anisotropy of the surface conductivity of metal at low temperatures and the non-tensorial anisotropy of the penetration depth of the electromagnetic field in superconductor, as arrived at in references 1-3, does not have sufficient experimental basis. The phenomena observed can be explained, at least qualitatively, on the basis of the above mentioned concept concerning the bond between the two fundamental oscillations of the coaxial resonator, with the aid of the usual tensorial anisotropic conductivity.

In any case, it should be most evident that there is a need for further and extensive investigations as to the anisotropy of surface conductivity at low temperatures before final conclusions as to its character can be formulated.

Translated by A. Andrews
15

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## The Problem of the Invalidity of One Statistical Treatment of Quantum Mechanics

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WIGNER and Szilard [1] have proposed a probability distribution in phase space of a quantum particle

$$F(q;p)=\frac{1}{2\pi}\int\psi^*\left(q-\frac{h\tau}{2}\right)e^{-i\tau p}\psi\left(q+\frac{h\tau}{2}\right)d\tau, \qquad (1)$$

satisfying the time-dependent equation

$$\frac{\partial F}{\partial t}+\frac{p}{m}\frac{\partial F}{\partial q}=\frac{i}{4\pi^2 h}\int\left[V\left(q-\frac{h\tau}{2}\right)-V\left(q+\frac{h\tau}{2}\right)\right]F(q;\eta)\,e^{i\tau(\eta-p)}d\eta d\tau. \qquad (2)$$

Here $q$, $p$, $m$ are coordinate, momentum and mass of of the particle; $V(q)$, its potential energy; $h$, Plank's constant; $t$, the time.

In an extension of this work [2] an interpretation of Eq. (2) has been given as the equation of a certain stochastic process of change of coordinate and momentum of a particle, i. e., a statistical treatment of quantum mechanics. For the validity of such a treatment it is necessary, in the first place, that $F(q;p)$, non-negative at a given moment of time, should remain non-negative at all later moments, proof of which was given by Bartlett (see Moyal [2]). However, in a recent work [3] it was correctly shown that $F$ in general does not preserve its sign with the passage of time. From this follows the conclusion of the invalidity of the quantum mechanical treatment given by Moyal.

It is necessary only to point out Bartlett's error. Bartlett supposed that a quantum system possesses a cyclic coordinate $\theta$ (it is obvious that it is always possible formally to incorporate into a given system an additional cyclic degree of freedom). He takes the general solution of the time-dependent equation for such a system in the form

$$F(q,\theta;p,g)=\sum_{\mu}e^{i\mu(t+\theta/\omega)}F_{\mu}(q;p,g), \qquad (3)$$

where $g$ and $\omega$ are the cyclic momentum and frequency; and $F_{\mu}$, certain constant functions. It is clear that if $F>0$ at a certain $t$ and arbitrary $\theta$, it will still be $>0$ at an arbitrary time. The error lies in the fact that the general solution of the time-dependent equation is

$$F(q,\theta;p,g)=\sum_{\mu_1\mu_2}e^{i\mu_1 t+i\mu_2\theta}F_{\mu_1\mu_2}(q;p,g). \qquad (4)$$

Therefore Bartlett's discussion necessarily applies only to a narrow class of solutions which actually preserve sign.

Translated by B. Leaf
16

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## The Fermi Theory of Multiple Particle Production in Nucleon Encounters

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IN calculating the statistical weights of various states, Fermi [1] applied the law of conservation of energy in exact form, but the law of conservation of momentum only in approximate form. The purpose of the present work is the exact application of the law of conservation of momentum for two limiting cases: the non-relativistic limit and the relativistic limit.

The statistical weight $S_n$ of a state for $n$ particles with zero spin is given by the expression

$$S_n = \left(\frac{V}{8\pi^3 \hbar^3}\right)^n \frac{dQ_n(E_0)}{dE_0}, \qquad (1)$$

where $V$ is the spatial volume and $Q$ the volume in momentum space*. Our problem then reduces to the calculation of

$$dQ_n \; dE_0 = W_n(E_0).$$

We first calculate $W_n(E_0, P_0)$, taking into account that the total momentum $P_0$ of the system is different from zero. In the non-relativistic case we can write

$$W_n(E_0, P_0) = \underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{3n} \delta\left(T - \frac{1}{2\mu}\sum_{i=1}^{n} p_r^2\right) \qquad (2)$$

$$\times\, \delta\left(P_{0x} - \sum_{r=1}^{n} p_{xr}\right) \times \delta\left(P_{0y} - \sum_{r=1}^{n} p_{yz}\right)$$

$$\times\, \delta\left(P_{0z} - \sum_{r=1}^{n} p_{zr}\right) \prod_{r=1}^{n} dp_{xr}\, dp_{yr}\, dp_{zr},$$

where $T$ is the total kinetic energy of the system.

Making use of the integral representation of the $\delta$ function, we write Eq. (2) in the form

$$W_n(E_0, P_0) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} \exp\{-iT\tau_1\}\, d\tau_1 \qquad (3)$$

$$\times \int_{-\infty}^{\infty} \exp\{-iP_{0x}\tau_2\}\, d\tau_2 \times \int_{-\infty}^{\infty} \exp\{-iP_{0y}\tau_3\}\, d\tau_3$$

$$\times \int_{-\infty}^{\infty} \exp\{-iP_{0z}\tau_4\}\, d\tau_4 \times \left[\iiint_{-\infty}^{\infty} \exp\left\{i\left[\frac{\tau_1 p^2}{2\mu} + \tau_2 p_x\right.\right.\right.$$

$$\left.\left.\left. + \tau_2 p_y + \tau_2 p_z\right]\right\} dp_x\, dp_y\, dp_z\right]^n.$$

We evaluate the integral

$$I_1 = \iiint_{-\infty}^{\infty} \exp\left\{i\left[\frac{\tau_1 p^2}{2\mu} + \tau_2 p_x + \tau_3 p_y + \tau_4 p_z\right]\right\}$$

$$\times\, dp_x dp_y dp_z.$$

Beginning with spherical coordinates and integrating over the angular variables, we obtain

$$I_1 = -\frac{2\pi i}{\tau} \int_{-\infty}^{\infty} x \exp\{i[\tau x + \alpha x^2]\}\, dx \qquad (4)$$

$$= \frac{\pi^{3/2}}{\sqrt{2}} \frac{i(1+i)\exp\{-i\tau^2/4\alpha\}}{\alpha^{3/2}};$$

$$\tau = \sqrt{\tau_2^2 + \tau_3^2 + \tau_4^2}, \qquad \alpha = \tau_1/2\mu.$$

We substitute Eq. (4) in (3) and again integrate over the angular variables in spherical coordinates:

$$W_n(E_0, P_0) = -\frac{(2\pi\mu)^{3/2 n}}{(2\pi)^3}\left[\frac{i(1+i)}{\alpha^{1/2}}\right]^n \frac{i}{P_0} \qquad (5)$$

$$\times \int \frac{\exp\{-iT\tau_1\}}{\tau_1^{3/2 n}}\, d\tau_1$$

$$\times \int_{-\infty}^{\infty} \tau \exp\left\{-i\left[\frac{n\mu\tau^2}{2\tau_1} - \tau P_0\right]\right\} d\tau$$

$$= \frac{(2\pi\mu)^{3/2(n-1)}}{n^{3/2}[3/2(n-1)-1]!}\left(T - \frac{P_0^2}{2n\mu}\right)^{[3/2(n-1)-1]}$$

This equation can easily be generalized to the case in which the particles possess different masses $\mu_1, \ldots, \mu_n$:

$$W_n(E_0, P_0) = \frac{(2\pi)^{3/2(n-1)}}{[3/2(n-1)-1]!} \qquad (6)$$

$$\times \left(\frac{\mu_1\mu_2\ldots\mu_n}{\mu_1 + \ldots + \mu_n}\right)^{3/2}$$

$$\times \left[T - \frac{P_0^2}{2(\mu_1 + \ldots + \mu_n)}\right]^{[3/2(n-1)-1]}$$

Further, we compute the function $W_n(E_0, P_0)$ for the relativistic limit. By analogy to (3) we can write

$$W_n(E_0, P_0) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} \exp\{-iE_0\tau_1\}\, d\tau_1 \qquad (7)$$

$$\times \int_{-\infty}^{\infty} \exp\{-iP_{0x}\tau_2\}\, d\tau_2$$

$$\times \int_{-\infty}^{\infty} \exp\{-iP_{0y}\tau_3\}\, d\tau_3 \int_{-\infty}^{\infty} \exp\{-P_{0z}\tau_4\}$$

$$\times\, d\tau_4 \left[\iiint_{-\infty}^{\infty} \exp\{i[\tau_1 \sqrt{p_x^2 + p_y^2 + p_z^2}\right.$$

$$+\tau_2 p_x + \tau_3 p_y + \tau_4 p_z]\} \, dp_x dp_y dp_z\Big]^n.$$

We now evaluate $I_2$, the integral contained in square brackets:

$$I_2 = -\frac{2\pi i}{\tau}\int_{-\infty}^{\infty} x \exp\{i[\tau x + \tau_1 |x|]\}\, dx \tag{8}$$

$$= -\frac{2\pi i}{\tau}\Big[\int_0^{\infty} x\,[\exp\{i(\tau_1+\tau)x\} - \exp\{i(\tau_1-\tau)x\}]\, dx\Big]$$

$$= -\frac{2\pi^2}{\tau}[\delta'_+(\tau_1+\tau) - \delta'_+(\tau_1-\tau)],$$

$\delta'_+$ is the derivative of the $\delta_+$ function. Finally,

$$I_2 = -8\pi i \tau_1 / (\tau^2 - \tau_1^2). \tag{9}$$

After substituting in Eq. (7) and integration over the angle variables, we get

$$W_n(E_0, P_0) = \frac{(-8\pi i)^n}{i(2\pi)^3 P_0}\int \exp\{-iE_0\tau_1\}\, \tau_1^n \tag{10}$$

$$\times\, d\tau_1 \int \frac{x \exp\{-iP_{0x}\}}{(\tau_1^2 - x^2)^{2n}}\, dx.$$

We introduce the new variables $y = \tau_1 + x$, $z = \tau_1 - x$ and carry out the calculation at the point $y = x = 0$, obtaining the following after some simple transformations:

$$W_n(E_0, P_0) = \frac{\pi^{n-1}}{2^{n-2}}\frac{(E_0^2 - P_0^2)^{n-2}}{P_0} \tag{11}$$

$$\times \sum_{r=0}^{n} C_n^r \frac{(E_0 - P_0)^r (E_0 + P_0)^{n-r}}{(n+r-2)!\,(2n-r-2)!}$$

$$\times\Big[\frac{E_0 + P_0}{2n - r - 1} - \frac{E_0 - P_0}{n + r - 1}\Big].$$

We now consider some applications of the formulas just developed. Frequently the phenomena which accompany particle collisions are studied in the center of mass coordinate system. In this case we must set $P_0 = 0$. Then we have

$$W_n(E_0, 0) = \frac{(2\mu\pi)^{3/2\,(n-1)}}{n^{3/2}[3/2\,(n-1) - 1]!}\, T^{[3/2\,(n-1)-1]} \tag{12}$$

in the limiting non-relativistic case, while in the limiting relativistic case we have

$$W_n(E_0, 0) = \left(\frac{\pi}{2}\right)^{n-1} E_0^{3n-4} \tag{13}$$

$$\times \sum_{r=0}^{(n-1)/2} \frac{C_n^r [3n - 2 - (n-2r)^2]}{(n+r-1)!\,(2n-2-1)!} \qquad (n \text{ odd})$$

or $$W_n(E_0, 0) = \left(\frac{\pi}{2}\right)^{n-1} E_0^{3n-4}.$$

$$\times \Bigg\{\sum_{r=0}^{(n/2)-1} \frac{C_n^r [3n - 2 - (n-2r)^2]}{(n+r-1)!\,(2n-r-1)!} + \frac{C_n^{n/2}}{(3/2\, n - 1)!\,(3/2\, n - 2)!}\Bigg\} \qquad (n \text{ even})$$

An important characteristic of the collision is the probability $w_n(E_0, p)$ that one particular particle, created in a collision in which $n$ particles are formed with a total energy $E_0$, has a momentum in the range $p$, $p+dp$. If we limit ourselves to statistical factors alone, then, omitting the multiplying factor $(V/8\pi^2\hbar^3)^n$, we can write the probability in the form

$$w_n(E_0, p)\, dp = 4\pi p^2 w_{n-1}\left[\left(T - \frac{p^2}{2\mu}\right), p\right] dp \tag{14}$$

(non-relativistic case) and

$$w_n(E_0, p)\, dp = 4\pi p^2 w_{n-1}\,[(E_0 - p), p]\, dp \tag{14'}$$

(relativistic case).

Making use of Eqs. (5) and (11) we obtain

$$w_n(E_0, p) = \frac{4\pi(2\pi\mu)^{3/2\,(n-2)} p^2}{(n-1)^{3/2}[3/2\,(n-2) - 1]!} \tag{15}$$

$$\times \left[T - \frac{np^2}{2(n-1)\mu}\right]^{[3/2\,(n-2)-1]} dp$$

(non-relativistic case).

If the particles have different masses,

$$w_n(E_0, p) = \frac{4\pi(2\pi)^{3/2\,(n-2)} p^2}{[3/2\,(n-2) - 1]!}\left(\frac{\mu_1 \ldots \mu_{n-1}}{\mu_1 + \ldots + \mu_{n-1}}\right)^{3/2} \tag{16}$$

$$\times \left\{T - \frac{p^2}{2}\left[\frac{\mu_1 + \ldots + \mu_n}{\mu_n(\mu_1 + \ldots + \mu_{n-1})}\right]\right\} dp.$$

Finally,

$$w_n(E_0, p)\,dp = 2\pi\left(\frac{\pi}{2}\right)^{n-2}[E_0(E_0-2p)^{n-3}p \quad (17)$$

$$\times\sum_{r=0}^{n-1}\frac{C_{n-1}^r E_0^{n-r-1}(E_0-2p)^r}{(n+r-3)!\,(2n-r-4)!}$$

$$\times\left[\frac{E_0}{2n-r-3}-\frac{E_0-2p}{n+r-2}\right]dp$$

(relativistic case)

It should be observed that after the completion of the present work, the paper of Lepore and Stuart[2] appeared in which similar problems were investigated.

Translated by R. T. Beyer
17

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[2] J. V. Lepore and R. N. Stuart, Phys. Rev. **94**, 1724 (1954)

* If the particles are identical, then the right side of Eq. (1) reduces to $n!$

## The Fermi Distribution at Absolute Zero, Taking into Account the Interaction of Electrons with Zero Point Vibrations of the Lattice

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THE method of Green's function developed in connection with problems of relativistic quantum field theory[1] can also be used in a number of other problems. In particular, the investigation of the distribution function for an electron gas which takes into account the interaction of the electrons with phonons is of considerable interest.

The Green's function is

$$G_{ss'}(x,y) = \frac{i}{\langle S\rangle_0}\langle T\{\psi_s(x)\,\psi^*_{s'}(y)\,S\}\rangle_0. \quad (1)$$

Here $S$ denotes the $S$ matrix, $x=\{x, x_0\}$, $s$, $s'$ are spin indices, $\psi_s(x)$ is the wave function of the electronic field in the representation of the interaction. The Green's function (1), being decomposed into an arbitrary complete set of functions $\phi_\lambda(x)$ $\phi^*_{\lambda'}(y)$ characterizes the electron distribution for $x_0 < y_0$, and the hole distribution for $x_0 > y_0$, in the terms of the parameter $\lambda$.

The $S$ matrix is given by the usual expression

$$S = T\exp\left\{i\int L(x)\,dx\right\}, \quad (2)$$

where $L$ is the Lagrangian of the interaction (in a system of units in which $\hbar = c = 1$, $c$ = speed of sound). For a system of electrons interacting with acoustic vibrations [2],

$$L = \{g\psi^*_s(x)\,\psi_s(x) + \rho(x)\}\,\varphi(x);\quad \varphi = \partial A(x)/\partial x_0, \quad (3)$$

where $g$ is a coupling constant and $\rho(x)$ is the "external charge density",

$$A(x) = \frac{1}{\sqrt{2\pi^3}}\int\frac{d\mathbf{f}}{|\mathbf{f}|}[b_\mathbf{f}\exp\{i\mathbf{f}\mathbf{x}-i|\mathbf{f}|x_0\} \quad (4)$$
$$+b^*_\mathbf{f}\exp\{-i\,\mathbf{f}\mathbf{x}+i\,|\mathbf{f}|\,x_0\}],$$

$b^*_\mathbf{f}$, $b_\mathbf{f}$ are the Bose operators of "creation" and "annihilation" of phonons.

Integration over $f$ is confined to the Debye limiting value of $f_0$. The equation for the Green's function can easily be found (for example, by the method given by Anderson[3]). It has the form

$$G_{s_1s_2}(x,y) = iK_{s_1s_2}(x,y) \quad (5)$$
$$-ig\int dz\,K_{s_1s'}(x,z)\,G_{s's_2}(z,y)\,a(z)$$
$$-\int dz\,dx'\,K_{s_1s'}(x,z)\,\Delta E_{s's''}(z,x')\,G_{s''s_2}(x',y).$$

Here the summation is carried out over the iterated spin indices;

$$a(z) = i\int F(z,z')\,\rho(z')\,dz'. \quad (6)$$

The functions $K_{ss'}(x,y)$ and $F(x,y)$ are the propagation functions of the "free" electron and phonon fields; $E$ is the analog of the mass operator

$$\Delta E_{s's''}(z,x') \quad (7)$$
$$= -ig\int dz'\,dx''\,\frac{\delta a(z')}{\delta\rho(z)}\,F(z',z)\,G_{s's''}(z,x'')$$
$$\times\,[\delta G^{-1}_{s'''s''}(x'',x')/\delta a(z')].$$

We have for the functions $K_{ss'}$ and $F$ (under the condition of complete degeneracy of the electron gas)

$$K_{s_1s_2}(x,y) = \langle T\{\psi_{s_1}(x)\,\psi^*_{s_2}(y)\}\rangle_0 \quad (8)$$
$$=\delta_{s_1s_2}\frac{i}{(2\pi)^4}\int dp\int_L dp_0\,\frac{\exp\{i(\mathbf{p},\mathbf{x}-\mathbf{y})-ip_0(x_0-y_0)\}}{p_0-(p^2/2m)}$$
$$\equiv\delta_{s_1s_2}K(x-y).$$