For example, with $\lambda \sim 1$mm and $N \sim 10^8$, we get $q_{max} \sim 30$. The existence of an initial velocity spread of the bunch when entering the undulator, and the impossibility of providing equally good injection conditions for each of the electrons of the bunch, shortens considerably the maximum possible length of the undulator. Therefore, an undulator with too great a number of spatial periods of the field is undesirable.

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Translated by D. Lieberman
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On the Paper "The Excitation Spectrum of a System of Many Particles"

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(Submitted to JETP editor February 7, 1955)
J. Exper. Theoret. Phys. USSR **28**, 749-750 (June, 1955)

In considering the correlation of identical particles, one should distinguish between the correlation of particles which are in the same spin state and the correlation of particles in different spin states. In reference 1 an approximation to the binary distribution was used appropriate for the calculation of the correlation of identical particles in the same spin state. Strictly speaking, this is realized only in the case of particles having no spin. Hence the results of reference 1 are completely valid in the case of spinless Bose particles. However, in the case of electrons, for example, it is necessary, generally speaking, to make several further considerations.

Let $\rho^{(+)}(q_1', q_1)$ and $\rho^{(-)}(q_1', q_1)$ be the density matrices for electrons with spin projections $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively. For particles in the same spin state, the coordinate part of the wave function is anti-symmetric. Hence the binary density matrix can be approximated as follows:

$$\rho_2^{(+,+)}(q_1', q_2'; q_1, q_2) = \rho^{(+)}(q_2', q_1)\,\rho^{(+)}(q_2', q_2) - \rho^{(+)}(q_1', q_2)\,\rho^{(+)}(q_2', q_1)\,. \quad (1)$$

In the case of particles in different spin states, the coordinate part of the wave function is symmetric; therefore*

$$\rho_2^{(+,-)}(q_1', q_2'; (q_1, q_2) = \rho^{(+)}(q_1', q_1)\,\rho^{(-)}(q_2', q_2) + \rho^{(+)}(q_1', q_2)\,\rho^{(-)}(q_2', q_1). \quad (2)$$

Relations (1) and (2) allow one to obtain the following equation for the quantum distribution function $f^{(+)}(\mathbf{q},\mathbf{p})$ of an electron with a positive spin projection. (The notation used is that adopted in reference 1):

$$\frac{\partial f^{(+)}}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f^{(+)}}{\partial \mathbf{q}} + \frac{i}{\hbar}\frac{1}{(2\pi)^3} \times \int d\mathbf{q}'d\vec{\tau}d\mathbf{p}'d\mathbf{p}''\left[U\left(\left|\mathbf{q}-\mathbf{q}'+\frac{\hbar\vec{\tau}}{2}\right|\right) - U\left(\left|\mathbf{q}-\mathbf{q}'-\frac{\hbar\vec{\tau}}{2}\right|\right)\right.$$
$$\times \left\{e^{i\vec{\tau}(\mathbf{p}''-\mathbf{p})} f^{(+)}(\mathbf{q},\mathbf{p}'')\,[f^{(+)}(\mathbf{q}',\mathbf{p}')+f^{(-)}(\mathbf{q}';\mathbf{p}')] + \exp\left[i\vec{\tau}\left(\frac{\mathbf{p}'+\mathbf{p}''}{2}-\mathbf{p}\right) + \frac{i(\mathbf{q}'-\mathbf{q})(\mathbf{p}'-\mathbf{p}'')}{\hbar}\right] f^{(+)}\left(\frac{\mathbf{q}+\mathbf{q}'}{2}-\frac{\hbar\vec{\tau}}{4},\mathbf{p}'\right)\right.$$
$$\left.\times\left[f^{(-)}\left(\frac{\mathbf{q}+\mathbf{q}'}{2}+\frac{\hbar\vec{\tau}}{4},\mathbf{p}''\right) - f^{(-)}\left(\frac{\mathbf{q}+\mathbf{q}'}{2}+\frac{\hbar\vec{\tau}}{4},\mathbf{p}''\right)\right]\right\} = 0. \quad (3)$$

The equation for $f^{(-)}$ is obtained from Eq. (3) by making the substitutions $(+) \rightarrow (-)$, and $(-) \rightarrow (+)$. Being interested in the fluctuations of the density, let us look at the equation for the function $f = f^{(+)} + f^{(-)}$. This equation is:

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial f}{\partial \mathbf{q}} + \frac{i}{\hbar}\frac{1}{(2\tau)^3}\int d\mathbf{q}'d\vec{\tau}d\mathbf{p}'d\mathbf{p}''\left[U\left(\left|\mathbf{q}-\mathbf{q}'+\frac{\hbar\vec{\tau}}{2}\right|\right) - U\left(\left|\mathbf{q}-\mathbf{q}'-\frac{\hbar\vec{\tau}}{2}\right|\right)\right] \times \left\{e^{i\vec{\tau}(\mathbf{p}''-\mathbf{p})} f(\mathbf{q},\mathbf{p}'')\,f(\mathbf{q}',\mathbf{p}')\right.$$
$$- \Phi\left(\frac{\mathbf{q}+\mathbf{q}'}{2}-\frac{\hbar\vec{\tau}}{4},\mathbf{p}'\right) \times \Phi\left(\frac{\mathbf{q}+\mathbf{q}'}{2}+\frac{\hbar\vec{\tau}}{4},\mathbf{p}''\right) \times \exp\left[i\vec{\tau}\left(\frac{\mathbf{p}'+\mathbf{p}''}{2}-\mathbf{p}\right)\right.$$
$$\left.\left. + i(\mathbf{q}'-\mathbf{p})(\mathbf{p}'-\mathbf{p}'')/\hbar\right]\right\} = 0, \quad (4)$$

Where $\Phi = f^{(+)} = f^{(-)}$. Equation (4) differs from the corresponding equation in the selfconsistent field approximation by the presence of terms containing Φ. If, in the equilibrium state $f^{(+)} = f^{(-)}$, then upon linearization of Eq. (4) the terms containing Φ contribute nothing whatever. Therefore the vibration spectrum in this case coincides with that obtained in the selfconsistent field approximation. In the case where $f^{(-)} = 0$, the results of paper 1 are obtained.

However, in addition to density fluctuations, for particles with spin, new excitations arise, characterized by the function Φ and similar to a spin wave. The equation for the function Φ, which is gotten from Eq. (3), is:

$$\frac{\partial\Phi}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial\Phi}{\partial\mathbf{q}} + \frac{i}{\hbar}\frac{1}{(2\pi)^3}\int dq'\,d\vec{\tau}\,dp'\,dp''\left[U\left(\left|q - q' + \frac{\hbar\vec{\tau}}{2}\right|\right) - U\left(\left|q - q' - \frac{\hbar\vec{\tau}}{2}\right|\right)\right] \times \left\{e^{i\vec{\tau}(p'-p)}\,\Phi(q, p'')\,f(q', p') - \Phi\left(\frac{q+q'}{2} + \frac{\hbar\vec{\tau}}{4}, p''\right) f\left(\frac{q+q'}{2} - \frac{\hbar\vec{\tau}}{4}, p'\right) \times \exp\left[i\vec{\tau}\left(\frac{p'+p''}{2} - p\right) - \frac{i(q'-q)(p'-p'')}{\hbar}\right]\right\} = 0. \qquad (5)$$

Note that for particles with spin one, one should write the opposite sign in front of the curly brackets in Eqs. (3), (4) and (5). If $\Phi = 0$ in the equilibrium state, then upon linearizing Eq. (5)

$$f = f_0 + \varphi_{\mathbf{k}} e^{i\mathbf{k}\mathbf{q}}, \quad \Phi = \Phi_{\mathbf{k}} e^{i\mathbf{k}\mathbf{q}}$$

we obtain the following equation, describing the spin fluctuation**,

$$\frac{\partial\Phi_{\mathbf{k}}}{\partial t} + \frac{i\mathbf{k}\mathbf{p}}{m}\Phi_{\mathbf{k}} \pm \frac{i}{\hbar}\Phi_{\mathbf{k}}\int dp'\,\nu\left(\left|\frac{p-p'}{\hbar}\right|\right) f_0 \times \left(p' - \frac{\hbar k}{2}\right) \mp \frac{i}{\hbar} f_0\left(p - \frac{\hbar k}{2}\right)\int dp'\,\nu \times \left(\left|\frac{p-p'}{\hbar}\right|\right)\Phi_{\mathbf{k}}(p') = 0. \qquad (6)$$

The upper sign goes with spin ½, the lower with spin 1. For the case of spin 1 and the temperature equal to zero, $f_0(p) = n_0\delta(p)$. In this case the solution to Eq. (6) is

$$\Phi_{\mathbf{k}}(p) = e^{-i\omega t}\,\delta\left(p - \frac{\hbar k}{2}\right) c(k),$$

where

$$\hbar\omega = \hbar^2 k^2/2m.$$

Thus the spin excitation spectrum of a degenerate Bose gas of spin one particles looks like the spectrum of noninteracting particles. Consequently one can say that (insofar as it is possible to consider liquid helium as a weakly ideal gas) the superfluidity of helium is explained, apparently, not only by the Bose statistics, which the He atoms obey, but also by their lack of spin.

In conclusion, I wish to thank Professor V. L. Ginzburg for a discussion of the results.

* For spin one particles, one should take the sum in Eq. (1) rather than the difference; and in Eq. (2) the difference rather than the sum.

** Such an equation is gotten for $f^{(-)}$ in the case where $f_0^{(-)} = 0$, which corresponds to the case of a system of particles whose spins in the ground state are oriented in one direction. In contrast to the ordinary theory of spin waves, Eq. (6) permits the calculation of the interaction not only with nearby particles, but also with those distantly situated.

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Translated by M. Rosen
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Detection of the Polarization of Beams of Fast Particles by Means of Nuclear Photoemulsions

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(Submitted to JETP editor February 7, 1955)
J. Exper. Theoret. Phys. USSR **28**, 761 (June, 1955)

THE experiments of scattering of high energy particles point to the presence of non-central forces in the interactions between nucleons. The presence of such forces results in polarization of beams of the scattered particles. The magnitude of this polarization is experimentally determined by means of the measurement of the asymmetry in double scattering. The value of asymmetry is generally determined from the equation:

$$\varepsilon(\theta) = \frac{I(\theta, \varphi = 0^\circ) - I(\theta, \varphi = 180^\circ)}{I(\theta, \varphi = 0^\circ) + I(\theta, \varphi = 180^\circ)},$$

where $I(\theta, \phi = 0)$ and $I(\theta, \phi = 180°)$ are the numbers of particles scattered in second scattering at angle θ to the left or right of the direction of motion of the polarized particles.

At the present time a large number of experiments is devoted to the observation of polarization of