2. *Formation of π^+ and π^- mesons by neutrons on carbon.* Measurements were made of the relative yield of π^+ and π^- mesons at an angle of 90° to the beam for the bombardment of carbon with neutrons. The neutrons were formed as a result of the action of 670 mev protons on the nuclei of beryllium[1]. The method of observing the mesons was similar to that described above. The ratio of the outputs was calculated on the bases of 26 $\pi^+ \to \mu^+$ disintegrations and 102 "stars", formed by the π^- mesons, and found to be equal to $\sigma(\pi^-)/\sigma(\pi^+) = 5.4 \pm 1.1$. This value, within the experimental error, corresponds to the ratio $\sigma(\pi^+)/\sigma(\pi^-)$ determined by bombarding carbon with protons which constitutes a confirmation of the principle of symmetry of charges.

It can be seen, as a result of measurements of the relative outputs of charged mesons formed by protons and neutrons, that in carbon nuclei π^+ and π^- mesons in $(p - n)$ collisions are formed and emitted at an angle of 90° with half the probability of formation of π^+ mesons in $(p - p)$ or π^- mesons in $(n\text{-}n)$ collisions. This indicates that there is a difference in the nature of the process of formation of charged mesons in the $(n - p)$ and $(p - p)$ systems.

3. *Formation of π^+ and π^- mesons in $(n - p)$ collisions.* A vessel containing liquid hydrogen was placed in the path of a neutron beam[8]. Charged mesons formed by the neutrons in hydrogen and leaving the target at an angle of 90° to the direction of the beam were registered by the method described above. As a result of the examination of the emulsion, there were recorded 131 $\pi^+ \to \mu^+$ disintegrations. In the same volume of the emulsion, there were observed 112 "stars" formed by π^- mesons. Thus, the ratio of the number of positive to negative mesons, after applying the correction for stars without rays, was equal to 0.9 ± 0.2. Fig. 3 shows the energy spectra of π^+ and π^- mesons formed by neutrons in hydrogen.

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"Acoustic Excitations" in Superconductors

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1. WE investigated the spectrum of elementary energy excitations in superconductors in the framework of the new phenomenological theory of superconductivity of Ginzburg and Landau. In their work [1,2], they made use of the system of equations for the "effective" wave function and the vector potential A:

$$ih\frac{\partial\Psi}{\partial t} = \frac{1}{2m}\left(-ih\nabla - \frac{e}{c}\mathbf{A}\right)^2\Psi + \frac{\partial F_{s0}}{\partial\Psi^*}, \qquad (1)$$

$$\Delta\mathbf{A} = \frac{2\pi ih}{mc^2}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) + \frac{4\pi e^2}{mc^2}\mathbf{A}\,|\Psi|^2, \qquad (2)$$

where $F_{s0} = F_{n0} + \alpha|\Psi|^2 + (\beta/2)|\Psi|^4$ is the free energy in the superconducting state, α and β are parameters, $|\Psi|^2$ is the concentration of superconducting electrons. At equilibrium,

$$|\Psi|^2 = n_{s0} = -(\alpha/\beta);\ \alpha = H_{\text{KM}}^2/4\pi n_{s0},$$

where H_{km} is the critical magnetic field for the mass particles. The term $ih(\partial\Psi/\partial t)$ is essential in the investigation of the excitation spectrum, since it is of the same order as the other terms in the equation. The term $1/c^2(\partial^2\mathbf{A}/\partial t^2)$ has been omitted, since $\omega \ll ck$ for the excitations under consideration here.

In the investigation of the excitation spectrum, it is appropriate to employ the equation for the quantum distribution function in addition to the equation for Ψ:

$$f(\mathbf{q},\mathbf{p}) = \frac{1}{(2\pi)^3}\int \Psi^*\left(\mathbf{q} - \tfrac{1}{2}h\vec{\tau}\right)\Psi\left(\mathbf{q} + \tfrac{1}{2}h\vec{\tau}\right)e^{-i\vec{\tau}\mathbf{p}}\,d\vec{\tau},$$

$$\frac{\partial f}{\partial t} = \frac{i}{h}\int\frac{1}{2m}\left(\vec{\eta} - \frac{e}{c}\mathbf{A}(\mathbf{r})\right)^2\left[f\left(\mathbf{r} + \tfrac{1}{2}h\vec{\tau},\ \vec{\eta} - \tfrac{1}{2}h\vec{\theta}\right)\right.$$

$$- f\left(\mathbf{r} - {}^1/_2 h\vec{\tau},\ \vec{\eta} + {}^1/_2 h\vec{\theta}\right)\right] \exp\left[i\vec{\tau}\left(\vec{\eta} - \mathbf{p}\right) + i\vec{\theta}\left(\mathbf{r} - \mathbf{q}\right)\right] d\vec{\tau} d\vec{\eta} d\mathbf{r} d\vec{\theta} - \frac{i}{h}\int \left[U\left(\mathbf{q} - {}^1/_2 h\vec{\tau}\right) - U\left(\mathbf{q} + {}^1/_2 h\vec{\tau}\right)\right] \times f\left(\mathbf{q}, \vec{\eta}\right) \exp\left[i\vec{\tau}\left(\vec{\eta} - \mathbf{p}\right)\right] d\vec{\tau} d\vec{\eta}, \tag{3}$$

where $U = \alpha\left[(\rho/n_{s0}) - 1\right]$, $\rho = \int f(\mathbf{q}, \mathbf{p})\, d\mathbf{p}$,

$$\Delta\mathbf{A} = -\frac{4\pi e}{mc}\int\left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right) f(\mathbf{q}, \mathbf{p})\, d\mathbf{p}, \quad \operatorname{div}\mathbf{A} = 0. \tag{4}$$

We have considered only a transverse field. The system of nonlinear equations (3) and (4) has a solution $f_0(\mathbf{p})$ which corresponds to a homogeneous distribution of superconducting electrons:

$$\int f_0(\mathbf{p})\, d\mathbf{p} = n_{s0}, \quad \mathbf{A} = 0.$$

We investigated a solution slightly shifted from the homogeneous one:

$$f = f_0(\mathbf{p}) + \varphi(\mathbf{q}, \mathbf{p}, t), \qquad \varphi \ll f_0, \quad \rho = n_{s0} + \rho_1(\mathbf{q}, t). \tag{5}$$

Substituting Eq. (5) in Eqs. (3) and (4), and neglecting quadratic terms in ϕ and $\mathbf{A}$, we obtained the system of linear equations for the determination of ϕ and $\mathbf{A}$:

$$\frac{\partial\varphi}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial\varphi}{\partial q} = -\frac{i}{h} \times \int \frac{e}{mc}(\mathbf{A}(\mathbf{r}, t)\,\mathbf{p})\left[f_0\left(\mathbf{p} - {}^1/_2 h\vec{\theta}\right) - f_0\left(\mathbf{p} + {}^1/_2 h\vec{\theta}\right)\right] \times \exp\left[i\vec{\theta}(\mathbf{r} - \mathbf{q})\right] d\vec{\theta} d\mathbf{r} - \frac{i\alpha}{h n_{s0}}\int\left[\rho_1\left(\mathbf{q} - {}^1/_2 h\vec{\tau}\right) - \rho_1\left(\mathbf{q} + {}^1/_2 h\vec{\tau}, t\right)\right] \times \exp\left[i\vec{\tau}\left(\vec{\eta} - \mathbf{p}\right)\right] f_0\left(\vec{\eta}\right) d\vec{\tau} d\vec{\eta}, \tag{6}$$

$$\Delta\mathbf{A} = \frac{\mathbf{A}}{\delta^2} - \frac{4\pi e}{mc}\int \mathbf{p}\varphi(\mathbf{q}, \mathbf{p})\, d\mathbf{p}, \quad \operatorname{div}\mathbf{A} = 0. \tag{7}$$

In the fundamental state $|\Psi|^2 = n_{s0}$, whence $f_0 = n_{s0}\delta(p)$ which corresponds to the state of total degeneracy in the Bose statistics. The expression for the excitation spectrum which is obtained from the requirements of the solvability of the system of equations (6) and (7), has the following form:

$$1 = \frac{\alpha}{n_{s0}}\int \frac{\left[f_0\left(\mathbf{p} - {}^1/_2\mathbf{P}\right) - f_0\left(\mathbf{p} + {}^1/_2\mathbf{P}\right)\right]}{E - (\mathbf{P}\mathbf{p}/m)}\, d\mathbf{p}, \tag{8}$$

where $E = h\omega$ is the energy of excitation, and $P = hk$ is the momentum of excitation. In obtaining Eq. (8), use was made of the transverse condition, and also of the fact that $f_0 = n_{s0}\delta(\mathbf{p})$. After integration over p, we obtain

$$E^2 = \frac{H^2_{\text{K.M}}}{4\pi m n_{s0}}\mathbf{P}^2 + \frac{\mathbf{P}^4}{4m^2}; \tag{9}$$

we found that $\alpha = H^2_{\text{km}} / 4\pi n_{s0}$.

For small momenta,

$$E = \left(H_{\text{K.M}}/\sqrt{4\pi m n_{s0}}\right)\mathbf{P}. \tag{10}$$

Thus we obtain an excitation of the acoustic type, the velocity of propagation of which is $v = H_{\text{km}}/\sqrt{4\pi m n_{s0}} \approx 10^5$ cm/sec.

The equation for the Fourier component of the vector potential $\mathbf{A}_\omega = \int \mathbf{A} e^{-i\omega t} dt$ has the following form:

$$\Delta\mathbf{A}_\omega = \frac{\mathbf{A}_\omega}{\delta^2} - \frac{4\pi e\alpha}{mc n_{s0}}\int \frac{\mathbf{k}\omega}{\omega^2 - (h^2k^4/4m^2)}\rho_{\mathbf{k}} e^{i\mathbf{k}\mathbf{q}} d\mathbf{k}. \tag{11}$$

It follows from Eq. (11) that the current

$$\mathbf{j} = \frac{e\alpha}{m n_{s0}}\int \frac{\mathbf{k}\omega}{\omega^2 - (h^2\mathbf{k}^4/4m^2)}\rho_{\mathbf{k}} e^{i\mathbf{k}\mathbf{q}} d\mathbf{k}, \tag{12}$$

which is associated with the excitations under consideration, is screened at a distance of $\delta \approx 10^{-5}$ cm. The mean value of the current is zero The spectrum which is obtained satisfies the criterion of superconductivity $v_c = [E/P]_{\min} > 0$. In this case, the critical velocity v_c, beginning at which the energy of the current can be transformed into the energy of excitation, is given by $v_c \approx H_{\text{km}}/\sqrt{4\pi m n_{s0}} \approx 10^5$ cm/sec.

2. In the normally conducting state, the electronic heat capacity increases linearly with temperature, i.e., $C_n = \gamma T$. In the superconducting state, the linear term is missing from the expression for the electronic heat capacity. Available experimental data show that in a number of cases the heat capacity of the electrons in the superconducting state is proportional to the cube of the temperature, i.e., $C_s \sim T^3$.

At small values of the momenta, the spectrum under examination has an acoustical character (phonons). The thermodynamic functions for an excitation of the phonon type are well known[3] and have the form

$$F = -\frac{\pi^2\varkappa^4}{90h^3v^3}T^4; \tag{13}$$

$$S = \frac{2\pi^2 \varkappa^4}{45 h^3 v^3} T^3; \quad C = \frac{2\pi^2 \varkappa^4}{15 h^3 v^3} T^3.$$

Here F is the free energy, S the entropy, C the heat capacity excited in the superconducting state. We thus obtain the correct temperature dependence for C. If the value of the velocity of the excitation, $v = H_{km}/\sqrt{4\pi m n_{s0}} \approx 10^5$ cm/sec is substituted in the expression for the heat capacity, then the correct order of magnitude is obtained. The correct value for the electronic heat capacity is obtained without the introduction of the concept of a large dielectric permeability.

If the particles described by the "effective" wave function obeyed Fermi statistics, then the velocity of the phonon type would have been 10^8 cm/sec. Consequently the calculated values of the thermodynamic functions are significantly less than those observed in the superconducting state. Moreover, for very small values of the parameter $\varkappa$ (for low concentration of superconducting electrons), excitations of the Fermi type would arise. Such excitations lead to the appearance of a linear term in T in the expression for the heat capacity.

In the calculation of the interaction, the linear parameter δ enters characteristically. We assume that destruction of the superconducting state by the thermal vibrations of the lattice takes place when $\lambda_{max} \approx \delta$, where λ_{max} is the wavelength which corresponds to the maximum number of phonons for a given temperature. Following the work of De Launay[4], we assume that

$$\lambda_{max} T \approx hs/\kappa,$$

where s is the velocity of sound in the lattice. We then get for the critical temperature the expression

$$\kappa T_k \approx hs/\delta \approx (H_{K,M}/\sqrt{4\pi m n_{s0}})\, s,$$

from which it follows that $T_k \approx 1°$ and $TM^{1/2} = \text{const}$.

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The Asymptote of Green's Function in Quantum Electrodynamics

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1. FROM the system of integral equations for Green's function in the presence of external sources of a photon field, one can show the relation*

$$G^{-1}(p, p' + s) - G^{-1}(p - s, p') = s_\mu \Gamma_\mu(p, p', s). \quad (1)$$

Equation (1) is a generalization of Ward's theorem, and, insofar as it follows from the exact equations, we shall in the future make much use of it. The system of renormalized equations for the Green's function in quantum electrodynamics may be written, to order e^2, as follows (we retain the notation of paper[1]):

$$\Gamma_\sigma(p, p - l, l) = \gamma_\sigma + \frac{e^2}{\pi i} \quad (2)$$

$$\times \int \{\Gamma_\mu(f, p - k, k)\, G(p - k)\, \Gamma_\sigma(p - k, p - k - l, l)$$

$$\times G(p - k - l)\, \Gamma_\nu(p - k - l, p - l, -k)$$

$$- \Gamma_\mu(p^0, p^0 - k, k)\, G(p^0 - k)$$

$$\times \Gamma_\sigma(p^0 - k, p^0 - k, 0)\, G(p^0 - k)$$

$$\times \Gamma_\nu(p^0 - k, p^0, -k)\}\, D_{\mu\nu}(k)\, d^4k,$$

$$\frac{\partial G^{-1}(p)}{\partial p_\mu} = \Gamma_\mu(p, p) \text{ or } G^{-1}(p) - G^{-1}(p - k) \quad (3)$$

$$= k_\mu \Gamma_\mu(p, p - k, k),$$

$$\frac{\partial D^{-1}(k^2)}{\partial k^2} = 1 + e^2 P = 1 + \frac{e^2}{6\pi} \mathrm{Sp}$$

$$\times \int \left\{ \frac{\partial}{\partial k_\nu} \left[\Gamma_\mu(p, p - k, k) \frac{\partial G(p - k)}{\partial k_\nu} \Gamma_\mu(p - k, p, -k) \right] \right.$$

$$- \frac{\partial}{\partial k^0_\nu} \left[\Gamma_\mu(p, p - k^0, k^0) \frac{\partial G(p - k^0)}{\partial k^0_\nu} \right. \quad (4)$$

$$\left. \left. \times \Gamma_\mu(p - k^0, p_1 - k^0) \right] \right\} G(p)\, d^4p,$$

where $D_{\mu\nu} = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) D(k^2) + \frac{k_\mu k_\nu}{k^4} d_l(k^2)$,