

## Investigation of High Frequency Discharges by the Probe Method

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A comparison is made of existing probe methods for the investigation of high-frequency discharges in He, Ne, Ar and H<sub>2</sub>. We consider changes in a discharge which arise when additional electrodes—counterprobes—are inserted in a high frequency discharge. An investigation is carried out of the distribution of temperature, electron concentration and space potential along the axis of the discharge tube, and also of the dependence of these parameters upon the gas pressure and the voltage of the high-frequency field at the exterior electrodes of the discharge tube.

### 1. INTRODUCTION

THE method of probes is a very widespread one for measuring the electrical parameters of the positive column of a gas discharge. Probe methods, adequately proven for the investigation of direct current gas discharges, have not been sufficiently examined for their application to high-frequency discharges.

As is well known, in probe measurements in a direct current discharge, the potential of the probe is referred to the potential of one of the electrodes. In high-frequency discharges, in view of the fact that there is a high-frequency varying potential on the electrodes, they cannot be used as reference points, or counterprobes.

In the researches in references 1,2 on investigation of high-frequency gas discharges by the method of probes, a third electrode at ground potential is used as a counterprobe. The potential of the probe can be referred to that of the counterprobe only under the condition that the potential of the region about the counterprobe does not depend upon the probe current. This is possible only if many more electrons and ions from the discharge fall on the counterprobe than on the probe. This condition requires a counterprobe of large surface area. However it is not known how a counterprobe of large dimensions will influence the regime of a high-frequency discharge. Taking these circumstances into account, Biberman and Panin<sup>3</sup> advanced a

method of two probes, and used it to determine the electron temperature and the concentration of charged particles, under the hypothesis that there was a Maxwellian distribution of electrons and ions in velocity. However the method of counterprobes has been used to determine not only the electron temperature and the concentration of charged particles but also the space potential. In addition, it gives criteria for a Maxwellian distribution of the electrons in velocity. Therefore the study of this method of investigating high-frequency discharges presents some interest.

The aims of the present work are:

- a) The comparison of data obtained by the experimental method of two probes with data obtained by the application of counterprobes.
- b) The experimental study of possible disturbances caused by introducing counterprobes in the discharge.
- c) The investigation of the distribution along the axis of the discharge tube of the electron temperature and concentration and the space potential for various gas pressures.

### 2. EXPERIMENTAL ARRANGEMENT

A gas discharge tube of length 70 cm and diameter 3.2 cm (Fig. 1a) was employed in carrying out the proposed undertaking. Inside the tube are placed two counterprobes: one, *f*, is fixed in the center of the tube near the probe, and the other, *m*, is movable. Both counterprobes are made of nickel foil with equal surface areas (30 cm<sup>2</sup>).

In the middle of the tube (at its axis) are located two identical platinum probes *P*, 4 mm apart, with diameters of 0.1 mm and lengths of 4 mm. The ratio of the area of the probes to the area of the counterprobe is approximately 1:2500. The probes

<sup>1</sup> Banerji and A. Ganguli, *Phil. Mag.* **11**, 410 (1931); **13**, 494 (1932); **15**, 676 (1933)

<sup>2</sup> H. Beek, *Z. Physik* **97**, 355 (1935)

<sup>3</sup> B. Biberman and L. Panin, *Zh. Tekhn. Fiz.* **21**, 12 (1951)

are situated strictly on the tube axis, in order that a possible gradient in the density of charged particles in the discharge should not affect the conditions under which particles strike the probe.

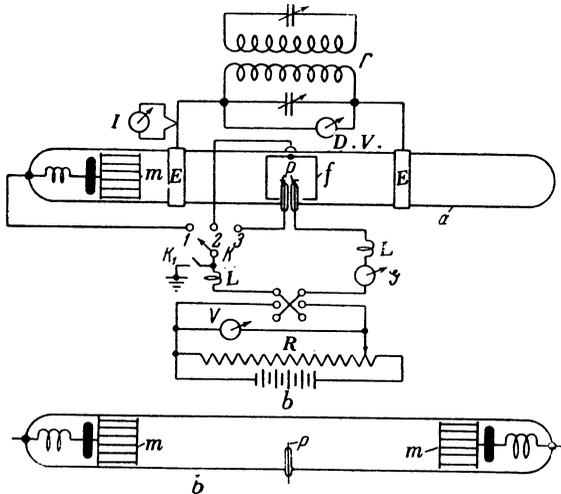


Fig. 1. Scheme for taking probe characteristics and types of discharge tubes.

In this manner we succeeded in getting a symmetric probe characteristic by means of the two probes method.

In investigating the distribution of electron temperature  $T_e$ , electron concentration  $n_e$  and space potential  $V_g$ , two movable counterprobes of identical form and dimensions are used (Fig. 1b). A cylindrical probe  $P$  is located in the center of the tube (perpendicular to the axis). Most of the discharge tubes for work with inert gases were unsoldered from the vacuum system. Absorbers were used to maintain the purity of the gas.

The scheme for probe measurements is presented in Fig. 1. Two external electrodes  $E$  in the form of rings surround the tube on the outside. Using the switch  $K$  it is possible to take the probe characteristics with the movable counterprobe (position 1), with the immovable counterprobe (position 2), and with two probes (position 3). While taking the probe characteristics the counterprobe is grounded separately, but in working with the two probes the ground is opened by means of the switch  $K_1$ . The chokes  $L$  are inserted in the probe circuit to suppress the high-frequency oscillations. In taking the probe characteristics for a field of frequency 5 mc, the discharge current in the tube is taken as a parameter. The current is measured by means of a milliammeter  $I$ . For higher frequencies (100 mc and above) the high-

frequency voltage between the external electrodes serves as parameter. The voltage is measured by means of a diode voltmeter  $D.V.$  assembled in the laboratory.

Linear extrapolation of the ion current is used to determine the electron current from the probe characteristic. The electron temperature is determined from the slope of a semi-logarithmic plot of the characteristic in the case of one-probe measurements, or computed from the formula of Biberman-Panin<sup>3</sup> in the case of two-probe measurements. The point on the semilogarithmic plot at which the graph of the characteristic begins to depart from a straight line is taken as the space potential.

### 3. COMPARISON OF ELECTRON TEMPERATURES FOUND BY ONE-AND TWO-PROBE METHODS

Measurements were performed in neon and helium to compare the values of the electron temperature gotten by means of the methods of one or two probes.

In Fig. 2 is shown the probe characteristic obtained by means of the probes (a), and also the volt-ampere and the semilogarithmic characteristics obtained by means of a movable counterprobe (b) and a central (immovable) counterprobe (c) respectively, with helium in the tube given in Fig. 1a.

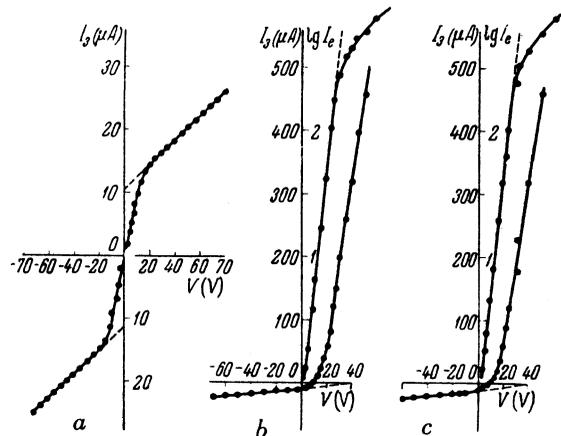


Fig. 2. Probe characteristics in He, obtained: (a) by the two-probe method, (b) by the one-probe method with the use of a fixed counterprobe  $f$ , (c) by the one-probe method with a movable counterprobe  $m$ .

The frequency of the field was 5 mc. These semi-logarithmic plots, the same for one counterprobe as the other, point in the direction of a Maxwellian

distribution of the electrons in velocity, which makes possible the use of the semilogarithmic characteristic as a general method of determining the electron temperature. However it must be pointed out that at low gas pressures cases were observed that depart from a straight-line semilogarithmic curve, indicating the absence of a Maxwellian distribution in electron velocity in a high-frequency discharge. The comparison of the two methods refers to those cases where there is a Maxwellian distribution of the electrons in velocity.

The results of the measurement of electron temperature by the various methods in helium and neon are given in Table 1.

From the data given in Table 1 the conclusion can be drawn that all three methods give the same value of the electron temperature. Some differences, evidently, can be put down to the account of experimental error. Thus it can be said that in the determination of the electron temperature by the the method of probes in a high-frequency discharge, counterprobes produce no essential modification.

#### 4. MODIFICATIONS PRODUCED BY COUNTERPROBES

In the work with the tube illustrated in Fig. 1a it was noticed that for low gas pressures the discharge in the vicinity of the central counterprobe was restricted to the axis of the tube. This circumstance, naturally, shows some distortion of the discharge by the movable counterprobe, which can be verified by the measurement of other parameters. To study these modifications by means of a movable counterprobe inserted in the discharge, a tube was used of the type presented in Fig. 1b.

Argon was selected for this work since the semi-logarithmic characteristic in argon has a sharp break that permits a high precision determination of space potential and the electron concentration along with the electron temperature. Measurements were taken for a field of frequency of 5 mc with a pressure of 0.23 mm Hg and a discharge current of 100 ma. In the discharge under the indicated conditions were observed three luminous parts separated by dark spaces.

The study of the effect of the counterprobe was carried out in the following manner. The external electrodes are stationed at a distance of 18 cm from each other, and the counterprobe at distances of 3 cm from the external electrodes, i.e., 24 cm from each other. Keeping the distance between the external electrode and counterprobe constant, the electrode and counterprobe are displaced along the tube axis so that the probe falls at successively different positions along the tube axis. Thus at each position along the axis of the discharge tube a probe characteristic is taken with either the left counterprobe, the right counterprobe, or finally both, connected as reference electrodes.

From the probe characteristics obtained for various points along the axis of the discharge tube, the electron temperature, the space potential, and the electron concentration are determined. These measurements are taken within the luminous parts of the discharge. The distribution obtained for the space potential is shown in Fig. 3. In the lower part of Fig. 3 is presented the external form of the working part of the discharge in which measurements were carried out (Fig. 1b). Curve 1 gives the distribution of space potential along the axis of the tube found in measurements with the right-

TABLE 1

Gas	Pressure (mm Hg)	Field Frequency mc	Electron Temperature (°K)		
			Two probes	Movable probe	Inmovable probe
Helium	0.23	5	52000	57500	58000
	—	—	50000	58000	57500
	—	—	48000	53000	53000
	—	—	55000	53000	54000
	0.4	5	50000	55000	55000
	—	—	47000	54000	53000
	—	—	48000	50000	50000
	—	—	49000	50000	50000
Neon	0.56	130	50000	46000	—
	—	—	49000	46000	—
	—	—	50000	45500	—

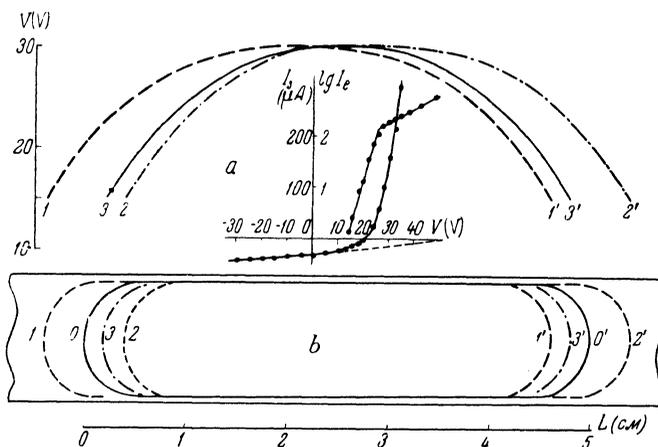


Fig. 3. Distribution of potential along the tube axis in A. The reference electrodes with which the distributions are gotten are: (1) right-hand counterprobe, (2) left-hand counterprobe, (3) both counterprobes connected simultaneously.

hand probe; curve 2, the left-hand probe; curve 3, both on simultaneously.

As is evident from the graphs shown, the space potential distribution is found to be symmetric with respect to the center of the discharge for the measurements taken with both probes on simultaneously, and somewhat asymmetric when the left or right-hand counterprobe was on separately. This phenomenon is connected with the change in form of the discharge depending on the location of the counterprobes. The change exhibited consists in the fact that when the right counterprobe is on, the working domain of the discharge shifts to the left, as shown by Fig. 1b, Nos. 1-1'. When the left counterprobe is on, the working part of the discharge shifts to the right, as shown by Nos. 2-2'. Finally, when both counterprobes are on, the working part (the luminous part) of the discharge is compressed at both sides, as shown by Nos. 3-3'. Nos. 0-0' show the form of this part of the discharge when the counterprobes are disconnected. The volt-ampere and semilogarithmic characteristics in argon, obtained for both counterprobes on simultaneously, are shown in the center of Fig. 3a.

It follows from the graphs that the presence of a counterprobe introduces a certain modification in the structure of a discharge. This modification is expressed in a change in position along the axis of the discharge tube of the luminous and dark domains of the discharge. The deformation of the discharge exhibited in our case was comparatively minor, not more than 3-4 mm at the two ends. Further it must be concluded that the use of two

counterprobes as reference electrodes, located symmetrically with respect to the interior electrode, is more convenient than the use of one counterprobe. In this case the internal structure of the discharge is kept symmetric with respect to the electrode.

##### 5. INVESTIGATION OF THE INTERNAL ELECTRICAL PARAMETERS IN A HIGH-FREQUENCY DISCHARGE IN ARGON, NEON AND HYDROGEN

An investigation of the distribution of electron temperature  $T_e$ , electron concentration  $n_e$ , and space potential  $V$  along the axis of the discharge tube was carried out in a tube of type b (Fig. 1) filled with argon, neon, and hydrogen.

In Fig. 4 graphs of the distribution of  $T_e$ ,  $n_e$  and  $V$  along the axis of the discharge tube are presented for argon at a pressure  $p = 0.07$  mm Hg and an external field of frequency 130 mc (with a voltage amplitude of 180 V at the external electrodes). Under these conditions the discharge was homogeneous.

From the graphs of Fig. 4 follow:

- The distribution of the electron temperature is symmetric about the electrode with its maximum value at the center of the discharge interval (Fig. 4a).
- The electron concentration (Fig. 4b) and the space potential (Fig. 4c) along the tube axis also are distributed symmetrically about the electrodes and achieve their maximum values at the center between them. A particularly strong variation of

space potential along the axis of the discharge tube was found by us in argon for a pressure of .07 mm Hg.

The configuration of the distribution curves of electron temperature and concentration and of space potential along the axis of the tube in general depends on the pressure and nature of the gas, on the diameter of the tube, and on the discharge current, but the curves are always symmetric about the electrodes.

Generally, in a homogeneous high-frequency discharge with not too great a distance between the electrodes, the distribution curve of electron temperature along the tube has its maximum value in the center between the electrodes. With increasing separation between the electrodes but constant high-frequency voltage amplitude, there is finally a change in the course of this curve.

The dependence of the distribution of electron temperature and concentration, and space potential, upon gas pressure was investigated with hydrogen in the discharge tube Fig. 1b (length 80 cm, diameter 35 mm, separation between external electrodes 13 cm, field frequency 130 mc). Measurements were carried out in the range of pressure from 0.1 to 1.1 mm Hg for a discharge current of 150 ma. Under these conditions the discharge in the tube

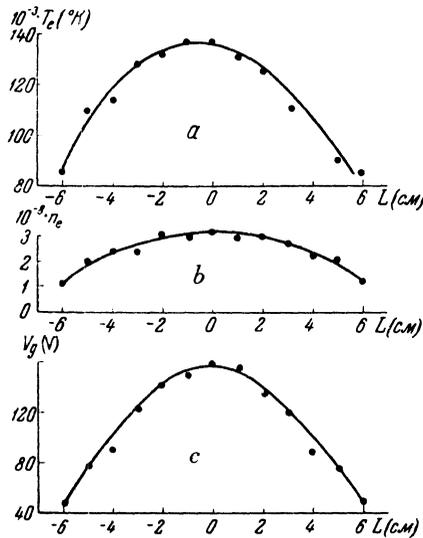


Fig. 4

Fig. 4. Distribution of (a) electron temperature, (b) electron concentration, (c) space potential along the tube axis in Ar at a frequency of 130 mc;  $p = 0.07$  mm Hg; amplitude of voltage between the external electrodes 180 V.

is homogeneous. During the measurements the probe was on the axis of the tube centrally between the electrodes. The results are shown in Fig. 5.

From the graphs of Fig. 5 it is evident that:

a) The electron temperature decreases with increasing pressure just as for DC discharges (curve a).

b) With rising pressure (approximately from 0.1 to 0.3 mm Hg) the electron concentration rises to some maximum value but then with a further increase in pressure the electron concentration begins to decrease monotonically (curve b).

c) With fixed discharge current and rising pressure, the space potential in the center of the region between the electrodes falls (curve c).

Fig. 6 gives the graphs of the dependence of temperature (a), concentration of electrons (b) and space potential (c) on the amplitude of the high-frequency voltage at the external electrodes for

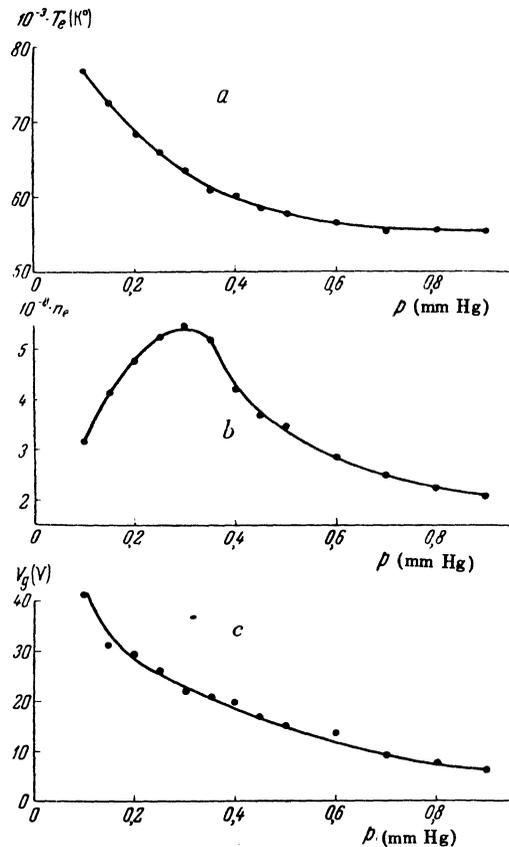


Fig. 5

Fig. 5. Dependence of  $T_e$ ,  $n_e$  and  $V_g$  upon pressure in  $H_2$ , at a frequency of 100 mc and a discharge current of 100 ma.

neon at pressures of 0.07 mm Hg (curve 1) and 0.56 mm Hg (curve 2), and with a field of frequency 130 mc. The measurements were taken in a discharge tube of the type shown in Fig. 1b, of length 80 cm and diameter 27 mm. From the graphs in Fig. 6 it follows that the electron temperature rises with increasing high-frequency voltage on the electrodes. The value of the electron concentration midway between the electrodes finally rises with increasing voltage, but the magnitude of the space potential, on the contrary, decreases with increasing voltage.

## 6. DISCUSSION OF RESULTS

In an investigation of high-frequency discharges by means of the method of probes, it was established that the electron temperatures determined both by the method of two probes and by the method of one probe with the use of a counterprobe, have the same value within the limits of experimental error. The use of a counterprobe as reference point leads to an insignificant redistribution of the electrical parameters of the discharge along the discharge tube. This gave us reason to carry out an investigation of high-frequency voltages in a tube of the type shown in Fig. 1b.

From Fig. 4, 5 and 6 for argon, hydrogen and neon it is evident that the temperature of the electrons in a high-frequency discharge decreases with rising gas pressure. Apparently this is explained basically by the same circumstances as in a DC discharge. However one should bear in mind that in a high-frequency discharge the frequency of the applied field also can have an influence upon the

value of  $T_e$ . In a high-frequency discharge at comparatively high pressures, where the frequency of the field is significantly less than the collision frequency of the electrons with the gas molecules, the character of the behavior of  $T_e$  with varying pressure does not differ essentially from that in a DC discharge. For higher field frequencies or comparatively low gas pressure, i.e., when the collision frequency is of the same order as or smaller than the field frequency, one can expect some difference between the behaviors of  $T_e$  in a high-frequency discharge and a DC discharge.

Table 2 compares the value of the electron temperature in a high-frequency discharge obtained by us with the value of the electron temperature in a DC discharge gotten by other authors<sup>4</sup>. As is evident from Table 2, in the case of A the value of  $T_e$  in a high-frequency discharge is significantly greater than in a DC discharge at the same pressure.

In all our measurements in the range of pressures from 0.07 to 1 mm Hg, for homogeneous discharges in hydrogen, argon and neon, with field frequencies from 5 to 130 mc. a Maxwellian distribution of electrons in velocity was obtained.

The intensity of the high-frequency field increases with increasing voltage amplitude at the electrodes. The energy which the electron acquires from the field on the average in a mean free path increases with increasing field intensity. Therefore the value of  $T_e$  in a high-frequency field increases with increasing voltage amplitude at the electrodes, other things being equal (Fig. 6). The very same result was obtained by Brasfield<sup>5</sup> in an investigation of high-frequency discharges by an

TABLE 2

Gas	Pressure (mm Hg)	$T_e \times 10^{-3}$ (°K)			
		DC discharge current		H.F. discharge current	
		50 ma	300 ma	50 ma	300 ma
Neon	0.07	57	53	60	—
	0.29	40.2	35	51	—
	0.56	35	37	48	—
Argon	0.07	26	22	—	100
	0.24	22	18	—	43.5
	0.96	16	12	—	40

optical method.

The value of the electron concentration is a complicated function of the gas pressure (Fig. 5,6). The nature of the dependence of electron concentration on gas pressure can be explained in the

<sup>4</sup> R. Seeliger and R. Hirschert, Ann. Physik. 11, 817 (1931)

<sup>5</sup> C. Brasfield, Phys. Rev. 35, 92 (1930)

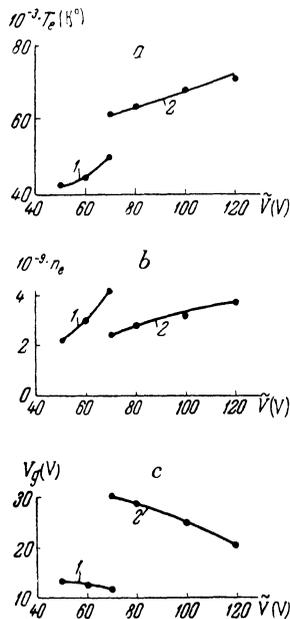


Fig. 6. Dependence of  $T_e$ ,  $n_e$  and  $V_g$  upon the amplitude  $\tilde{V}$  of the high-frequency voltage in Ne: (1)  $p = 0.07$  mm Hg, (2)  $p = 0.56$  mm Hg; frequency = 130 mc.

following manner. In going from high pressure to less, the basic cause of the rise in electron concentration is the increase in energy of the electrons over a mean free path; this increase in energy increases the probability of ionization of a molecule by electron collisions. In going to the low pressures, there is a decrease in electron concentration as a result of a large loss of electrons, on account of diffusion and also on account of the decreased probability of encounters between electrons and gas molecules.

The experimental curve we obtain for the dependence of electron concentration on gas pressure for hydrogen recalls in its shape the curve obtained theoretically<sup>6</sup> for the dependence of the conductivity of an ionized gas on pressure at ultra-high frequencies. It likewise follows from the curves for the distribution of electron concentration in its dependence on pressure in hydrogen, that at pressures at which the electron concentration has its maximum value, the frequency of collisions of electrons with gas molecules is of the same order as the frequency of the applied field. This is also in accord with the theoretical results of some authors<sup>6,7</sup> who have investigated the conductivity of high-frequency discharges. The electron con-

centration increases with increasing high-frequency field intensity.

Investigating the distribution of space potential along the axis of the discharge tube, we verified the result of Banerji and Ganguli<sup>1</sup> on the existence, in some cases, of a large steady field in a high-frequency discharge. It was shown the magnitude of the steady field strongly depends on the gas pressure, the intensity of the high-frequency field and the diameter of the discharge tube. With a rise in gas pressure and high-frequency field intensity, there is a decrease in the steady field on account of the decrease in the volume of positive charge on the axis of the discharge tube between the electrodes. The increase of the steady field in a high-frequency field with decreasing gas pressure also occurs as a result of more intensive flow of electrons from the discharge region because of increased processes of diffusion to the wall of the discharged tube, both on account of the greater mean free path of the electrons and on account of the greater mean energy of the electrons.

The flow of positive ions from the discharge region as a result of diffusion processes is significantly less. Therefore there will result an accumulation of positive space charge at the axis of the discharge tube as long as the increase of positive charge is not stopped as a result of the establishment of ambipolar diffusion of electrons and ions to the walls of the discharge tube.

## RESULTS

1. A comparison is made of existing probe methods for investigating high-frequency discharges. Measurements are made by the two-probe method, and by a one-probe method with the use of a counterprobe.

The results of the measurement show that both methods give the same result for the electron temperature within the limits of experimental error. The use of a counterprobe as a reference point leads to an insignificant redistribution of the electrical parameters of the high-frequency discharge along the axis of the tube. This resolves the contradiction between references 1 and 2.

2. An investigation is made of the dependence of temperature, electron concentration and space potential at the axis of the discharge tube upon the pressure of the gas.

3. The distribution of temperature, electron concentration and space potential along the axis of the tube are obtained. In a homogeneous high-frequency discharge these parameters are symmetric with respect to the electrodes and have their maximum values between the electrodes. The distributions of temperature, electron concentration

<sup>6</sup> F. Adler, J. Appl. Phys. **20**, 1125 (1940)

<sup>7</sup> H. Margenau, Phys. Rev. **69**, 508 (1946)

and space potential along the tube depend essentially on the gas pressure, diameter of the discharge tube and high-frequency field intensity.

L. A. Rosnovskaia took part in carrying out

some of the measurements.

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### The Absorption and Emission of X-Rays in Ferromagnetic Metals

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The problem of absorption and emission of x-rays by a ferromagnetic is examined in the framework of interaction of the inner and outer electrons.

1. There has been to date no theoretical examination of the absorption and emission of x-rays by ferromagnetic substances. In view of the importance of a study of the subject, we shall attempt to solve the problem in a very rough, qualitative approximation.

In the absorption of x-rays by inner shell electrons it is essential that the frequency of the x-ray be sufficient to excite the electron into an unoccupied level above the conduction levels. By examining the fine structure of the absorption band edge, one can hope to obtain some detailed information about the highest electron energy levels in the metal.

Since the initial electron level is part of an inner shell, it can be taken as infinitely narrow. Let  $\psi_0$  be the normalized wave function of the initial state,  $\hbar\omega_{\xi',0}$  --- the energy difference between this state and a state of conductivity  $k$ . The optical conductivity, which determines absorption, is given by the equation<sup>1</sup>

$$nk\nu = \sigma = -\frac{\pi e^2}{m^2 \hbar \omega G^3 a^3} \sum_{ss'} \int \frac{\rho_0(\xi, s') - \rho_0(\xi, s)}{|\text{grad}_{\xi} \omega_{\xi\xi'}|} \times |p_x(\xi, \xi')|^2 du dv. \quad (1)$$

Here  $s$  is the sum index over initial states, and  $s'$  the sum index over final states of the metal electrons in the first Brillouin zone. Since, in the present case, the initial levels are essentially the discrete levels of isolated atoms, the corresponding wave function is different from zero only in a small region. This makes the summation over  $s$  easy. If there are  $N_0$  atoms per unit volume, then we have for the optical conductivity:

<sup>1</sup>A. V. Sokolov, J. Exper. Theoret. Phys. USSR 25, 9 (1953)

$$(\rho_0(\xi, s') = 0, \quad \text{and} \quad \rho_0(\xi, s) = G^3 / 8\pi^3)$$

$$\sigma = nk\nu = \frac{\pi e^2 \hbar N_0 G^3}{8\pi^3 m^2 \hbar \omega a^3} \Omega \int \{[\nabla_{\xi} E(\xi)]^{-1} \times |p_x(\xi, 0)|^2\}_{\omega_{\xi_0}=\omega} du dv, \quad (2)$$

where we take into account the relation

$$\omega_{\xi_0} = [E(\xi) - E(0)] / \hbar, \quad (3)$$

$E(0) = \text{const}$ , is the energy of the  $K$  electron in the atom; the summation over  $s'$  in the first zone is replaced by multiplication by the number of states in unit volume of  $k$ -space.

2. With the removal of an electron from an inner shell, a conduction electron may make the transition to the vacant lower level, simultaneously emitting an x-ray quantum. Because of this, a study of the emission spectra of a metal serves as a source of information about the energy levels occupied by conduction electrons<sup>2</sup>. The intensity of x-rays emitted in the frequency interval  $\omega$  to  $\omega + d\omega$  is

$$I(\omega) d\omega = \frac{4N_0 \hbar^2 \omega^2 e^2}{3c^3 m^2} d\omega \sum_{s'} \left| \int \psi_k^* \nabla \psi_0 d\tau \right|^2, \quad (4)$$

where the summation is carried over all occupied states of the conduction electrons with energy corresponding to a frequency  $\omega_0$  such that  $\omega \leq \omega_0 \leq \omega + d\omega$ , i.e.,  $\hbar d\omega = dE$ .

Instead of a summation over all possible values of  $k$ , it is possible to write an integral over  $(G/2\pi)^3 \Omega_0 dk_1 dk_2 dk_3$ ; we then obtain

$$I(\omega) d\omega = \frac{N_0 \hbar^2 \omega^2 e^2 G^3 \Omega_0}{6\pi^3 m^2 c^3} dE \int \left( \frac{\partial E}{\partial k_1} \right)^{-1} \quad (5)$$

<sup>2</sup>A. Wilson, *Quantum Theory of Metals*, pp. 42-44