The cross section σ_0 is practically independent of energy. Figure 3 shows the total cross section σ_s and σ_0 for different deformations z of the nucleus. The total cross section for excitation of the lth rotational level of the nucleus differs from zero only for even l and diminishes quickly with increasing l. For deformations of the nucleus which are not too great and for $l \geq 2$,

$$\sigma_l \sim 2\pi b^2 \frac{a_l^2}{2^{2l} l},$$

where a_l is the coefficient of resolution

$$\xi(x) = \sum_{l=0}^{\infty} a_l x^l.$$

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[2] A. Akhiezer and I. Pomeranchuk, *Several Questions in Nuclear Theory*, Moscow, 1950

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The Scattering of Fast Neutrons by Non-Spherical Nuclei. I

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ONE examines the scattering of fast neutrons by the black nucleus, which has the form of a body of revolution and spin equal to zero. The solution of the problem of scattering in the adiabatic approximation[1] may be obtained in the following manner. The ψ-function of the system satisfies the Schroedinger equation:

$$(H_0 + T)\psi = E\psi. \tag{1}$$

where H_0 is the Hamiltonian operator of the system consisting of the neutron and the fixed target nucleus; T is the operator for the rotational energy of the nucleus. It is assumed that the energy of the incident neutron is significantly greater than the rotational energy of the nucleus. For this reason, it is possible , in the zero approximation in Eq. (1), to drop the operator T and set up the ψ-function in the form:

$$\psi \approx u_{\mathbf{k}}(\mathbf{r}, \omega)\, \varphi_{n_0}(\omega), \tag{2}$$

where $H_0 u_{\mathbf{k}} = E_k u_{\mathbf{k}}$; $T\phi_n = \epsilon_n \phi_n$; E_k is the energy of the incident neutron; ϵ_n represents the rotational energy levels of the nucleus. The operator H_0 operates only on the radial coordinate $\mathbf{r}$ of the nucleus, and does not operate on the angular coordinate ω, which determines the orientation of the nucleus. By this means one examines, in the adiabatic approximation, neutron scattering on the fixed target nucleus, which was in the rotational state $\phi_{n_0}(\omega)$ with energy ϵ_{n_0} prior to the scattering interaction.

The operator for the rotational energy of the nucleus has the form[2]

$$T = -(\hbar^2/2I)\,\Delta_\omega,$$

where $\Delta\omega$ is the Laplace operator on the unit sphere, I is the moment of inertia of the nucleus with respect to the principal axis, parpendicular to the axis of symmetry. The eigenfunctions $\phi_n(\omega)$, describing the rotation of the nucleus, appear as the spherical functions $Y_{lm}(\omega)$; the rotational levels of the nucleus are determined by the formula:

$$\varepsilon_l = \frac{\hbar^2 l(l+1)}{2I}; \tag{3}$$

When r approaches ∞, the ψ-function (2) has the form:

$$\psi \sim \left[e^{ikr} + \frac{e^{ikr}}{r} f(\omega, \Omega) \right] \varphi_{n_0}(\omega), \tag{4}$$

where $f(\omega, \Omega)$ is the scattering amplitude on the fixed target nucleus in the direction Ω, dependent on the orientation ω of the nucleus. Resolving the quantity $f(\omega, \Omega)\phi_{n_0}(\omega)$ into a series by functions $\phi_n(\omega)$ we obtain:

$$\psi \sim e^{ikr}\varphi_{n_0}(\omega) + \frac{e^{ikr}}{r} \sum_n F_{nn_0}(\Omega)\,\varphi_n(\omega), \tag{5}$$

where

$$F_{nn_0}(\Omega) = \int d\omega\, \varphi_n^*(\omega)\, f(\omega, \Omega)\, \varphi_{n_0}(\omega). \tag{6}$$

The ψ-function (5) describes the system before the scattering interaction as well as the scattering processes, as a result of which the nucleus is left in distinct rotational states $\phi_n(\omega)$. Energy is not conserved in the approximation being used, because the particles, scattered with distinct excited rotational states, all have the identical wave vector $\mathbf{k}$.

With the aid of the expression (5) for the ψ-functions, it is possible to determine the effective cross sections. The differential cross section for scattering in the direction Ω with the nth excited rotational state summed over all orientations of the nucleus, is

$$\sigma_n(\Omega) = |F_{nn_0}(\Omega)|^2. \qquad (7)$$

In particular, the elastic scattering cross section is determined by the diagonal element $F_{n_0 n_0}(\Omega)$. Since $\sigma_n(\Omega) \equiv \sigma_{lm}(\Omega)$, the cross section for excitation of the lth rotational level of the nucleus has the form:

$$\sigma_l(\Omega) = \sum_{m=-l}^{l} \sigma_{lm}(\Omega). \qquad (8)$$

The summed cross section for scattering in the direction Ω with distinct excited rotational states is determined by the formula:

$$\sigma_s = \sum_n \sigma_n(\Omega),$$

or

$$\sigma_s = \int d\omega\, |f(\omega, \Omega)\, \varphi_{n_0}(\omega)|^2. \qquad (9)$$

The corresponding total cross sections are obtained by integrating the differential cross sections over all directions Ω. The expression for the total cross section for all the scattering processes including absorption, is found by the well-known[3] formula:

$$\sigma_t = \frac{4\pi}{k} \operatorname{Im} \int d\omega\, |\varphi_{n_0}(\omega|^2 f(\omega, \Omega)|_{\theta=0}, \qquad (10)$$

where θ is the angle of scattering. The absorption cross section is the difference between the total cross section and the cross section found by integrating the summed cross section σ_s over the angles Ω: $\sigma_c = 0_t - \sigma_s$.

The scattering amplitude $f(\omega, \Omega)$ is computed quasi-classically under the assumption that the wave length of the neutron is much smaller than the size of the nucleus R ($kR \gg 1$). It is likewise assumed that the black nucleus has the shape of an ellipsoid of revolution, either oblate or prolate. A physical (optical) analogy is used for the computation of $f(\omega, \Omega)$: the diffraction of light by a black ellipsoid. Using the principle of supplementary screening[4] we obtain

$$f(\omega, \Omega) = \frac{(kb)^2}{ik}\, \xi(x) \frac{J_1(t)}{t}; \qquad (11)$$

$$t = kb\theta \sqrt{\xi^2(x)\cos^2(\varphi - \Phi) + \sin^2(\varphi - \Phi)},$$

where $\xi(\chi) = \sqrt{z^2 + (1 - z^2)\chi^2}$; $z = a/b$, b is the radius of maximum circular cross section of the ellipsoid, a is half of its axis of symmetry; $\chi = \cos\theta$. The angles θ, ϕ determine the direction of the axis of symmetry ω of the ellipsoid; the angles θ, ϕ determine the direction of scattering Ω. The deformation parameter z assumes values from zero to infinity. For a spherical nucleus, $z = 1$.

To obtain the conditions of applicability of the formulas given for the effective cross section, one computes the corrections of the next approximation to the ψ-functions (2) and by their aid one finds the corrections to the cross sections.

The conditions of applicability follow from the requirement of smallness of these corrections. The condition of applicability of the adiabatic approximation for the computation of the cross section σ_l for the excited rotational level of the nucleus has the form $\quad \varepsilon_l/E_k \ll 1, \quad l \neq 0.$

In accord with Eq. (3), this condition is violated when l is large and when there is little deformation of the nucleus ($z \to 1$), since for a spherical nucleus the moment of inertia I becomes zero.

The condition of applicability of the adiabatic approximation for the computation of the elastic cross section has the form

$$\frac{\Delta\varepsilon}{E_k} \int_0^1 P_2(x)\, \xi(x)\, dx \ll 1,$$

where $\Delta\epsilon$ is the effective unmonochromatic condition (effective energy spread) of the neutrons scattered with distinct excited rotational states:

$$\Delta\varepsilon = \frac{\hbar^2 l(l+1)}{2I}, \quad l \sim 2.$$

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