circularly polarized radiation.

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Green's Function in Scalar Electrodynamics in the Region of Small Momenta

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IN the usual perturbation theory, in addition to the fundamental divergences which occur at high energies of the virtual quanta, there arise divergences in the integration over the virtual quanta with q^2, close to zero. This divergence bears the name of the infrared "catastrophe", and is connected with the improper application of perturbation theory to the given processes. The study of any particular process presents no difficulty if the Green's function and the operator of the peak part are computed. Inasmuch as we do not have to take into account the virtual electron-positron pairs at $k^2 \sim m^2$, the photon Green's function will be the Green's functions of the free photon field.