

example, one can retain a smaller number of terms.

### 3. GENERAL REMARKS

For the particular case of proton emission, Eq. (8) reduces to the Richards formula<sup>2</sup> (his Case B) when one sets  $\alpha_0 = \beta_0 = \theta_0$  with  $\theta_0$  being small. Further if one sets  $\alpha_0 = \beta_0 = \pi/2$  Eq. (8)

coincides with the formula developed in<sup>1</sup> for  $\theta_0 = \pi/2$ .

It gives me pleasure to express my thanks to Doctor Vestmeier who suggested the problem and to M. M. Agrest for many valuable discussions.

<sup>2</sup> H. T. Richards, Phys. Rev. **59**, 796 (1941)

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## On the Theory of Crystal Growth

(Based on the article by I. V. Salli<sup>1</sup>)

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**I**N the article "On the Theory of Crystal Growth" by I. V. Salli<sup>1</sup>, there appears a sequence of incorrect equations, which tends to invalidate the author's conclusions to a large extent.

The author's discussion is based on Eq. (5), the equation for the linear growth velocity  $u$  of a spherical nucleus of a new phase, and on the subsequent Eqs. (6), (7), (7a) and (7b), derived from it. After a necessary correction is made (the coefficient 2 is superfluous in all expressions for  $u$ , since it appeared due to an incorrect assumption in the computation of  $q$ , on p. 209<sup>1</sup>, where the volume of a sphere was taken to be  $2/3 \pi r^3$ ) Eq. (5) takes on the following form:

$$u = Dv \left( \Delta - \frac{a}{r} \right) \left( \frac{1}{x} + \frac{1}{r} \right).$$

This and the subsequent equations, which are correct for an isotropic liquid, are applied to the analysis of different cases of crystal growth without any investigation of the validity of such an application. Thus, based on the fact that the coefficient of surface tension  $\sigma$  appears in the expression for  $u$  only through  $a$  ( $a = 2\sigma MvC \sim /RT$ ), the author makes the following completely unwarranted conclusion regarding the role of the coefficient of surface tension in the crystal

growth process: "One of the most important consequences of Eq. (6) is that surface tension affects the growth velocity only in the early stages of growth" (p. 210). As is well known from actual cases, the strong dependence of crystal growth velocity on direction remains true also for large crystals. For example, large as well as small crystals of sodium chloride have a cubic form. The cube faces are the planes having the slowest growth velocity and the smallest of all possible values of surface energy. Analogous conditions exist for other substances also.

In his attempts to confirm the correctness of his conclusions, Salli cites as an example the formation of platelets of cementite and their conversion to globules. Actually, he gives merely the appearance of agreement between the theory developed by him and actual observations. At that, he accomplished this only by a very peculiar interpretation of the coefficients  $\sigma$  and  $r$ : for  $\sigma$  he assumes surface tension in a direction normal to the crystal boundary (as if it were possible to speak of surface tension directed perpendicularly to the surface!), and for  $r$  — the linear dimension of a given part of the crystal (?) (see pp. 210, 211). Actually,  $r$  can only mean the radius of curvature at a given point on the crystal surface.

The fallacy in I. V. Salli's approach to the solution of the problems that he raises lies in his fundamental disregard of the specific properties of crystals as compared to those of isotropic bodies. In his opinion, crystallographic properties can play a role only in the very early growth stages of the

<sup>1</sup> I. V. Salli, J. Exper. Theoret. Phys. USSR **25**, 208 (1953)

crystal. In his explanation of the occurrence of one or another crystal habit, he adds the following principle to the above-mentioned equations :  
 “. . . the growing crystallization nucleus assumes the growth form which assures the most rapid development of the crystallized substance” (p. 208). This assumption appears as the only general principle determining crystal growth. It would have been possible to agree with him if, as the author assumes, the most prominent crystal faces were the most rapidly growing ones. In fact, the reverse is true; the most prominent crystal faces are the slowest growing ones<sup>2</sup>. For this reason, the occurrence of a many-faced habit can in no way be made to agree with the principle of fastest growth rate. Pursuant to this, he attempts to connect the many-faced crystal habits shown by the author (pp. 211, 212, case 2a) with the above stated principle and with the constitution of their interior; however, all the arguments pertaining to case 2a can be completely carried over to the formation of drops of a liquid. In this case, one reaches the conclusion that in the presence of certain fully determinable conditions a growing drop of liquid must assume a many-faced habit! It is interesting to cite the opinion of Shubnikov<sup>3</sup> in this connection: “. . . two tendencies, to a certain extent contradictory, appear simultaneously during crystal growth; a tendency to minimize the free energy and a tendency to most rapidly complete the process”. The explanation of one or

another crystal growth property based on only one of these tendencies is possible only within narrow limits; the author of the article under discussion, however, has gone far beyond these limits.

Without discussing every weak point in the author's discussion, let us consider just one characteristic example where two parts of the same sentence are completely contradictory. On page 212<sup>1</sup> one reads: “After boundary formation, the growth velocity does not depend on the surface tension and the final habit is determined by the Curie-Wulff law”. But the Curie-Wulff principle has as its consequence that (we cite reference 2, p. 204): “the boundary growth velocity is proportional to the individual surface energies of these boundaries with regard to the mother liquor”. It is clear, that the final crystal habit can in no way be determined by the Curie-Wulff law if the growth of the crystals does not obey this law.

The disregard for the role of crystal properties becomes the more clear in the tabulation of “fundamental factors, determining the kinetics and the form of crystal growth”, placed at the end of the article under discussion.. Here, there is not one word regarding the function of the structural properties of the crystal; the discussion concerns itself only with the constitution of the crystal's environment. Such a one-sided approach to the problem of crystal growth cannot possibly yield good results.

<sup>2</sup> V. D. Kuznetsov, *Crystals and Crystallization*, GITTL, 1953, p. 101

<sup>3</sup> A. V. Shubnikov, *Crystal Genesis*, Academy of Science, USSR publication, 1953, p. 101