Letters to the Editor

On the Theory of Langmuir Probes

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IN the last few years there have appeared a number of articles[1-3] in which several premises of the theory of Langmuir probes have been subject to criticism. The purpose of the present paper is to compare Langmuir's theory with the exact theory of spherical probes developed in reference 4 for low pressures and for the condition of negative probe potential. Under these conditions, Langmuir's basic assumptions[5,6] are as follows:

1) The actual potential distribution is replaced by potential $\varphi(r)$ which differs from zero in a certain range of radius r_s (in the sheath), and equals zero in the remaining space. Plasma outside the sheath is undisturbed.

2. Within the sheath there are only ions. This determines the sheath boundary. Potential distribution inside the sheath is in accordance with the three-halves-power law with zero initial velocities.

3. Pressure is assumed to be low and, for given probe potential, whether a particle arrives at the probe is determined by the threshold parameter p and by particle velocity at the sheath boundary. If r_m is the minimum distance from the probe reached by an ion, then

$$p^2(r_m v) = r_m^2\left[1-\frac{2e\varphi(r_m)}{Mv^2}\right], \tag{1}$$

where e and M are ion charge and mass. In the case of attraction, $e\,\varphi(r_m) < 0$. In accordance with assumption 1, $p = r_m$ when $r_m > r_s$. Depending on $\varphi(r)$ and v, the relation between p and r_m may be monotonic or nonmonotonic. This is shown in the graph for a fixed $\varphi(r)$ and for several values of v, where curves a, b, c, d correspond to descending values of v. The graph shows that above a certain velocity the curves exhibit a minimum which lies within the sheath. The variable p which is determined by Eq. (1) acts as a target parameter only to the minimum point; and only the right side of the $p(r_m)$ curves has a physical meaning. If the target parameter is smaller than the minimum value of Eq. (1), then the motion is of threshold value, and the particles are captured by the probe. The radius of the threshold sphere, r_0, (i.e., the value of r_m at which Eq. (1) is at a minimum), differs from probe radius and is a function of velocity v.

It is shown in reference 4 that for any (monotonic) potential, current into the probe is determined by the minimum value of Eq. (1), and equals

$$F = \int_0^\infty 4\pi p^2(r_0 v)\, f(v)\, v^3\, dv, \tag{2}$$

where $f(v)$ is the number of ions in a unit phase volume at a large distance from the probe.

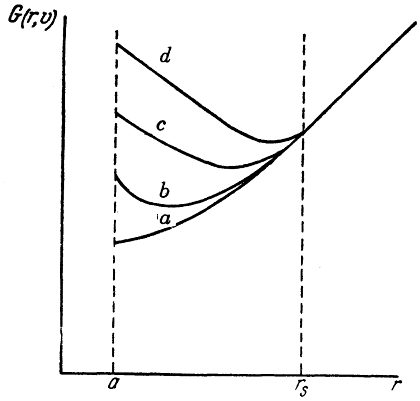
In Langmuir theory[5], current to the probe is determined by probe potential and sheath dimensions, and is independent of potential within the sheath. This is a consequence of the assumption that when the square of the radial velocity of an ion, formally calculated on the basis of the law of conservation, is not negative on the probe surface, then it is also not negative elsewhere. This in turn leads to the assumption that Eq. (1) has a minimum on the probe surface, i.e., that there is no threshold motion. However, in Langmuir's model with a sheath, Eq. (1) is known not to be monotonic, and its minimum is not located on the probe, at least at sufficiently small velocities (threshold motion). This contradiction in Langmuir theory becomes obvious from the fact that, to assure a particle reaching the probe, aside from the requirement that the square of radial velocity on the probe surface must be positive, the additional independent requirement is needed that radial velocity on sheath surface must be positive. The first condition is equivalent to p^2 being smaller than the right side of Eq. (1) on the surface of the probe; and the second, that p^2 is smaller on sheath surface. If the minimum value of Eq. (1) occurred on the probe, then the second condition would be a consequence of the first condition. Actually, during derivation, occasions arise when the second condition is met but not the first. This is especially clearly brought out in the form of Langmuir theory used in reference 6, where the expression for ion probe current is based on the assumption that all ions may be divided into two groups:

$$v > \sqrt{-2e\varphi(a)/M\left[\left(\frac{r_s}{a}\right)^2 - 1\right]} \quad (3a)$$

(for these velocities an absence of threshold motion is assumed, i.e., $r_0 = a$); and

$$v < \sqrt{-2e\varphi(a)/M\left[\left(\frac{r_s}{a}\right)^2 - 1\right]} \quad (3b)$$

(for these velocities it is assumed that the motion has a threshold value, and that the threshold sphere coincides with sheath boundary r_s).



Condition (3a) leads to the conclusion that $p(r_s) > p(a)$. Curves a and b in the graph satisfy this condition. However, with curve b there is threshold motion, and $r_0 > a$. From condition (3b) it follows that $p(r_s) < p(a)$. Curves c and d satisfy this requirement. It is obvious from the curves that in this case $r_0 < r_s$. Since for curves of type b, $p^2(r_0 v) < p^2(av)$, and for curves of type c, $p^2(r_0 v) < p^2(r_s v)$, therefore Eq. (2) shows that calculation of current F in a Langmuir model with a sheath, using the above method, results in values of ion probe current known to be too high. As a matter of fact, even calculations using Langmuir's equations result in ion current values smaller than those obtained by experiment. This is due to the fact that assumption 1, which does not allow for a field outside the sheath, and which assumes that ion velocity at the sheath boundary is equal to velocity at a large distance from the probe, constitutes a poor approximation for ions. An approximate expression for ion current, taking into account the field outside the sheath, is given in reference 3.

Assumption of three-halves-power law potential distribution inside the sheath is proper when ions inside the sheath are moving radially. Since, in Langmuir's model, the threshold sphere lies within the sheath, use of the three-halves-power law is not proper. At the same time, for assumptions made in reference 3, the threshold sphere lies outside the charged sheath, and three-halves-power law can be applied.

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[5] H. Mott-Smith and I. Langmuir, Phys. Rev. 28, 727 (1952)

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Adiabatic Process at High Temperatures

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In connection with the effect of thermal ionization on the thermal properties of gases at high temperatures[1], it is of interest to consider an an adiabatic process, taking the thermal ionization into account. In this case, the original equation for an adiabatic process will have the following form:

$$pdV + dU' + I_1 dN_1 = 0 \quad (1)$$

where dV is the increase in volume, I_1 is the energy of single ionization, dU' is the increase in internal energy of the gas, dN_1 the increase in the number of ions in the heating of the gas.

The expression pdV can be found from the Mendeleev-Clapeyron equation:

$$pdV = \frac{(N + N_1)\, kTdV}{V}. \quad (2)$$

The internal energy of the gas can be written in the following form:

$$U' = 3/2\,(N + N_1)kT. \quad (3)$$

Denoting the degree of ionization by $x = N_1/N$ and substituting Eqs. (2), (3) in Eq. (1), we obtain the equation

$$\frac{3}{2}(1 + x)\frac{dT}{T} + \left(\frac{3}{2} + \frac{I_1}{kT}\right) dx + (1 + x)\frac{dV}{V} = 0. \quad (4)$$