On the Paper of V. I. Karpman, " The Problem of the Connection Between the Method of Regularization and the Theory of Particles with Arbitrary Spin " [1]

IU. A. IAPPA
Leningrad State University

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THE work of Karpman referred to in the title is devoted to a criticism of our note "The connection between the theory of regularization and the theory of particles with arbitrary spin "[2]. Karpman stated in his work that our results are in error. This letter is written to show that this statement of Karpman's is fundamentally the result of a misunderstanding. It was shown in reference 2 that the invariant δ - function introduced into the determination of the transformation relation of the second quantization for the fields described by the relativistically invariant equations in the case of finite size, satisfies a number of conditions, for each part of which there are conditions of regularization introduced by Pauli and Villers. This fact, the establishment of which is the basic result of the note[2], was not disproved,but rather confirmed in Karpman's comment. It may be presumed that this circumstance has a well-known physical interest in that the theory of particles having a mass spectrum represents, up to now, the only case in the theory of quantized fields, for which it has been shown that the "regularization" condition is obtained as a consequence of a certain conception of the properties of particles (the consequence of the idea that particles have a mass spectrum). In this sense the condition of regularization is obtained here more naturally, from the physical viewpoint, than in the theory with higher order derivatives,or even in thetheory of " compensation " fields. This was precisely the point we made.

Karpman[1] showed that the adjustable relation, in spite of the presence of the regularization condition mentioned, following from the same structure of theory, contains certain divergent terms. However,this fact does not to any degree contradict the presence of a regularization condition in the case of particles with a mass spectrum. By the word "regularization " in reference 2 was implied only the presence of a number of regularization conditions. In order to be able to consider a concrete expression for physical quantities, it is necessary to solve all complex problems of normalization and regularization for particles with a mass spectrum. Reference 2 was not by any means designed to do this, as is evident from the nature of the note itself. The author regrets that there is an incorrect statement in the article, namely: ". . . analysis of the function $C(x)$ is characterized by an expansion of the function Δ_R into a series of terms vanishing as $x_s x^s \to 0$, and, in this manner, the function $C(x)$ itself is regularized . . . . ", which can lead to an error relative to the meaning of the term "regularization ". All of Karpman's criticisms arise at this point.

It follows that the results of our earlier work relate not only to the establishment of the existence of proper regularization in the theory of particles with a mass spectrum, but to a new derivation of the adjustment relations of second quantization in that theory. These adjustment relations, brought out in reference 2, are a simpler method than was used previously by Pauli[3]; the result is a little different from that of Pauli. This difference is connected with the circumstance that in reference 2 the adjustment relations are subject to the natural requirement that they be the same in equations of the first order as in higher order equations, which requirement the wave function satisfies. In Karpman's work, nothing was said about this circumstance, which seems strange, inasmuch as to obtain the conditions of regularization of the adjustment relations in reference 1, it is necessary to go over to the viewpoint given in our work[2] with certain differences indicated above. It is also not clear, granting the basic results of our work, and its consequences, why Karpman did not mention this in the text of his work, taking notice only of the apparently erroneous results, and that in a footnote.

In conclusion, we present the derivation of Eq. 9 of reference 2, determining the operator F in the adjustment relations of the second quantization for a particle with mass spectrum. Let the equation for the eigenvalues of the matrix L^0 have the form:

$$L^0 \psi_r^{(0)} = \lambda_r \psi_r^{(0)}. \qquad (1)$$

Then, using the condition $S^{-1} L^{\mu} S t^0_{\mu} = L^0$,

[1] V. I. Karpman, Doklady Akad. Nauk SSSR 89, 257 (1953)

[2] Iu. A. Iappa, Doklady Akad. Nauk SSSR 86, 51 (1952)

[3] W. Pauli, *Relativistic Theory of Elementary Particles*

we get

$$L^{\mu} t^{0}{}_{\mu} S\psi_r^{(0)} = \lambda_r S\psi_r^{(0)}; \quad (2)$$

$$S\psi_r^{(0)} \equiv \psi_r; \quad L^{\mu} t^{0}_{\mu} \psi_r = \lambda_r \psi_r; \quad (2')$$

or, since it is possible to put

$$t^0_{\mu} = p_{\mu} / \sqrt{p^{\lambda} p_{\lambda}} \ \text{(cf. reference 3)}, \quad (3)$$

$$L^{\mu} p_{\mu} \psi_r = \lambda_r \sqrt{p^{\lambda} p_{\lambda}} \psi_r.$$

Equation (3) has the form of an equation which is used to determine the eigenvalues and eigenfunctions of the matrix $L^{\mu}p_{\mu}$. The quantities $\varkappa_r = \lambda_r \sqrt{p^{\lambda}p_{\lambda}}$ would be, correspondingly, the roots of the characteristic equation that can be constructed by means of the matrix $L^{\mu}p_{\mu}$.
We will keep in mind the fact, established by Gel'fand and Iaglom, that to each value $+\lambda_r$ there corresponds a value $-\lambda_r$ in the spectrum of the matrix L^{μ}; and consequently to each $+\lambda_r\sqrt{p^{\lambda}p_{\lambda}}$ must correspond a $-\lambda_r\sqrt{p^{\lambda}p_{\lambda}}$ in the spectrum of the matrix $L^{\mu}p_{\mu}$. Accordingly, the minimal polynomial of the matrix $L^{\mu}p_{\mu}$. should have the form:

$$\Delta(\varkappa) = \prod_{i=1}^{s} (\varkappa^2 - \lambda_i p^{\lambda} p_{\lambda}) \quad (4)$$

or

$$\Delta(\varkappa) = \varkappa^{2s} + a_2 p^{\lambda} p_{\lambda} \varkappa^{2s-2} + \quad (5)$$

$$\ldots + a_{2s-2} (p^{\lambda} p_{\lambda})^{s-1} \varkappa^2 + a_{2s} (p^{\lambda} p_{\lambda})^s.$$

To derive the formula determining the operator F, we can then repeat, with appropriate changes, the derivation of the formula that determines the form of the "associated matrix". We will make the initial assumption that null eigenvalues are missing. We consider the expression $\phi(\kappa, x)$ of the formula

$$\varphi(\varkappa, x) = \frac{\Delta(\varkappa) - \Delta(x)}{\varkappa - x}. \quad (6)$$

After dividing in Eq. (6) we obtain

$$\varphi(\varkappa, x) = \varkappa^{2s-1} + x\varkappa^{2s-2} + (x^2 + a_2 p^{\lambda} p_{\lambda}) \varkappa^{2s-3} + \quad (7)$$

$$\cdots + [x^{2s-1} + a_2 p^{\lambda} p_{\lambda} x^{2s-3} + \cdots + a_{2s-2} (p^{\lambda} p_{\lambda})^{s-1} x].$$

The matrix $L^{\mu}p_{\mu}$ has as a root the minimal polynomial

$$\Delta(L^{\mu} p_{\mu}) = \Delta(-L^{\mu} p_{\mu}) = 0. \quad (8)$$

According to Eqs. (6) and (8) we have

$$(L^{\mu} p_{\mu} + \varkappa)\, \varphi(\varkappa, -L^{\mu} p_{\mu}) = \Delta(\varkappa), \quad (9)$$

That is, (see reference 2, Eq. (8)) one can obtain

$$F = \varphi(\varkappa, -L^{\mu} p_{\mu}) \quad (10)$$

or

$$F = (-L^{\mu} p_{\mu})^{2s-1} + \varkappa(-L^{\mu} p_{\mu})^{2s-2} \quad (11)$$

$$+ (\varkappa^2 + a_2 p^{\lambda} p_{\lambda})(-L^{\mu} p_{\mu})^{2s-3} + \cdots$$

$$\cdots + [\varkappa^{2s-1} + a_2 p^{\lambda} p_{\lambda} \varkappa^{2s-3} + \cdots + a_{2s-2} (p^{\lambda} p_{\lambda})^{s-1} \varkappa]\, E.$$

It is easily seen that when there is an n-fold zero eigenvalue, Eq. (9) is solved by use of the operator $F^1 = (L^{\mu}p_{\mu})^n F$.

The operator F could be investigated in relation to the completion of the theorem of Pauli for the adjustment relations of the second quantization, which theorem is obtained by its help. The author hopes to return to a consideration of this question. It might be remarked that at present in the work of both Soviet (for example, see reference 3) and foreign (Bhaba, le Coutre) authors, there are several points of view on the problem of second quantization of the field of particles with a mass spectrum, so that the physical content of the corresponding problems is far from clear.

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