# Calculation of the Ultimate Thickness of an Emulsion Layer in the Investigation of Nuclear Processes by Means of the Photographic Method, II.

A. A. BENE

(Submitted to JETP editor May, 1954)
J. Exper. Theoret. Phys. USSR 28, 706-710 (1955)

Herein is derived the correction formula used in the determination of the total number of tracks which are formed by a nuclear reaction within a photographic emulsion when the number of tracks which are completely within the emulsion layer are known. One considers only those tracks which lie within the angles $|\alpha| < \alpha_0$ and $|\beta| < \beta_0$ where $\alpha_0$ is the maximum track inclination with respect to the emulsion plane and $\beta_0$ is the maximum inclination of the projection of the track onto the emulsion plane with respect to the direction of the incident particles.

An additional assumption is made that the beam of incident particles lies parallel to the emulsion plane.

## 1. INTRODUCTION

BY irradiating a photographic emulsion with nuclear particles of differing energies one obtains, within the emulsion, tracks of secondary particles. In addition a primary particle of given energy can be identically referred to a track of a secondary particle, however, only in the case that the track is totally within the emulsion. In a previous paper[1] a correction formula was obtained which permits one to determine the total number of tracks that characterize primary particles of specific energy when one knows the number of tracks which are completely within the emulsion. In this case the authors considered all tracks distributed about the direction of the incident particle within a maximum angle of $\theta_0$ . In many cases, however, it appears impossible to determine with sufficient accuracy the number of long tracks which possess large inclinations relative to the emulsion plane, i.e., relative to the plane in the visual field of a microscope. In this connection, we develop, in the present communication, an analogous correction formula which estimates the total number of tracks from the number of tracks within an angle $\alpha$ ( $\alpha \leq \alpha_0$ ). The second limiting condition is that the angle $\beta$ formed by the projection of the track onto the plane of the emulsion and the direction of the incident particle does not exceed the value $\beta_0$ (compare Fig. 1 and Fig. 2).

In these instances when $\alpha_0$ is essentially small, $\beta_0$ can be assigned a large value, since its magnitude does not influence the accuracy in the measurement of long tracks. The direction of the incident particle is assumed to be parallel to the emulsion plane.

[1] A. A. Bene and M. M. Agrest, J. Exper. Theoret. Phys. USSR 27, 557 (1954)

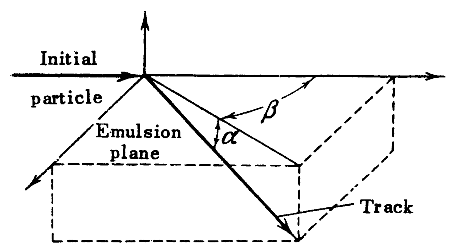


FIG. 1. Specification of the path of a track by the angles $\alpha$ and $\beta$.

## 2. DEVELOPMENT OF CORRECTION FORMULAS

Let $\sigma$ represent the total number of secondary particles which are scattered within the given region of $-\alpha_0 \leq \alpha \leq \alpha_0$ ; $-\beta_0 \leq \beta \leq \beta_0$ , and let $\sigma'$ be the number of secondary tracks which terminate in the emulsion averaged relative to $y$ over the entire emulsion thickness $d$ . In this case $y$ is the depth of the point of nuclear interaction beneath the surface of the emulsion. The ratio $W$ of the number of tracks falling completely within the emulsion to the total number of tracks, has the form

$$W = \frac{N_1}{N} = 1 - \frac{\sigma'}{\sigma}. \tag{1}$$

*Derivation of the expression for* $\sigma$. Let us now consider the expression for $\sigma$. It is easily seen that this expression has the form

$$\sigma = \int_0^{2\pi} d\varphi \int_0^{\theta_1(\varphi)} f(\theta) \sin\theta \, d\theta, \tag{2}$$

where $\theta$ and $\phi$ are the usual spherical coordinates (Fig. 2) and $f(\theta)$ is the intensity of the secondary particles in the scattering angle $\theta$ which is measured in the laboratory system of coordinates.

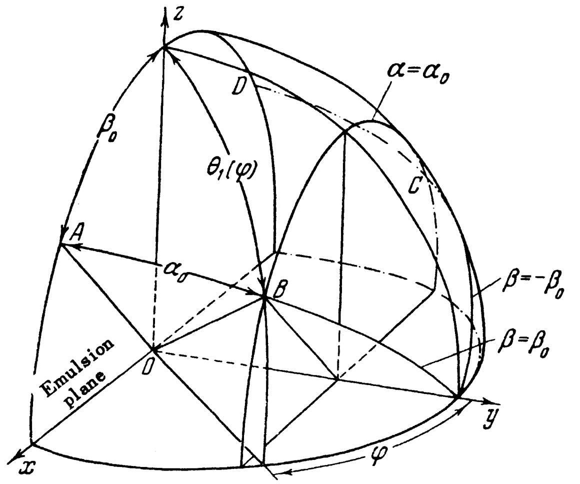


FIG. 2. The unit sphere with the region $\sigma \leq \alpha \leq \alpha_0$ and $-\beta_0 \leq \beta \leq \beta_0$ (surface $ABCD$) within which is scattered half of all the secondary particles under consideration. The figure is symmetric about plane $xz$ (the emulsion plane). The direction of the incident particle is along the $z$ axis. $O$ is the point of the nuclear interaction.

By considering Fig. 2 one can show that

$$\cos\theta = \cos\alpha\cos\beta. \tag{3}$$

$$\operatorname{tg}\varphi = \operatorname{ctg}\alpha\sin\beta.$$

equation (2) then transforms to

$$\sigma = 4\int_0^{\beta_0} d\beta \int_0^{\alpha_0} f(\alpha, \beta)\cos\alpha\, d\alpha, \tag{4}$$

where $f(\alpha, \beta)$ is the intensity function $f(\theta)$ transformed with the aid of the first equation of (3).

To perform the integration in Eq. (4) we assume that the function $f(\theta)$ can be represented sufficiently accurately by

$$f(\theta) = a_0 + a_1\cos\theta + a_2\cos^2\theta$$

or

$$f(\alpha, \beta) = a_0 + a_1\cos\alpha\cos\beta + a_2\cos^2\alpha\cos^2\beta. \tag{5}$$

With this substitution we obtain (6)

$$\frac{1}{4}\sigma = a_0K_0\sin\alpha_0 + \frac{1}{2}a_1K_1(\alpha_0 + \sin\alpha_0\cos\alpha_0) + a_2K_2\left(\sin\alpha_0 - \frac{1}{3}\sin^3\alpha_0\right),$$

where $K_i$ are functions of $\beta_0$ and are given below [see Eq. (10)].

*Derivation of the Expression for $\sigma'$.* In order to develop the expression for $\sigma'$ we must consider the distribution of path lengths $S(\theta) = S(\alpha, \beta)$ of the secondary particles that are under under consideration (Fig. 3). In this figure the circles $\alpha = \alpha_0$ and $\beta = \beta_0$ were obtained by a central projection from the unit sphere (see Fig. 2) onto the surface $S = S(\theta)$, where the center of projection is the origin of coordinates. In addition, two locations of the emulsion surface which are essential for the derivation of $\sigma'$ are shown in the Fig. 3.

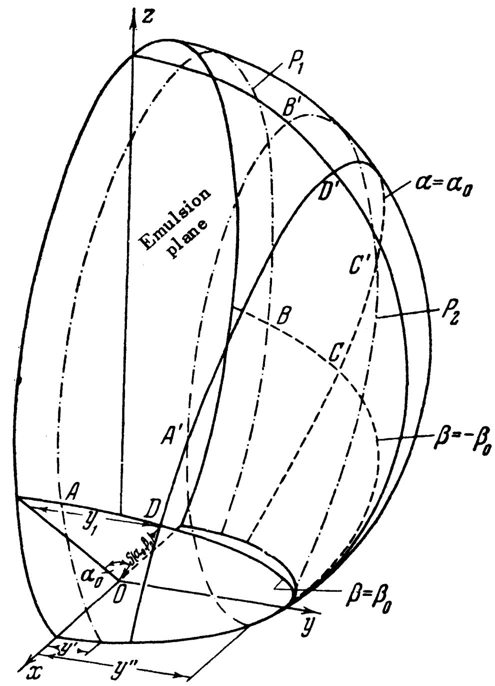


FIG. 3. The dependence of the path lengths $S$ of the secondary particles on the angles of scattering is shown. The incident particle is directed along the $z$ axis. The nuclear reaction occurs at the point $O$. Two positions of the surface layer of the emulsion are illustrated (circles $P_1$ and $P_2$) with $y = y' < y_1$ and $y = y'' > y_1$. Tracks which leave the emulsion lie in the regions $ABCD'DA$ or $A'B'C'D'A'$ for $y = y'$ or $y = y''$ respectively.

For a given value $y$ the number of tracks $\sigma_1(y)$ which pass out through the surface of the emulsion is given by

$$\sigma_1(y) = 2\int_0^{\beta_0} d\beta \int_{\alpha_1}^{\alpha_0} f(\alpha, \beta)\cos\alpha\, d\alpha,$$

if

$$0 \leqslant y \leqslant y_1 = S(\alpha_0, \beta_0)\sin\alpha_0$$

and by

$$\sigma_1(y) = 2\int_0^{\beta_1} d\beta \int_{\alpha_1}^{\alpha_0} f(\alpha, \beta)\cos\alpha\, d\alpha,$$

if

$$y \geqslant y_1.$$

In this case $\alpha_1$ is the value of $\alpha$ which specifies the location of the emulsion surface for a given value of $y$ and $\beta$. The values of $\alpha_1$ and $\beta_1$ are determined by the relations:

$$\sin\alpha_1 = y / S(\alpha_1, \beta);$$
$$S(\alpha_0, \beta_1) = y / \sin\alpha_0.$$

We shall evaluate $\sigma'$($\sigma_1$ averaged over $y$) only for the case $d \geq S(\alpha_0, 0)$ (case $B$ [1]) since, thanks to the existence of modern thick photo-emulsions, this condition is satisfied for most experiments.

If we consider that the tracks can break through on both sides of the emulsion, we obtain

$$\frac{d}{4}\sigma' = \int\limits_0^{y_1} dy \int\limits_0^{\beta_0} d\beta \int\limits_{\alpha_1}^{\alpha_0} f(\alpha, \beta)\cos\alpha\, d\alpha + \int\limits_{y_1}^{S(\alpha_0, 0)\sin\alpha_0} dy \int\limits_0^{\beta_1} d\beta \int\limits_{\alpha_1}^{\alpha_0} f(\alpha, \beta)\cos\alpha\, d\alpha, \tag{7}$$

where $d$ is the emulsion thickness. Equation (7) is correct only for those cases with $\alpha_0 \leq \bar{\alpha}$, where $\bar{\alpha}$ is the value of $\alpha$ for which the function $\psi(\alpha) = S(\alpha, 0)\sin\alpha$ is a maximum (see reference 1, Fig. 3).

If we assume that the path length $S$ depends upon the scattering angle in the manner of

$$S(\theta) = A_0 + A_1\cos\theta + A_2\cos^2\theta$$

or

$$S(\alpha, \beta) = A_0 + A_1\cos\alpha\cos\beta + A_2\cos^2\alpha\cos^2\beta,$$

we obtain

$$W = 1 - \frac{1}{d}\left\{S(\alpha_0, \beta_0)\sin\alpha_0 + \frac{4}{\sigma}I\right\}. \tag{8}$$

The quantity $I$ has the form

$$\begin{aligned} I = {} & P_1\sin^2\alpha_0 + P_2\sin^4\alpha_0 + P_3\sin^6\alpha_0 \\ & + P_4\sin^2\alpha_0\cos\alpha_0 + P_5\sin^4\alpha_0\cos\alpha_0 \\ & + P_6\alpha_0\sin\alpha_0 + P_7\alpha_0\sin\alpha_0\cos\alpha_0 \\ & + P_8\alpha_0\sin\alpha_0\cos^2\alpha_0 - P_9\cos\alpha_0 + P_9, \end{aligned} \tag{9}$$

where the coefficients are

$$P_1 = \frac{1}{8}Z_{22}K_5 + \frac{1}{2}Z_{02}K_8 + \frac{1}{2}Z_{11}[K_0 - K_2] - \frac{1}{2}Z_{00}K_0 - \frac{1}{2}Z_{20}K_2,$$

$$P_2 = -\frac{1}{6}Z_{22}[K_5 - K_4] - \frac{1}{2}Z_{02}\left[K_8 - \frac{1}{2}K_2\right] + \frac{1}{4}Z_{11}[3K_2 - 2K_0] + \frac{1}{12}Z_{20}K_2,$$

$$P_3 = \frac{1}{24}Z_{22}K_5,$$

$$P_4 = Z_{12}\left[K_1 - \frac{11}{10}K_3\right] + Z_{21}\left[K_6 - \frac{1}{5}K_3\right] + Z_{01}K_7 - \frac{1}{6}Z_{10}K_1,$$

$$P_5 = -Z_{12}\left[K_1 - \frac{13}{10}K_3\right] - \frac{1}{3}Z_{21}K_6,$$

$$P_6 = -\frac{1}{2}Z_{10}K_1,$$

$$P_7 = \frac{1}{2}Z_{11}[K_0 - 2K_2],$$

$$P_8 = \frac{1}{2}Z_{12}[2K_1 - 3K_3],$$

$$P_9 = \frac{1}{3}K_1[Z_{10} + Z_{01}] + \frac{1}{5}K_3[Z_{12} + Z_{21}].$$

here

$$Z_{ik} = a_i A_k$$

and

$$\begin{aligned} K_0 &= \beta_0, \\ K_1 &= \sin\beta_0, \\ K_2 &= \frac{1}{2}\beta_0 + \frac{1}{4}\sin 2\beta_0, \\ K_3 &= \frac{3}{4}\sin\beta_0 + \frac{1}{12}\sin 3\beta_0, \\ K_4 &= \frac{3}{8}\beta_0 + \frac{1}{4}\sin 2\beta_0 + \frac{1}{32}\sin 4\beta_0, \\ K_5 &= -\left[\frac{1}{2}\beta_0 + 2\beta_0\cos 2\beta_0 + \frac{3}{8}\sin 4\beta_0\right], \\ K_6 &= \frac{13}{40}\sin\beta_0 - \frac{1}{2}\beta_0\cos\beta_0 - \frac{3}{40}\sin 3\beta_0, \\ K_7 &= \frac{1}{3}\sin\beta_0 - \beta_0\cos\beta_0, \\ K_8 &= K_2 - 2\beta_0\cos^2\beta_0. \end{aligned} \tag{10}$$

Regardless of the fact that in practice $\alpha_0$ is small, all powers containing $\alpha$ are retained in Eq. (9) since the coefficients $Z_{ik}$ can be large for different cases. In the consideration of a specific example, one can retain a smaller number of terms.

### 3. GENERAL REMARKS

For the particular case of proton emission, Eq. (8) reduces to the Richards formula[2] (his Case B) when one sets $\alpha_0 = \beta_0 = \theta_0$ with $\theta_0$ being small. Further if one sets $\alpha_0 = \beta_0 = \pi/2$ Eq. (8) coincides with the formula developed in[1] for $\theta_0 = \pi/2$.

It gives me pleasure to express my thanks to Doctor Vestmeier who suggested the problem and to M. M. Agrest for many valuable discussions.

[2] H. T. Richards, Phys. Rev. 59, 796 (1941)

Translated by A. Skumanish
125

SOVIET PHYSICS JETP VOLUME 1, NUMBER 3 NOVEMBER, 1955

# On the Theory of Crystal Growth

(Based on the article by I. V. Salli[1])

A. V. BELIUSTIN
*Gorki Physico-Technical Institute*

(Submitted to JETP editor October 27, 1953)
J. Exper. Theoret. Phys. USSR 28, 725-726 (June, 1955)

IN the article "On the Theory of Crystal Growth" by I. V. Salli[1], there appears a sequence of incorrect equations, which tends to invalidate the author's conclusions to a large extent.

The author's discussion is based on Eq. (5), the equation for the linear growth velocity $u$ of a spherical nucleus of a new phase, and on the subsequent Eqs. (6), (7), (7a) and (7b), derived from it. After a necessary correction is made (the coefficient 2 is superfluous in all expressions for $u$, since it appeared due to an incorrect assumption in the computation of $q$, on p. 209[1], where the volume of a sphere was taken to be $2/3\,\pi r^3$) Eq. (5) takes on the following form:

$$u = Dv\left(\Delta - \frac{a}{r}\right)\left(\frac{1}{x} + \frac{1}{r}\right).$$

This and the subsequent equations, which are correct for an isotropic liquid, are applied to the analysis of different cases of crystal growth without any investigation of the validity of such an application. Thus, based on the fact that the coefficient of surface tension $\sigma$ appears in the expression for $u$ only through $a$ ($a = 2\sigma MvC \sim /RT$), the author makes the following completely unwarrented conclusion regarding the role of the coefficient of surface tension in the crystal growth process: "One of the most important consequences of Eq. (6) is that surface tension affects the growth velocity only in the early stages of growth" (p. 210). As is well known from actual cases, the strong dependence of crystal growth velocity on direction remains true also for large crystals. For example, large as well as small crystals of sodium chloride have a cubic form. The cube faces are the planes having the slowest growth velocity and the smallest of all possible values of surface energy. Analogous conditions exist for other substances also.

In his attempts to confirm the correctness of his conclusions, Salli cites as an example the formation of platelets of cementite and their conversion to globules. Actually, he gives merely the appearance of agreement between the theory developed by him and actual observations. At that, he accomplished this only by a very peculiar interpretation of the coefficients $\sigma$ and $r$: for $\sigma$ he assumes surface tension in a direction normal to the crystal boundary (as if it were possible to speak of surface tension directed perpendicularly to the surface!), and for $r$ — the linear dimension of a given part of the crystal (?) (see pp. 210, 211). Actually, $r$ can only mean the radius of curvature at a given point on the crystal surface.

The fallacy in I. V. Salli's approach to the solution of the problems that he raises lies in his fundamental disregard of the specific properties of crystals as compared to those of isotropic bodies. In his opinion, crystallographic properties can play a role only in the very early growth stages of the

[1] I. V. Salli, J. Exper. Theoret. Phys. USSR 25, 208 (1953)